Lab 12. Vibrating Strings

Goals

- To experimentally determine the relationships between the fundamental resonant frequency of a vibrating string and its length, its mass per unit length, and the tension in the string.
- To introduce a useful graphical method for testing whether the quantities x and y are related by a "simple power function" of the form $y = ax^n$. If so, the constants a and n can be determined from the graph.
- To experimentally determine the relationship between resonant frequencies and higher order "mode" numbers.
- To develop one general relationship/equation that relates the resonant frequency of a string to the four parameters: length, mass per unit length, tension, and mode number.

Introduction

Vibrating strings are part of our common experience. Which as you may have learned by now means that you have built up explanations in your subconscious about how they work, and that those explanations are sometimes self-contradictory, and rarely entirely correct.

Musical instruments from all around the world employ vibrating strings to make musical sounds. Anyone who plays such an instrument knows that changing the tension in the string changes the pitch, which in physics terms means changing the resonant frequency of vibration. Similarly, changing the thickness (and thus the mass) of the string also affects its sound (frequency). String length must also have some effect, since a bass violin is much bigger than a normal violin and sounds much different. The interplay between these factors is explored in this laboratory experiment.

You do not need to know physics to understand how instruments work. In fact, in the course of this lab alone you will engage with material which entire PhDs in music theory have been written. Knowing that various waves on a string are harmonic multiples does not make you understand how they sound when played together.

Water waves, sound waves, waves on strings, and even electromagnetic waves (light, radio, TV, microwaves, etc.) have similar behaviors when they encounter boundaries from one type of material (called a medium) to another. In general all waves reflect part of the energy and transmit some into the new medium. In some cases the amount of energy transmitted is very small. For example a water wave set up in your bathtub moves down the length of the tub and hits the end. Very little energy is transmitted into the material of the tub itself and you can observe a wave of essentially the same size as the "incident" wave being reflected. The clamps at the ends of a string provide similar boundaries for string waves such that virtually all the energy of the wave is reflected back and the wave travels from one end to the other. The wave "bounces" back and forth. If waves are sent down a string of some length at a constant frequency, then there will be certain frequencies where the reflected waves and the waves being generated on the string interfere constructively. That is, the peaks of the incident waves and the peaks of the reflected waves coincide spatially and thus add together. When this occurs, the composite wave no longer "travels" along the string but appears to stand still in space and oscillate transversely. This is called a "standing wave." A marching band that is marching "in place" but not moving is a fair analogy. You can demonstrate this phenomenon with a stretched rubber band or jump rope. These standing waves occur only at particular frequencies, known as resonant frequencies, when all the necessary conditions are satisfied. These necessary conditions depend on the factors mentioned above, such as whether the string is clamped tightly at the ends or not (i.e., the boundary conditions), the length of the string, its mass per unit length, and the tension applied to the string.

With this in mind, we will systematically explore how the resonant frequency depends on three of the four factors listed above. In all cases our strings are clamped or held tightly at both ends; we consistently use the same boundary conditions. Finally, we will search for a single equation that describes the effect of length, tension, and mass per unit length on the resonant frequency.

Equipment set up

A schematic diagram of the set up is shown in Figure 12.1. Connect the speaker unit to the output terminal (marked with a wave symbol) and the ground terminal (marked with the ground symbol) of the Pasco Model 850 interface unit. The interface unit can be configured to produce a voltage that varies sinusoidally at a known frequency. In the Experimental Setup window, click on the image of the output terminal (marked with a wave symbol). In the window that appears, make sure that the waveform pull down menu is set to Sine Wave. Use the frequency and voltage windows to set the frequency and output voltages, respectively. Keep the output voltage below 4.5 V. Select the Current Limit of 0.55 Amps (only available on Channel 1). Click the "Auto" button (which toggles the Auto function off), then click the "On" button to start the voltage generator.

This voltage drives an audio speaker mechanism that lacks the diaphragm that normally produces the sound. You will nevertheless hear some sound from the speaker drive mechanism. This sound can be irritating, so use the minimum voltage required to make a good measurement. This speaker drive oscillates in synchrony with the drive voltage and is connected to the string via an "alligator" clip.

Caution: Do not apply loads greater than 10 kg to the end of the string!

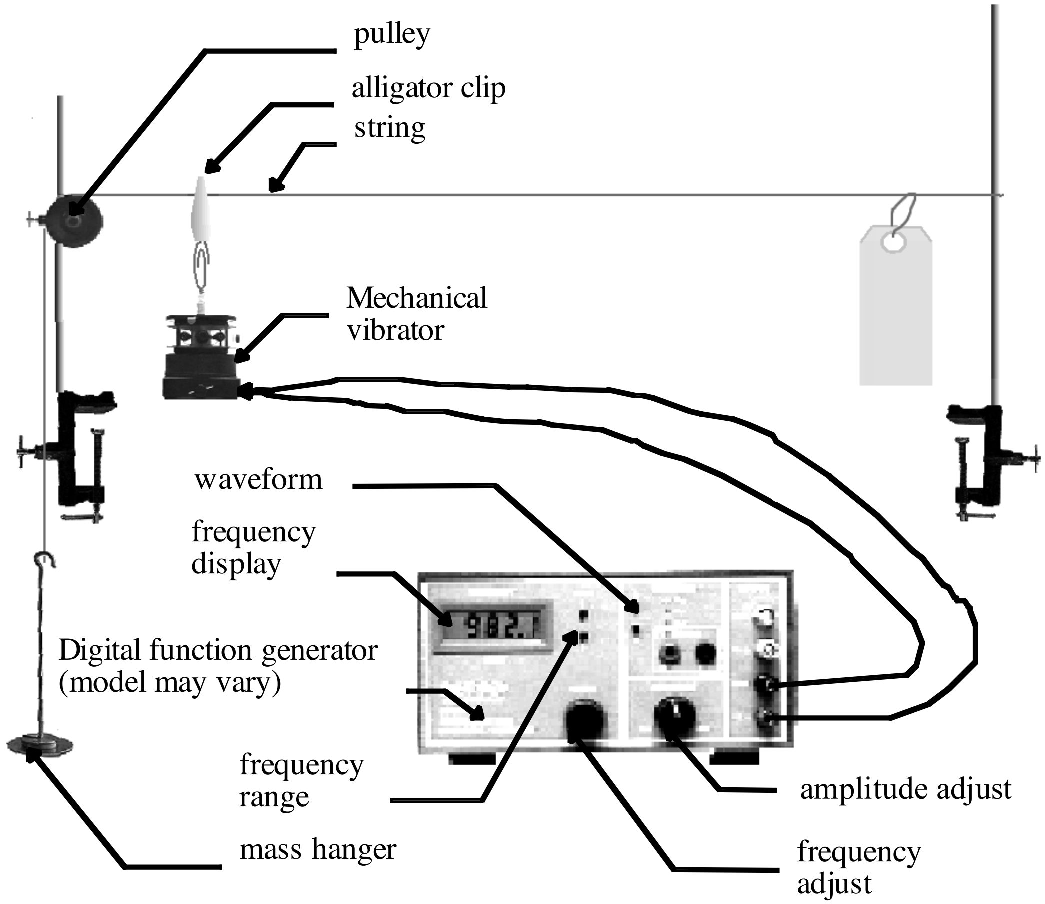

Figure 12.1. Typical apparatus for the vibrating string experiment. The Pasco Model 850 interface unit can be used to control the mechanical vibrator in place of the digital function generator.

Effect of string length on resonant frequency

Start with the 1.3 g/m string (see the tag attached to the end of string) and hang a total mass of 5 kg, including the mass of the mass hanger, on the end of the string. Determine the fundamental resonant frequency for five or six different string lengths. Length is measured from the clip of the wave generator to the point where the string touches the pulley.

Plucking the string with your finger near the middle point excites a vibration of the string, primarily in its fundamental resonant mode (also called the first harmonic). Pluck the string and note how the string vibrates. The vibration of the string stops a short time after you pluck it because of energy losses due to air friction. The speaker drive allows you to pump energy into the vibrating system at the same rate that it is lost, so that the vibration can be maintained for as long you wish.

When you pluck the string, only the resonant modes will manifest. This is because all non-resonant modes will tend to self-cancel upon reflection. However, when driving the string vibration with an external force, you can vibrate the string at non-harmonic modes. The string will vibrate strongly only at certain well-defined frequencies which are multiples of the harmonics though. By adjusting the frequency of the speaker drive slowly while watching the string you should be able to find the frequency that makes the string vibrate in its fundamental resonant mode. You can recognize the fundamental resonant frequency easily because the whole middle portion of the string oscillates up and down like a jump rope; the fundamental resonance can be thought of as the "jump rope mode." For best results you must continue adjusting the speaker drive until you have found the "middle" of the resonance, where the amplitude of the vibration is maximized.

Note that the distance from the alligator clip to the top of the pulley where the string is held tightly determines the length of the vibrating string. The alligator clip does vibrate slightly but the string behaves very nearly as if the clip defines a clamped end. (The motion of the alligator clip cannot be ignored for very heavy strings. For these, you may have to visually locate the point near the alligator clip which appears to be clamped and doesn't vibrate. This location on the string which stays relatively stationary is called a "node" while the position on the string which vibrates the most each cycle is called an "anti-node.")

Make sure that the string lengths that you test are approximately uniformly spaced between 0.4 m and the maximum string length possible given the length of the table. By graphical means determine a mathematical function for the fundamental resonant frequency, f, as a function of L, where L is the length of the vibrating string as determined by the placement of the alligator clip. Do you get a linear graph if you plot f on the y-axis and L on the x-axis? Instead, try plotting f on the y-axis and $1/L$ on the x-axis. What important property of the wave on the string can be determined from this graphical analysis? The units of the slope of this graph (assuming it is linear) provide information on what this quantity might be. Explain your reasoning!

Effect of string tension on resonant frequency

Keep using the same string for each of these first three sets of experiments. This will allow you to re-measure any data which seems to warrant closer inspection due to inconsistencies. Determine the fundamental resonant frequency of the string as the total mass on the end of the string is increased from 1.0 to 10.0 kg. The weight of the hanging mass will be equal to the tension in the string, T. Graphically determine whether the relationship between the fundamental resonant frequency, f, and the string tension, T, is a simple power function. Refer to the Uncertainty-Graphical Analysis Supplement in the lab manual.

Effect of harmonic mode number on resonant frequency

Still using the 1.3 g/m string and now 3 kg hanging mass, set the length of the string to at least 1.5 m. So far you have looked at the fundamental frequency, or first harmonic, of the string vibration. The second harmonic (mode number $n = 2$) will have a "jump rope" mode on each half of the string but they will oscillate in opposite directions. The center of the "jump rope" effect is called an anti-node, and the stationary part of the string between them is called a node. At the fundamental frequency you have one anti-node, and each end of the clamped string is a node, this gives you half of a wave shape. The second harmonic has one full wave shape, so you have two anti-nodes and three nodes (counting the two clamped ends).

Increase the driver frequency until you find this resonance and record it. The third harmonic will have three anti-nodes on the string, etc. At the very least you should collect the data for $n = 1, 2, 3$, and 4. If time allows, determine frequencies for even higher n values. As you increase your number of anti-nodes, the amplitude will decrease, making it harder to observe the anti-node formation. Placing a contrasting solid color background behind the string can improve visibility. Increasing the voltage of the wave driver can also increase the amplitude. **Do not exceed 6V output.**

Determine the relationship $f(n)$ between the resonant frequency, f, and the mode number, n, by graphical means.

Effect of string mass-per-unit-length on resonant frequency

We test string mass last because up until this point you have not had to change the string which is mounted, and are thus able to go back and retest any previous data. Check your data again right now before you change the string to be certain you do not want to run any more trials with the current string density.

For this set of experiments, use the maximum string length employed previously and hang a total of 5 kg on the end of the string. Test the four the strings in the box, noting the mass per unit length (μ) indicated on the attached cards. Find the fundamental oscillation frequency for each of the strings at your station. Remember that you already took one data point while observing the effect of string length. Normally we would determine graphically what the relationship between fundamental frequency, f, and the mass per unit length, μ, that is $f(\mu)$, is. But, with only four data points it is not possible to distinguish clearly between the various best fit lines to determine which is the ideal fit to your graph. This relationship should match a power series. Find the equation for frequency as a function of mass/unit length. Refer to the Uncertainty-Graphical Analysis Supplement in this lab manual for details.

Note that by having a relationship between mass per unit length, you are directly showing a second link to frequency and string length. In the first part of this experiment you had determined a relationship between the length and the frequency. The length dependence of this linear density would have influenced your earlier results, causing your best fit line to stray from being a "comfortable" linear fit as a result.

Summary

Summarize your findings clearly and succinctly. Can you write a single mathematical function that encapsulates all the relationships that you have discovered? That is $f(T, \mu, L, n)$. Note that taking the sum of the four relations you determined above will not work. This is blatantly obvious if you perform a unit analysis on the equation, as you are not allowed to add values together if they have different units.

Compare your experimental results with those theoretically predicted in your textbook. (This is sometimes included in a section on musical instruments.) Show that the textbook formula is dimensionally correct (meaning analyze the units). Be quantitative in your comparisons (meaning analyze the numbers).

	No Effort	Progressing	Expectation	Scientific
SL.A.a Is able to analyze the experiment and recommend improvements Labs: 2, 3, 5, 11, 12	No deliberately identified reflection on the efficacy of the experiment can be found in the report	Description of experimental procedure leaves it unclear what could be improved upon.	Some aspects of the experiment may not have been considered in terms of shortcomings or improvements, but some are identified and discussed.	All major shortcomings of the experiment are identified and reasonable suggestions for improvement are made. Justification is provided for certainty of no shortcomings in the rare case there are none.
SL.B.a Is able to explain operation and limitations of measurement tools Labs: 2, 3, 6, 11, 12	At least one of the measuring tools used in lab lacks a clear identification of precision/limitation	All measuring tools are identified with mention of the precision/limitation of each tool, but no details on how measurements are performed	All measuring tools identified with precision/limitation of each tool listed. Description of how to measure using some tools may be incorrect/vague, or precision may not be adequately justified.	All measuring tools are identified with proper precision values and thorough discussion of limitations. Descriptions on how to make measurements are complete and could be understood by readers with no prior familiarity with the measuring tools.
SL.B.b Is able to explain patterns in data with physics principles Labs: 2, 3, 6, 8, 11, 12	No attempt is made to explain the patterns in data	An explanation for a pattern is vague, OR the explanation cannot be verified through testing, OR the explanation contradicts the actual pattern in the data.	An explanation is made which aligns with the pattern observed in the data, but the link to physics principles is flawed through reasoning or failure to understand the physics principles.	A reasonable explanation is made for the pattern in the data. The explanation is testable, and accounts for any significant deviations or poor fit.
SL.B.c Is able to explain steps taken to minimize uncertainties and demonstrate understanding through performance where able. Labs: 4, 5, 8, 9, 12	No explicitly identified attempt to minimize uncertainties and no attempt to describe how to minimize uncertainties present	No explicitly identified attempt to minimize uncertainties is present, but there is a description of how to minimize experimental uncertainty.	An attempt is made and explicitly identified for minimizing uncertainty in the final lab results, but the method is not the most effective.	The uncertainties are minimized in an effective way.
CT.A.d "Is able to identify sources of uncertainty " Labs: 4, 5, 8, 9, 12 No attempt is made to identify experimental uncertainties.	descworst	An attempt is made to identify experimental uncertainties, but many sources of uncertainty are not addressed, described vaguely, or incorrect.	Most experimental uncertainties are correctly identified. But there is no distinction between random and experimental uncertainty.	All experimental uncertainties are correctly identified. There is a distinction between experimental uncertainty and random uncertainty.

	No Effort	Progressing	Expectation	Scientific
CT.B.a Is able to describe physics concepts underlying experiment Labs: 2, 3, 6, 9, 11, 12	No explicitly identified attempt to describe the physics concepts involved in the experiment using student's own words.	The description of the physics concepts underlying the experiment is confusing, or the physics concepts described are not pertinent to the experiment for this week.	The description of the physics concepts in play for the week is vague or incomplete, but can be understood in the broader context of the lab.	The physics concepts underlying the experiment are clearly stated.
CT.B.b Is able to identify dependent and independent variables Labs: 2, 3, 6, 12	No attempt to explicitly identify any variables as dependent or independent	Some variables identified as dependent or independent are irrelevant to the hypothesis/experiment, or some variables relevant to the experiment are not identified	The variables relevant to the experiment are all identified. A small fraction of the variables are improperly identified as dependent or independent.	All physical quantities relevant to the experiment are identified as dependent and independent variables correctly, and no irrelevant variables are included in the listing.
QR.A Is able to perform algebraic steps in mathematical work. Labs: 3-5, 7-12	No equations are presented in algebraic form with known values isolated on the right and unknown values on the left.	Some equations are recorded in algebraic form, but not all equations needed for the experiment.	All the required equations for the experiment are written in algebraic form with unknown values on the left and known values on the right. Some algebraic manipulation is not recorded, but most is.	All equations required for the experiment are presented in standard form and full steps are shown to derive final form with unknown values on the left and known values on the right. Substitutions are made to place all unknown values in terms of measured values and constants.
QR.B Is able to identify a pattern in the data graphically and mathematically Labs: 2, 3, 6, 9, 11, 12	No attempt is made to search for a pattern, graphs may be present but lack fit lines	The pattern described is irrelevant or inconsistent with the data. Graphs are present, but fit lines are inappropriate for the data presented.	The pattern has minor errors or omissions. OR Terms labelled as proportional lack clarity - is the proportionality linear, quadratic, etc. Graphs shown have appropriate fit lines, but no equations or analysis of fit quality	The patterns represent the relevant trend in the data. When possible, the trend is described in words. Graphs have appropriate fit lines with equations and discussion of any data significantly off fit.
QR.C Is able to analyze data appropriately Labs: 2-12	No attempt is made to analyze the data.	An attempt is made to analyze the data, but it is either seriously flawed, or inappropriate.	The analysis is appropriate for the data gathered, but contains minor errors or omissions	The analysis is appropriate, complete, and correct.

	No Effort	Progressing	Expectation	Scientific
IL.A Is able to record data and observations from the experiment Labs: 1-12	"Some data required for the lab is not present at all, or cannot be found easily due to poor organization of notes. "	"Data recorded contains errors such as labeling quantities incorrectly, mixing up initial and final states, units are not mentioned, etc. "	Most of the data is recorded, but not all of it. For example measurements are recorded as numbers without units. Or data is not assigned an identifying variable for ease of reference.	All necessary data has been recorded throughout the the lab and recorded in a comprehensible way. Initial and final states are identified correctly. Units are indicated throughout the recording of data. All quantities are identified with standard variable identification and identifying subscripts where needed.
IL.B Is able to construct a force diagram Labs: 1-12	No force diagrams are present.	Force diagrams are constructed, but not in all appropriate cases. OR force diagrams are missing labels, have incorrectly sized vectors, have vectors in the wrong direction, or have missing or extra vectors.	Force diagram contains no errors in vectors, but lacks a key feature such as labels of forces with two subscripts, vectors are not drawn from the center of mass, or axes are missing.	The force diagram contains no errors, and each force is labelled so that it is clearly understood what each force represents. Vectors are scaled precisely and drawn from the center of mass.
WC.A Is able to create a sketch of important experimental setups Labs: 2, 4, 5, 7, 8, 10-12	No sketch is constructed.	Sketch is drawn, but it is incomplete with no physical quantities labeled, OR important information is missing, OR it contains wrong information, OR coordinate axes are missing.	Sketch has no incorrect information but has either a few missing labels of given quantities, or subscripts are missing/inconsistent. Majority of key items are drawn with indication of important measurements/locations.	Sketch contains all key items with correct labeling of all physical quantities and has consistent subscripts. Axes are drawn and labeled correctly. Further drawings are made where needed to indicate precise details not possible in the scale of initial sketch.
WC.B Is able to draw a graph Labs: 2, 3, 5-9, 11, 12	No graph is present.	A graph is present, but the axes are not labeled. OR there is no scale on the axes. OR the data points are connected.	"A graph is present and the axes are labeled, but the axes do not correspond to the independent (X-axis) and dependent (Y-axis) variables or the scale is not accurate. The data points are not connected, but there is no trend-line. "	The graph has correctly labeled axes, independent variable is along the horizontal axis and the scale is accurate. The trend-line is correct, with formula clearly indicated.

Print this page. Tear in half. Each lab partner should submit their half along with the lab report and then retain until the end of semester when returned with evaluations indicated by TA.

Lab 12 Vibrating Strings:

Name: ______________________ Lab Partner: ______________________

EXIT TICKET:

☐ Quit Capstone and any other software you have been using.
☐ Leave only the 1 kg mass hanger on the end of the 1.3 g/m string.
☐ Straighten up your lab station. Put all equipment where it was at start of lab.
☐ Required Level of Effort.
  ☐ Complete the pre-lab assignment ☐ Arrive on time
  ☐ Work well with your partner ☐ Complete the lab or run out of time

SL.A.a	
SL.B.a	
SL.B.b	
SL.B.c	
CT.A.d	

CT.B.a	
CT.B.b	
QR.A	
QR.B	

QR.C	
IL.A	
IL.B	
WC.A	
WC.B	

Lab 12 Vibrating Strings:

Name: ______________________ Lab Partner: ______________________

EXIT TICKET:

☐ Quit Capstone and any other software you have been using.
☐ Leave only the 1 kg mass hanger on the end of the 1.3 g/m string.
☐ Straighten up your lab station. Put all equipment where it was at start of lab.
☐ Required Level of Effort.
  ☐ Complete the pre-lab assignment ☐ Arrive on time
  ☐ Work well with your partner ☐ Complete the lab or run out of time

SL.A.a	
SL.B.a	
SL.B.b	
SL.B.c	
CT.A.d	

CT.B.a	
CT.B.b	
QR.A	
QR.B	

QR.C	
IL.A	
IL.B	
WC.A	
WC.B	