Physics 550

Problem Set 4: Matrix Mechanics 2

Name: ______________________________________

Instructions / Notes / Suggestions:

- Each problem is worth five points.
- In order to receive credit, you must show your work.
- Circle your final answer.
- Unless otherwise specified, answers should be given in terms of the variables in the problem and/or physical constants and/or cartesian unit vectors $(\hat{\mathbf{e}}_x, \hat{\mathbf{e}}_x, \hat{\mathbf{e}}_z)$ or radial unit vector $\hat{\mathbf{r}}$.
- Staple your work.

Problem #1:

Consider the (matrix) function $\mathbf{g}$ of the canonical matrices $\mathbf{q}$ and $\mathbf{p}$:

$$\mathbf{g}(\mathbf{pq}) = a\mathbf{q}^2\mathbf{p} + b\mathbf{pq}^2 + c\mathbf{qpq}$$

where a, b, and c are scalars.

Derive the Heisenberg equation of motion for $\mathbf{g}$.

Note that you may make reasonable assumptions on $\mathbf{H}$, e.g., about its $\mathbf{q}$ and $\mathbf{p}$ dependencies, if needed.

Problem #2:

The (quantum) Hamiltonian $\mathbf{H}$ for the harmonic oscillator can be written in terms of the canonical matrices $\mathbf{q}$ and $\mathbf{p}$:

$$\mathbf{H}(\mathbf{pq}) = \frac{\mathbf{p}^2}{2m} + \frac{1}{2}m\omega^2\mathbf{q}^2$$

Assume the initial conditions $\mathbf{q}(0) = \mathbf{q}_i$ and $\mathbf{p}(0) = \mathbf{p}_i$.

(a) Find the equation of motions for $\mathbf{q}$ and $\mathbf{p}$.

(b) Find the equation of motion for $\mathbf{H}$.

(c) Find equations for $\mathbf{q}(t)$ and $\mathbf{p}(t)$.

Problem #3:

Let $\mathbf{B}$ be a real and symmetric 2×2 matrix:

$$\mathbf{B} = \begin{pmatrix} 0 & b \\ b & c \end{pmatrix}$$

(a) Diagonalize $\mathbf{B}$.

(b) Give a mathematical interpretation of the matrix elements obtained in part (a).

Assume that $\mathbf{B}$ corresponds to the (quantum) Hamiltonian for some system.

(c) Give a physical interpretation of the matrix elements obtained in part (a).

Problem #4:

Let **A** be a real and diagonal 2×2 matrix whose diagonal elements are equal.

Let **B** be a real and symmetric 2×2 matrix:

$$\mathbf{B} = \begin{pmatrix} 0 & b \\ b & c \end{pmatrix}$$

(a) Show that **A** and **B** commute.

(b) Find the eigenvalues and eigenvectors of **A**.

(c) Find simultaneous eigenvectors of **A** and **B**, if such exist.

Problem #5:

In quantum mechanics, the Pauli equation describes spin-1/2 particles, taking into account their interaction with an external electromagnetic field.

In the Pauli equation occurs the (Pauli) matrices:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

(a) Find the eigenvalues and eigenvectors of σ_1.

(b) Find the eigenvalues and eigenvectors of σ_2.

(c) Show whether there is a unitary matrix that diagonalizes both σ_1 and σ_2.

Problem #6:

Consider the 2×2 (quantum) Hamiltonian matrix:

$$\mathbf{H}(\mathbf{pq}) = \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}$$

Let $\mathbf{g}$ be a 2×2 matrix that represents some dynamical quantity, which at time $t = 0$ is:

$$\mathbf{g}(\mathbf{pq}) = \begin{pmatrix} 3 & 1 \\ 1 & 4 \end{pmatrix}$$

Assume above and in what follows that units work themselves out.

(a) Find the spectrum of $\mathbf{H}$.

(b) Find the spectrum of $\mathbf{g}$ at $t = 0$.

(c) Find the spectrum of $\mathbf{g}$ at $t = 2$.