Physics 550

Problem Set 3: Matrix Mechanics 1

Name: ______________________________

Instructions / Notes / Suggestions:

- Each problem is worth five points.
- In order to receive credit, you must show your work.
- Circle your final answer.
- Unless otherwise specified, answers should be given in terms of the variables in the problem and/or physical constants and/or cartesian unit vectors $(\hat{\mathbf{e}}_x, \hat{\mathbf{e}}_x, \hat{\mathbf{e}}_z)$ or radial unit vector $\hat{\mathbf{r}}$.
- Staple your work.

Problem #1:

Let **A** be a diagonal and real 2×2 matrix whose diagonal elements are not equal.

Find the Hermitian matrices that commute with **A**.

Problem #2:

The (classical) definition of angular momentum is $\mathbf{L} = \mathbf{r} \times \mathbf{p}$, where $\mathbf{r}$ and $\mathbf{p}$ are position and momentum vectors, respectively. Note that $\mathbf{L}$ too is a vector.

The expression for $\mathbf{L}$ can be carried over to quantum mechanics, by reinterpreting $\mathbf{r}$ and $\mathbf{p}$ as operators. Thus, $\mathbf{L}$ too becomes a (vector) operator: the orbital angular momentum operator.

(a) Explicitly evaluate $[L_x, L_y]$.

(b) Using the result from part (a) and symmetry arguments, obtain expressions for $[L_y, L_z]$ and $[L_x, L_z]$.

(c) Compare your results from parts (a) and (b) to (classical) Poisson brackets.

Problem #3:

Like any vector, a (squared) magnitude can be defined for the orbital angular momentum operator:

$$\mathbf{L}\cdot\mathbf{L} = \mathbf{L}^2 \equiv L_x^2 + L_y^2 + L_z^2$$

$\mathbf{L}^2$ is another operator.

Evaluate $[\mathbf{L}^2, L_x]$.

Problem #4:

For all functions $F(\mathbf{q}_i)$ and $G(\mathbf{p}_i)$ that can be expressed as power series in their argument:

$$[\mathbf{q}_i, G(\mathbf{p}_i)] = i\hbar\frac{\partial G}{\partial \mathbf{p}_i} \qquad [\mathbf{p}_i, F(\mathbf{q}_i)] = -i\hbar\frac{\partial F}{\partial \mathbf{q}_i}$$

Explicitly verify these formulas.

Problem #5:

The Baker–Campbell–Hausdorff (BCH) formula is very important in quantum mechanics.

The BCH formula for the product of the exponentials of two operators **A** and **B** is:

$$e^{\mathbf{A}}e^{\mathbf{B}} = e^{\mathbf{A}+\mathbf{B}+\frac{1}{2}[\mathbf{A},\mathbf{B}]+\ldots}$$

where ... represents terms that are at least cubic in **A** and **B**, and involve nested commutators.

Show that the above formula holds up to second order in operator multiplication.