Lab 3. Projectile Motion

Goals

- To determine the launch speed of a projectile and its uncertainty by measuring how far it travels horizontally before landing on the floor (called the range) when launched horizontally from a known height.
- To predict and measure the range of a projectile when the projectile is fired at an arbitrary angle with respect to the horizontal.
- To predict the initial firing angle of the launcher for a prescribed range value.
- To determine quantitatively whether the measured ranges in (2) and (3) are consistent with the desired range values.

Introduction

When objects undergo motion in two (or even three) dimensions rather than in just one, the overall motion can be analyzed by looking at the motion in any two (or three) mutually perpendicular directions and then putting the motions "back together," so to speak. In the case of projectiles, the horizontal and vertical directions are usually chosen. Why is this choice made? Ignoring the effects of air resistance, an object moving vertically near the surface of Earth experiences a constant acceleration. We know this by experiment. Likewise an object moving horizontally experiences zero acceleration. Any other choice of perpendicular directions would have nonzero, constant values of acceleration in both directions. When we write the descriptions of the motion in mathematical terms, the horizontal/vertical choice of directions results in the simplest description.

Under what conditions can the effects of air resistance be ignored? One condition is that the object's speed is not too high, since the effect of the air resistance increases with speed. If two objects are the same size and shape, the lighter one of the two will experience the larger effect on its motion due to the air. (Imagine a ping-pong ball and a steel ball bearing of the same size.) In designing this lab, care has been taken to ensure that air resistance has a negligible effect on the trajectory of the projectile. When conditions are such that air resistance cannot be ignored, the motion is more complicated.

Mathematical preliminaries—Equation for range

To accomplish the first two of our stated goals, we need a general mathematical relationship between the horizontal range of the projectile and the initial height, initial velocity, and launch angle. See Figure 3.1. You will need to solve the appropriate kinematics equations for motion with constant acceleration in the horizontal and vertical directions simultaneously. Rather than writing the equations in terms of the angle, θ, it is suggested that you use the symbols v_{0x} and v_{0y}, where $v_{0x} = v_0 \cos\theta$ and $v_{0y} = v_0 \sin\theta$, to simplify the algebra. You need to solve for the range, R, in terms of v_{0x}, v_{0y}, h, and g. The details of this derivation must be included in your lab notes.

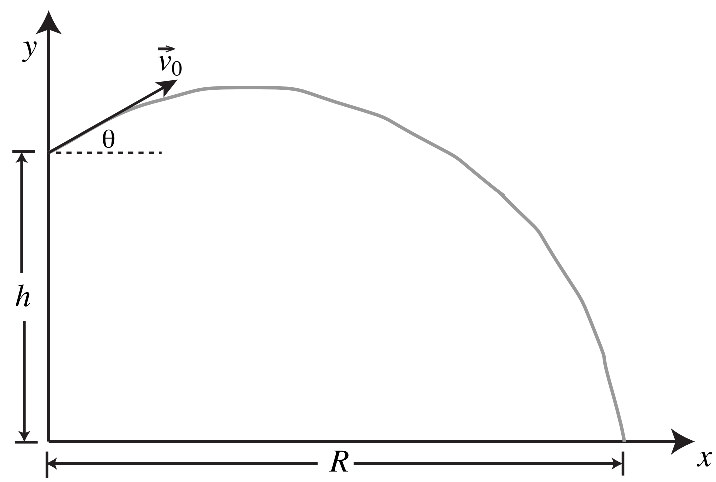

Figure 3.1. Coordinate system for calculating the range, R.

Instructions and precautions for using the ball launcher

Warning: Never look down the barrel of a launcher. Wear eye protection until all the groups have finished launching projectiles.

1. Make sure that the launcher is attached securely to the table so it does not move when the launcher is fired. Make sure the launcher is at the proper angle by using the built-in plumb bob on the side of the launcher. Note that the angle measured by this plumb bob is the angle between the “barrel” of the launcher and the horizontal.
2. Since the projectiles will be hitting the floor, use a second plumb bob to locate and mark the position on the floor (blue tape works) directly below the launch point of the projectile. This indicates the initial horizontal position of the ball at floor level so the range (horizontal distance traveled by the ball) can be measured later. You will have to measure the height to get the vertical distance. Clearly indicate in a diagram how you measured the height (from where to where). If you are not sure how the height should be measured, please discuss it with your TA. Note that "Clearly indicate" here means that you should be describing/drawing to within the limit of the accuracy of your measuring instrument (a meter stick, so 0.1mm). Have a justification for why you chose each end of the ruler at which to measure.

3. To launch the projectile, load the ball into the projectile launcher. Use the rod to push the ball into the launch tube to one of the first two out of the three preset launch positions (short, medium, or long range. Do not use Long Range). You will hear a click as you reach each position. Notify others nearby and across the room before firing the ball. Stand out of the way and fire the launcher by pulling on the string attached to its trigger on the top. To minimize the force applied by the string to the launch tube, pull the string at right angles to the launch tube (straight up). You may need to use your other hand to stabilize the launch tube (grip the tube and frame to prevent it from rocking at launch).
4. To record the position where the projectile strikes the floor, tape a white paper target to the thin hard-board sheet (about 0.3 m $\times$ 0.5 m in size) at your lab station. Place the sheet and target at the approximate place where the ball lands. When you are ready to record some landing points, lay a piece of carbon paper (carbon side down) on top of the target. Do not put tape on the carbon paper. The ball will leave a dark smudge on the white paper where it lands. If necessary you can tape the hard-board sheet to the floor to keep it from moving, but avoid the indiscriminate use of tape on the floors. Indicate on your target which marks have already been recorded into your notes to avoid confusion in future measurements.

Determining the initial speed of the projectile

1. Simplify your general equation for the range for the case when $\theta = 0$ (horizontal launch). Then solve for v_0 in terms of R, h, and g.
2. Set the launcher to fire horizontally, that is, to launch at an angle of zero degrees. Care with this angle setting can significantly improve your results later in the lab.
3. Starting with the medium range launch setting, fire the projectile (using the four steps in the previous section) a couple times noting where the projectile lands. Center the paper target as best you can where the ball will land. Now use the carbon paper to record the landing position of four or five launches using the same initial conditions.
4. From your data determine the average range, R, of the ball. Use this average distance to calculate the average initial speed of the ball as it was launched.
5. Repeat the same procedure for the short range setting on the launcher.

Range for nonzero launch angles

1. Choose a launch angle between 30° and 40°. Using the values of the initial speed of the ball measured above and your general equation for the range, calculate the horizontal distance (range) from the launch point to where the ball should land for the short and medium range settings using the initial launch angle that you have chosen. (Do not use the long range setting.)
2. For the short and medium range settings, place a paper target on the floor at the calculated position and fire the projectile. If the projectile misses the target completely, check your calculations and/or discuss it with your TA. If the projectile does hit the target, then repeat several times to get a good average experimental range value and its corresponding standard deviation to compare with your calculated range.
3. Compare your predicted range values with the experimental range values using your ex-

perimental standard deviations. Assume that your predicted range, $R_{predicted}$ has zero uncertainty. Then check if your measurement is consistent with your prediction if $t' = |R_{measured} - R_{predicted}|/\sigma(R_{measured}) < 3$. If you find that $t' > 3$, check your calculations and consider carefully what systematic errors may be present in your experiment.

Launch angle to achieve a given range

1. Ask your TA to assign a value of horizontal distance (range) for your group.
2. Calculate a suitable angle at one of the range settings for launching the projectile to the target set at the assigned distance. The relationship giving the initial launch angle in terms of the other parameters is:

$$\tan\theta = \frac{v_0^2}{gR} \pm \left[\left(\frac{v_0^2}{gR}\right)^2 - 1 + \frac{2v_0^2 h}{gR^2}\right]^{1/2} \tag{3.1}$$

3. Now set the target and do the experiment with your TA present to observe. Were you able to hit the target? If you have trouble, check your calculations. Is your calculator in radian or degree mode? Get assistance from your TA, if necessary. Again, compare your experimental range value to the range value assigned by your TA. If not, check your calculations and your procedure.

Conclusion

Summarize all your results, preferably in a table showing the measured and calculated quantities with their uncertainties. Clearly display your comparisons between predicted values and experimental values. Are you convinced that the theoretical predictions made by separating the horizontal and vertical motions agree with experiment, at least within the calculated uncertainties of the experiment? Your answers must be based on your experimental results and the calculated uncertainties of the quantities you are comparing. Do not make vague statements that are not directly supported by your calculations and measurements.

	No Effort	Progressing	Expectation	Scientific
SL.A.b Is able to identify the hypothesis for the experiment proposed Labs: 3, 4, 6, 7, 9, 12	No deliberately identified hypothesis is present in the first half page or so of notes	An attempt is made to state a hypothesis, but no clearly defined dependent and independent variable, or lacking a statement of relationship between the two variables	A statement is made as a hypothesis, it contains a dependent and independent variable along with a statement of relationship between the two variables. This statement appears to be testable, but there are some minor omissions or vague details.	The hypothesis is clearly stated and the direct link to the experiment at hand is apparent to any reasonably informed reader.

	No Effort	Progressing	Expectation	Scientific
SL.A.c Is able to determine hypothesis validity Labs: 3, 4, 6, 7, 9, 12	No deliberately identified attempt to use experimental results to validate hypothesis is present in the sections following data collection.	A statement about the hypothesis validity is made, but it is not consistent with the data analysis completed in the experiment	A statement about the hypothesis validity is made which is consistent with the data analysis completed in the experiment. Assumptions which informed the hypothesis and assumptions not validated during experimentation are not taken into account.	A statement about the hypothesis validity is made which is consistent with the data analysis and all assumptions are taken into account.
SL.B.c Is able to explain steps taken to minimize uncertainties and demonstrate understanding through performance where able. Labs: 3, 4, 7-9, 11	No explicitly identified attempt to minimize uncertainties and no attempt to describe how to minimize uncertainties present	No explicitly identified attempt to minimize uncertainties is present, but there is a description of how to minimize experimental uncertainty.	An attempt is made and explicitly identified for minimizing uncertainty in the final lab results, but the method is not the most effective.	The uncertainties are minimized in an effective way.
CT.A.a Is able to compare recorded information and sketches with reality of experiment Labs: 2-7, 9, 12	No sketches present and no descriptive text to explain what was observed in experiment	Sketch or descriptive text is present to inform reader what was observed in the experiment, but there is no attempt to explain what details of the experiment are not accurately delivered through either representation.	Sketch and descriptive text are both present. The sketch and description supplement one another to attempt to make up for the failures of each to convey all observations from the experiment. There are minor inconsistencies between the two representations and the known reality of the experiment from the week, but no major details are absent.	Sketch and description address the shortcomings of one another to convey an accurate and detailed record of experimental observations adequate to permit a reader to place all data in context.
CT.A.b Is able to identify assumptions used to make predictions Labs: 3, 4, 6, 7, 9, 12	No attempt is made to identify any assumptions necessary for making predictions	An attempt is made to identify assumptions, but the assumptions stated are irrelevant to the specific predicted values or apply to the broader hypothesis instead of the specific prediction	Relevant assumptions are identified regarding the specific predictions, but are not properly evaluated for significance in making the prediction.	Sufficient assumptions are correctly identified, and are noted to indicate significance to the prediction that is made.

	No Effort	Progressing	Expectation	Scientific
CT.A.c Is able to make predictions for each trial during experiment Labs: 3, 4, 6, 7, 9, 12	Multiple experimental trials lack predictions specific to those individual trial runs.	Predictions made are too general and could be taken to apply to more than one trial run. OR Predictions are made without connection to the hypothesis identified for the experiment. OR Predictions are made in a manner inconsistent with the hypothesis being tested. OR Prediction is unrelated to the context of the experiment.	Predictions follow from hypothesis, but are flawed because relevant experimental assumptions are not considered and/or prediction is incomplete or somewhat inconsistent with hypothesis or experiment.	A prediction is made for each trial set in the experiment which follows from the hypothesis but is hyper-specific to the individual trial runs. The prediction accurately describes the expected outcome of the experiment and incorporates relevant assumptions.
CT.A.d "Is able to identify sources of uncertainty " Labs: 3, 4, 7-9, 11 No attempt is made to identify experimental uncertainties.	descworst	An attempt is made to identify experimental uncertainties, but many sources of uncertainty are not addressed, described vaguely, or incorrect.	Most experimental uncertainties are correctly identified. But there is no distinction between random and experimental uncertainty.	All experimental uncertainties are correctly identified. There is a distinction between experimental uncertainty and random uncertainty.
QR.A Is able to perform algebraic steps in mathematical work. Labs: 2-4, 6-12	No equations are presented in algebraic form with known values isolated on the right and unknown values on the left.	Some equations are recorded in algebraic form, but not all equations needed for the experiment.	All the required equations for the experiment are written in algebraic form with unknown values on the left and known values on the right. Some algebraic manipulation is not recorded, but most is.	All equations required for the experiment are presented in standard form and full steps are shown to derive final form with unknown values on the left and known values on the right. Substitutions are made to place all unknown values in terms of measured values and constants.
QR.C Is able to analyze data appropriately Labs: 1-12	No attempt is made to analyze the data.	An attempt is made to analyze the data, but it is either seriously flawed, or inappropriate.	The analysis is appropriate for the data gathered, but contains minor errors or omissions	The analysis is appropriate, complete, and correct.
IL.A Is able to record data and observations from the experiment Labs: 1-12	"Some data required for the lab is not present at all, or cannot be found easily due to poor organization of notes. "	"Data recorded contains errors such as labeling quantities incorrectly, mixing up initial and final states, units are not mentioned, etc. "	Most of the data is recorded, but not all of it. For example measurements are recorded as numbers without units. Or data is not assigned an identifying variable for ease of reference.	All necessary data has been recorded throughout the the lab and recorded in a comprehensible way. Initial and final states are identified correctly. Units are indicated throughout the recording of data. All quantities are identified with standard variable identification and identifying subscripts where needed.

	No Effort	Progressing	Expectation	Scientific
IL.B Is able to construct a force diagram Labs: 1-12	No force diagrams are present.	Force diagrams are constructed, but not in all appropriate cases. OR force diagrams are missing labels, have incorrectly sized vectors, have vectors in the wrong direction, or have missing or extra vectors.	Force diagram contains no errors in vectors, but lacks a key feature such as labels of forces with two subscripts, vectors are not drawn from the center of mass, or axes are missing.	The force diagram contains no errors, and each force is labelled so that it is clearly understood what each force represents. Vectors are scaled precisely and drawn from the center of mass.
WC.A Is able to create a sketch of important experimental setups Labs: 1, 3, 4, 6, 7, 10-12	No sketch is constructed.	Sketch is drawn, but it is incomplete with no physical quantities labeled, OR important information is missing, OR it contains wrong information, OR coordinate axes are missing.	Sketch has no incorrect information but has either a few missing labels of given quantities, or subscripts are missing/inconsistent. Majority of key items are drawn with indication of important measurements/locations.	Sketch contains all key items with correct labeling of all physical quantities and has consistent subscripts. Axes are drawn and labeled correctly. Further drawings are made where needed to indicate precise details not possible in the scale of initial sketch.

Print this page. Tear in half. Each lab partner should submit their half along with the lab report and then retain until the end of semester when returned with evaluations indicated by TA.

Lab 3 Projectile Motion:

Name: ______________________________ Lab Partner: ______________________________

EXIT TICKET:

- ☐ Return the projectile and the carbon paper to the TA Table.
- ☐ Remove all tape from the floor.
- ☐ Wrap the plumb bob string around the cardboard spool.
- ☐ Store the plumb bob and string in its plastic bag.
- ☐ Return the goggles to their plastic bags.
- ☐ Place the plumb bob, tape measure, goggles and rulers in your equipment basket.
- ☐ Loosen the thumbscrew on the launcher so it can move freely.
- ☐ Quit any software you have been using.
- ☐ Straighten up your lab station. Put all equipment where it was at start of lab.
- ☐ Required Level of Effort.
  - ☐ Complete the pre-lab assignment
  - ☐ Arrive on time
  - ☐ Work well with your partner
  - ☐ Complete the lab or run out of time

SL.A.b		CT.A.b		QR.C	
SL.A.c		CT.A.c		IL.A	
SL.B.c		CT.A.d		IL.B	
CT.A.a		QR.A		WC.A	

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