Lab 8. Work and Energy

Goals

- To apply the concept of work to each of the forces acting on an object pulled up an incline at constant speed.
- To compare the total work on an object to the change in its kinetic energy as a first step in the application of the so-called Work-Energy Theorem.
- To relate the work done by conservative forces to the concept of potential energy.
- To apply the concept of conservation of mechanical energy, where mechanical energy is defined as the sum of kinetic and potential energy, to a system where the work done by nonconservative forces is zero or cancelled out, as in this experiment.

Introduction

The notion of "work" has a special meaning in physics. When the applied force is constant in magnitude and direction, and the motion is along a straight line, the formula for work reduces to:

$$W = Fd\cos\theta = (F\cos\theta)d = F(d\cos\theta) \tag{8.1}$$

where F is the magnitude of the force, d is the magnitude of the displacement, and θ is the angle between the force vector and the displacement vector. Since magnitudes are always positive, F and d are always positive, and the sign of the work is determined by the factor of $\cos\theta$.

If the force is not constant, then one must sum the work done over each of a series of very small displacements, where the force is approximately constant over each small displacement. In calculus, this process is described in terms of integration.

The concept of work is most useful for point particles in the presence of conservative forces (no friction). Because work is a scalar and forces are vectors, problems that can be solved using the work concept are usually easier to solve by using work than by using Newton's Second Law.

Work done on a cart moving at constant velocity

Carefully place the wooden block on edge under the end of the track opposite the pulley so that the track is inclined at an angle of 4–5° to the horizontal (About a 4" raise, so your 2x4 block should be set up appropriately). “Hooking” the small rubber feet on the bottom of the track over the edge of the block will keep the track from slipping and changing the angle if the track is bumped. Determine the angle of the ramp to within about one-tenth of a degree. A protractor can’t be read accurately enough; use trigonometry. $\theta = \arctan \frac{rise}{run}$ (Be sure you know if the answer is in degrees or radians, a quick check is to enter $cos(90)$ and check if your answer is 0 or not). Measure the mass of the cart and the mass of one of the black steel bars. A steel bar will be placed in the cart for this experiment.

The final thing to prepare for the lab is to properly zero your spring scale. You will likely need to zero the spring scale multiple times during the lab session. To zero your spring scale, you hold it at the angle which you will be measuring, and then slide the aluminum plate so that the unloaded scale reading is zero grams.

Work done by you on the cart with spring scale parallel to track

Using the small spring scale held parallel to the ramp, pull the cart with the steel bar on board at a slow *constant* velocity up the ramp a distance of 0.5 m. To know that your velocity is constant, you either go "by feeling" and accept significant error, or you use the velocity readings from Capstone and the "Photogate with Pulley" to keep track of how constant your velocity is Using Capstone like this does add another force to your system, making your calculations more complicated, but your results more accurate. If you area "going by feeling" then your indication that the acceleration is constant is that the spring scale will have a stable reading the entire time you are pulling. The indication that your constant acceleration is zero will be that your velocity has not changed from the start to the finish of the motion (something very hard for people to distinguish clearly with only 1 meter of travel).

From the definition of work in your textbook or the Introduction above, calculate the work done by you on the cart as you pull the cart up the ramp. Be careful of units! The gram readings of the spring scale must be converted to newtons.

Repeat the measurement as you carefully lower the cart down the ramp at constant velocity.

Work done by you on the cart with spring scale inclined 60° to track

Pull the cart up the track through the 0.5 m distance at constant velocity while holding the spring scale at an angle of 60° with respect to the ramp. If you hold the spring scale along the 30-60-90 plastic triangle provided, you can keep the scale at the required 60° inclination. Again calculate the work done by you on the cart as you pull the cart up the ramp.

Repeat the measurement as you carefully lower the cart down the ramp at constant velocity.

Compare the work with a 60° incline against that parallel to the track and comment on the results.

Work done by gravity on the cart as it moves up and down the ramp

Draw a free-body force diagram of the cart being pulled up the ramp. (Ignore friction.) You have already computed the work you did as the cart was pulled up the ramp. Now calculate the work done by each of the other forces. Show the steps of your analysis completely and be careful of signs.

Repeat the free-body diagram and work calculations for the cart as it moves down the ramp. Use a table to show the values of the work done by each force acting on the cart for the 0° and 60° orientations of the spring balance. Sum the values of the work to find the total work done by all the forces acting on the cart for each of the two cases.

When a net force begins to act on an object at rest, the object begins to move. One can argue mathematically (see your textbook for the details) that the work done on the object (neglecting friction) is equal to the change in its kinetic energy if we define the kinetic energy to be $mv^2/2$, where m is the mass of the object and v is its speed. Remember that the net force on the cart is zero when it moves with constant velocity.

Based on the Work-Energy Theorem, what total work would you expect for each case? Did your calculated total work values behave in accordance with these expectations? Make quantitative comparisons between your expected and experimental results. Explain.

Applying the Work-Energy theorem to an accelerating cart

Before beginning this investigation, level the track and take the steel bar out of the cart. Then add or subtract paper clips on the end of the string as necessary to cancel the frictional forces acting on the pulley and cart as the cart moves toward the pulley end of the track. When you have achieved the correct balance between the weight of the paper clips and the friction, the net force on the cart will be very close to zero. Then the acceleration of the cart should also be very close to zero. (What will the velocity-time graph look like if the acceleration is zero?) Give the cart a gentle push toward the pulley and use Capstone with the "Photogate with Pulley" sensor to measure the acceleration. Adjust the number of paper clips as necessary to "fine-tune" the apparatus, so that the average acceleration is as close to zero as possible.

Place a 20-g mass on the end of the string in addition to the paper clips. The net force on the system (cart plus hanging mass) is now the weight of the 20-g mass. As the cart moves and the 20-g hanging mass descends, work is done by the gravitational force on the cart-hanging mass system. Is work done by any other forces acting on the system? Remember that we have "cancelled out" the frictional force with the paper clips, so friction need not be considered here.

Move the cart to the end of the track opposite the pulley and release it from rest. Click the "Start" button in Capstone a couple seconds before releasing the cart. This ensures that you get some data before the cart is released. With the "Photogate with Pulley" sensor, Capstone defines the position of the cart at the beginning of data collection to be zero. This will be helpful below.

Take appropriate data to address the question of whether the total work done on the system is equal to the change in kinetic energy. Note that the computer can calculate the total work done since the system was released from rest and the instantaneous value of the kinetic energy in real time as the cart moves. Use the Capstone "Calculator" tool from the Tools Palette to define expressions for the work done on the system and for the kinetic energy. Your TA can assist if necessary. **Be sure to show the reasoning used to get these expressions in your lab notes.** These defined quantities can then be displayed on a graph just like other measured quantities. Displaying the work and the kinetic energy on the same graph provides the simplest method for comparing the two as a function of time.

Print out the results and discuss your findings. Be sure to address the issue of "change in kinetic energy" versus just "kinetic energy." Are they ever the same?

Kinetic and potential energy

A slightly more complicated arrangement is produced by raising the end of the track opposite the pulley to make an angle of about 2° with the horizontal. (Laying the block of wood flat on the table under the feet of the track should be adequate.) With the 20-g mass still hanging from the end of the string, the net force on the system of the cart and hanging mass now involves more than one force. (Note: The assumption that the friction force is not changed much by raising the ramp to a small angle should be very good.) Again address the question, does the total work done by the forces equal the change in the kinetic energy of the system? Collect appropriate data to determine the validity of this hypothesis.

Potential energy and the conservation of energy

Where does the idea of "potential energy" come from? In many ways potential energy is an intuitive concept from everyday experience. For example, if you are hit by a falling apple, you know instinctively (or by experience?) that the damage it does depends on the height from which it falls. We might even be tempted to think about the notion of "conservation of energy." While the apple is falling and losing energy of position (potential energy), is it possible that the energy of motion (kinetic energy) increases so that the sum of the two energies remains constant? One approach to answering this question is to assume that the sum of the kinetic and potential energies remains the same, and then try to discover how the potential energy would have to be defined to obey such a conservation law.

In the previous exercises you have hopefully shown that the work (W) done on an object by a net force is equal to the change in kinetic energy (KE) of that object. Mathematically we say:

$$W = \Delta KE = KE_{final} - KE_{initial} \tag{8.2}$$

If our energy conservation idea is to be correct, then something like potential energy (denoted PE) must exist so that the sum of KE and PE is constant:

$$KE_{final} + PE_{final} = KE_{initial} + PE_{initial} \quad \text{, or} \tag{8.3}$$

$$KE_{final} - KE_{initial} = PE_{initial} - PE_{final} \quad \Rightarrow \quad \Delta KE = -\Delta PE \tag{8.4}$$

The energy conservation idea from Equation 8.4 can be reconciled with the work-energy relationship from Equation 8.2 only if the change in PE is equal to the negative of the work done. That is,

$$\Delta PE = -W \tag{8.5}$$

This relationship cannot be used in the presence of forces like friction since the position of an object in space does not uniquely specify the work done by friction in the process of moving the object to that position; thus the potential energy cannot be uniquely defined either. Fortunately for us, the work done by the gravitational force and the electric force, two of the most common forces in nature, are both defined uniquely as objects move from one place to another. (Refer to your textbook for a discussion of conservative forces, such as gravity and electric forces, and non-conservative forces, such as friction.) Let's apply Equation 8.5 to a simple case of an object of mass, m, near the earth's surface falling vertically from a position, $y+h$, to a lower position, y. Since the gravitational force is downward, the work done by gravity is positive and equal to mgh. Therefore

$$\Delta PE = PE_y - PE_{y+h} = -mgh \quad \text{or} \quad PE_{y+h} - PE_y = mgh \tag{8.6}$$

Interestingly, Equation 8.6 only specifies that the difference in the potential energy from the initial to the final state must be mgh. This can be satisfied by setting PE_{y+h} to mgh and setting PE_y to zero. An equally valid solution would be to choose $PE_{y+h} = mg(y+h)$ and $PE_y = mgy$. In this case the zero of potential energy will occur when $y = 0$. There is a certain amount of latitude in choosing the "zero" of potential energy. This is always the case!

If an object moves horizontally near the earth's surface, the gravity force has no component along the direction of the displacement. Thus no work is done, and the potential energy of the object does not change. In cases where both horizontal and vertical displacements occur, only the vertical displacement leads to a change in potential energy of the body.

Plotting the cart's total mechanical energy as a function of time

Use Capstone's Calculator Tool to calculate and plot the sum of the kinetic and potential energies of the cart/mass system as the cart moves down the track. Is the total mechanical energy of the system is conserved?

Conclusion

Summarize all your findings carefully and succinctly. Where possible, discuss your results in terms of the measurement uncertainties.

	No Effort	Progressing	Expectation	Scientific
SL.A.b Is able to identify the hypothesis for the experiment proposed Labs: 4, 5, 7, 8, 10	No deliberately identified hypothesis is present in the first half page or so of notes	An attempt is made to state a hypothesis, but no clearly defined dependent and independent variable, or lacking a statement of relationship between the two variables	A statement is made as a hypothesis, it contains a dependent and independent variable along with a statement of relationship between the two variables. This statement appears to be testable, but there are some minor omissions or vague details.	The hypothesis is clearly stated and the direct link to the experiment at hand is apparent to any reasonably informed reader.
SL.A.c Is able to determine hypothesis validity Labs: 4, 5, 7, 8, 10	No deliberately identified attempt to use experimental results to validate hypothesis is present in the sections following data collection.	A statement about the hypothesis validity is made, but it is not consistent with the data analysis completed in the experiment	A statement about the hypothesis validity is made which is consistent with the data analysis completed in the experiment. Assumptions which informed the hypothesis and assumptions not validated during experimentation are not taken into account.	A statement about the hypothesis validity is made which is consistent with the data analysis and all assumptions are taken into account.
SL.B.c Is able to explain steps taken to minimize uncertainties and demonstrate understanding through performance where able. Labs: 4, 5, 8, 9, 12	No explicitly identified attempt to minimize uncertainties and no attempt to describe how to minimize uncertainties present	No explicitly identified attempt to minimize uncertainties is present, but there is a description of how to minimize experimental uncertainty.	An attempt is made and explicitly identified for minimizing uncertainty in the final lab results, but the method is not the most effective.	The uncertainties are minimized in an effective way.
CT.A.a Is able to compare recorded information and sketches with reality of experiment Labs: 3-8, 10	No sketches present and no descriptive text to explain what was observed in experiment	Sketch or descriptive text is present to inform reader what was observed in the experiment, but there is no attempt to explain what details of the experiment are not accurately delivered through either representation.	Sketch and descriptive text are both present. The sketch and description supplement one another to attempt to make up for the failures of each to convey all observations from the experiment. There are minor inconsistencies between the two representations and the known reality of the experiment from the week, but no major details are absent.	Sketch and description address the shortcomings of one another to convey an accurate and detailed record of experimental observations adequate to permit a reader to place all data in context.

	No Effort	Progressing	Expectation	Scientific
CT.A.b Is able to identify assumptions used to make predictions Labs: 4, 5, 7, 8, 10	No attempt is made to identify any assumptions necessary for making predictions	An attempt is made to identify assumptions, but the assumptions stated are irrelevant to the specific predicted values or apply to the broader hypothesis instead of the specific prediction	Relevant assumptions are identified regarding the specific predictions, but are not properly evaluated for significance in making the prediction.	Sufficient assumptions are correctly identified, and are noted to indicate significance to the prediction that is made.
CT.A.c Is able to make predictions for each trial during experiment Labs: 4, 5, 7, 8, 10	Multiple experimental trials lack predictions specific to those individual trial runs.	Predictions made are too general and could be taken to apply to more than one trial run. OR Predictions are made without connection to the hypothesis identified for the experiment. OR Predictions are made in a manner inconsistent with the hypothesis being tested. OR Prediction is unrelated to the context of the experiment.	Predictions follow from hypothesis, but are flawed because relevant experimental assumptions are not considered and/or prediction is incomplete or somewhat inconsistent with hypothesis or experiment.	A prediction is made for each trial set in the experiment which follows from the hypothesis but is hyper-specific to the individual trial runs. The prediction accurately describes the expected outcome of the experiment and incorporates relevant assumptions.
CT.A.d "Is able to identify sources of uncertainty " Labs: 4, 5, 8, 9, 12 No attempt is made to identify experimental uncertainties.	descworst	An attempt is made to identify experimental uncertainties, but many sources of uncertainty are not addressed, described vaguely, or incorrect.	Most experimental uncertainties are correctly identified. But there is no distinction between random and experimental uncertainty.	All experimental uncertainties are correctly identified. There is a distinction between experimental uncertainty and random uncertainty.
QR.A Is able to perform algebraic steps in mathematical work. Labs: 3-5, 7-12	No equations are presented in algebraic form with known values isolated on the right and unknown values on the left.	Some equations are recorded in algebraic form, but not all equations needed for the experiment.	All the required equations for the experiment are written in algebraic form with unknown values on the left and known values on the right. Some algebraic manipulation is not recorded, but most is.	All equations required for the experiment are presented in standard form and full steps are shown to derive final form with unknown values on the left and known values on the right. Substitutions are made to place all unknown values in terms of measured values and constants.
QR.C Is able to analyze data appropriately Labs: 2-12	No attempt is made to analyze the data.	An attempt is made to analyze the data, but it is either seriously flawed, or inappropriate.	The analysis is appropriate for the data gathered, but contains minor errors or omissions	The analysis is appropriate, complete, and correct.

	No Effort	Progressing	Expectation	Scientific
IL.A Is able to record data and observations from the experiment Labs: 1-12	"Some data required for the lab is not present at all, or cannot be found easily due to poor organization of notes. "	"Data recorded contains errors such as labeling quantities incorrectly, mixing up initial and final states, units are not mentioned, etc. "	Most of the data is recorded, but not all of it. For example measurements are recorded as numbers without units. Or data is not assigned an identifying variable for ease of reference.	All necessary data has been recorded throughout the the lab and recorded in a comprehensible way. Initial and final states are identified correctly. Units are indicated throughout the recording of data. All quantities are identified with standard variable identification and identifying subscripts where needed.
IL.B Is able to construct a force diagram Labs: 1-12	No force diagrams are present.	Force diagrams are constructed, but not in all appropriate cases. OR force diagrams are missing labels, have incorrectly sized vectors, have vectors in the wrong direction, or have missing or extra vectors.	Force diagram contains no errors in vectors, but lacks a key feature such as labels of forces with two subscripts, vectors are not drawn from the center of mass, or axes are missing.	The force diagram contains no errors, and each force is labelled so that it is clearly understood what each force represents. Vectors are scaled precisely and drawn from the center of mass.
WC.A Is able to create a sketch of important experimental setups Labs: 2, 4, 5, 7, 8, 10-12	No sketch is constructed.	Sketch is drawn, but it is incomplete with no physical quantities labeled, OR important information is missing, OR it contains wrong information, OR coordinate axes are missing.	Sketch has no incorrect information but has either a few missing labels of given quantities, or subscripts are missing/inconsistent. Majority of key items are drawn with indication of important measurements/locations.	Sketch contains all key items with correct labeling of all physical quantities and has consistent subscripts. Axes are drawn and labeled correctly. Further drawings are made where needed to indicate precise details not possible in the scale of initial sketch.
WC.B Is able to draw a graph Labs: 2, 3, 5-9, 11, 12	No graph is present.	A graph is present, but the axes are not labeled. OR there is no scale on the axes. OR the data points are connected.	"A graph is present and the axes are labeled, but the axes do not correspond to the independent (X-axis) and dependent (Y-axis) variables or the scale is not accurate. The data points are not connected, but there is no trend-line. "	The graph has correctly labeled axes, independent variable is along the horizontal axis and the scale is accurate. The trend-line is correct, with formula clearly indicated.

Print this page. Tear in half. Each lab partner should submit their half along with the lab report and then retain until the end of semester when returned with evaluations indicated by TA.

Lab 8 Work and Energy:

Name: ____________________ Lab Partner: ____________________

EXIT TICKET:

- ☐ Quit Capstone and any other software you have been using.
- ☐ Straighten up your lab station. Put all equipment where it was at start of lab.
- ☐ Required Level of Effort.
  - ☐ Complete the pre-lab assignment ☐ Arrive on time
  - ☐ Work well with your partner ☐ Complete the lab or run out of time

SL.A.b		CT.A.b		IL.A	
SL.A.c		CT.A.c		IL.B	
SL.B.c		CT.A.d		WC.A	
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CT.A.a		QR.A		WC.B	
		QR.C			