Lab 4. Projectile Motion

Goals

- To determine the launch speed of a projectile and its uncertainty by measuring how far it travels horizontally before landing on the floor (called the range) when launched horizontally from a known height.
- To predict and measure the range of a projectile when the projectile is fired at an arbitrary angle with respect to the horizontal.
- To predict the initial firing angle of the launcher for a prescribed range value.
- To determine quantitatively whether the measured ranges in (2) and (3) are consistent with the desired range values.

Introduction

When objects undergo motion in two (or even three) dimensions rather than in just one, the overall motion can be analyzed by looking at the motion in any two (or three) mutually perpendicular directions and then putting the motions "back together," so to speak. In the case of projectiles, the horizontal and vertical directions are usually chosen. Why is this choice made? Ignoring the effects of air resistance, an object moving vertically near the surface of Earth experiences a constant acceleration. We know this by experiment. Likewise an object moving horizontally experiences zero acceleration. Any other choice of perpendicular directions would have nonzero, constant values of acceleration in both directions. When we write the descriptions of the motion in mathematical terms, the horizontal/vertical choice of directions results in the simplest description.

Under what conditions can the effects of air resistance be ignored? One condition is that the object's speed is not too high, since the effect of the air resistance increases with speed. If two objects are the same size and shape, the lighter one of the two will experience the larger effect on its motion due to the air. (Imagine a ping-pong ball and a steel ball bearing of the same size.) In designing this lab, care has been taken to ensure that air resistance has a negligible effect on the trajectory of the projectile. When conditions are such that air resistance cannot be ignored, the motion is more complicated.

Mathematical preliminaries—Equation for range

To accomplish the first two of our stated goals, we need a general mathematical relationship between the horizontal range of the projectile and the initial height, initial velocity, and launch angle. See Figure 4.1. You will need to solve the appropriate kinematics equations for motion with constant acceleration in the horizontal and vertical directions simultaneously. Rather than writing the equations in terms of the angle, θ, it is suggested that you use the symbols v_{0x} and v_{0y}, where $v_{0x} = v_0 \cos\theta$ and $v_{0y} = v_0 \sin\theta$, to simplify the algebra. You need to solve for the range, R, in terms of v_{0x}, v_{0y}, h, and g. The details of this derivation must be included in your lab notes.

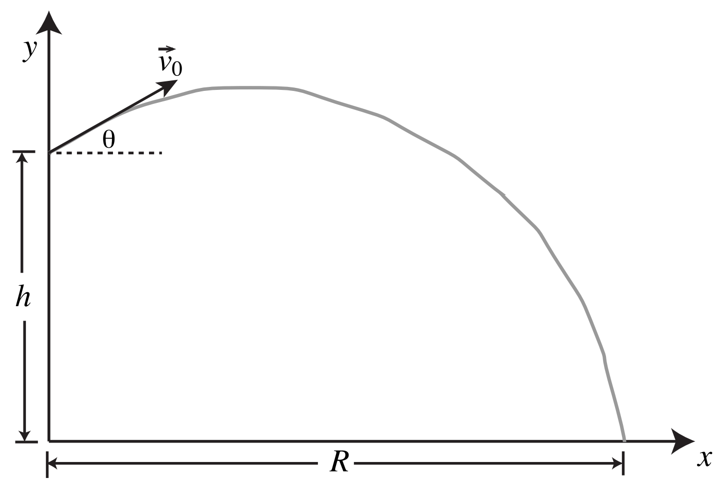

Figure 4.1. Coordinate system for calculating the range, R.

Instructions and precautions for using the ball launcher

Warning: Never look down the barrel of a launcher. Wear eye protection until all the groups have finished launching projectiles.

1. Make sure that the launcher is attached securely to the table so it does not move when the launcher is fired. Make sure the launcher is at the proper angle by using the built-in plumb bob on the side of the launcher. Note that the angle measured by this plumb bob is the angle between the “barrel” of the launcher and the horizontal.
2. Since the projectiles will be hitting the floor, use a second plumb bob to locate and mark the position on the floor (blue tape works) directly below the launch point of the projectile. This indicates the initial horizontal position of the ball at floor level so the range (horizontal distance traveled by the ball) can be measured later. You will have to measure the height to get the vertical distance. Clearly indicate in a diagram how you measured the height (from where to where). If you are not sure how the height should be measured, please discuss it with your TA. Note that "Clearly indicate" here means that you should be describing/drawing to within the limit of the accuracy of your measuring instrument (a meter stick, so 0.1mm). Have a justification for why you chose each end of the ruler at which to measure.

3. To launch the projectile, load the ball into the projectile launcher. Use the rod to push the ball into the launch tube to one of the first two out of the three preset launch positions (short, medium, or long range. Do not use Long Range). You will hear a click as you reach each position. Notify others nearby and across the room before firing the ball. Stand out of the way and fire the launcher by pulling on the string attached to its trigger on the top. To minimize the force applied by the string to the launch tube, pull the string at right angles to the launch tube (straight up). You may need to use your other hand to stabilize the launch tube (grip the tube and frame to prevent it from rocking at launch).
4. To record the position where the projectile strikes the floor, tape a white paper target to the thin hard-board sheet (about 0.3 m $\times$ 0.5 m in size) at your lab station. Place the sheet and target at the approximate place where the ball lands. When you are ready to record some landing points, lay a piece of carbon paper (carbon side down) on top of the target. Do not put tape on the carbon paper. The ball will leave a dark smudge on the white paper where it lands. If necessary you can tape the hard-board sheet to the floor to keep it from moving, but avoid the indiscriminate use of tape on the floors. Indicate on your target which marks have already been recorded into your notes to avoid confusion in future measurements.

Determining the initial speed of the projectile

1. Simplify your general equation for the range for the case when $\theta = 0$ (horizontal launch). Then solve for v_0 in terms of R, h, and g.
2. Set the launcher to fire horizontally, that is, to launch at an angle of zero degrees. Care with this angle setting can significantly improve your results later in the lab.
3. Starting with the medium range launch setting, fire the projectile (using the four steps in the previous section) a couple times noting where the projectile lands. Center the paper target as best you can where the ball will land. Now use the carbon paper to record the landing position of four or five launches using the same initial conditions.
4. From your data determine the average range, R, of the ball. Use this average distance to calculate the average initial speed of the ball as it was launched.
5. Repeat the same procedure for the short range setting on the launcher.

Range for nonzero launch angles

1. Choose a launch angle between 30° and 40°. Using the values of the initial speed of the ball measured above and your general equation for the range, calculate the horizontal distance (range) from the launch point to where the ball should land for the short and medium range settings using the initial launch angle that you have chosen. (Do not use the long range setting.)
2. For the short and medium range settings, place a paper target on the floor at the calculated position and fire the projectile. If the projectile misses the target completely, check your calculations and/or discuss it with your TA. If the projectile does hit the target, then repeat several times to get a good average experimental range value and its corresponding standard deviation to compare with your calculated range.
3. Compare your predicted range values with the experimental range values using your ex-

perimental standard deviations. Assume that your predicted range, $R_{predicted}$ has zero uncertainty. Then check if your measurement is consistent with your prediction if $t' = |R_{measured} - R_{predicted}|/\sigma(R_{measured}) < 3$. If you find that $t' > 3$, check your calculations and consider carefully what systematic errors may be present in your experiment.

Launch angle to achieve a given range

1. Ask your TA to assign a value of horizontal distance (range) for your group.
2. Calculate a suitable angle at one of the range settings for launching the projectile to the target set at the assigned distance. The relationship giving the initial launch angle in terms of the other parameters is:

$$\tan\theta = \frac{v_0^2}{gR} \pm \left[\left(\frac{v_0^2}{gR} \right)^2 - 1 + \frac{2v_0^2 h}{gR^2} \right]^{1/2} \tag{4.1}$$

3. Now set the target and do the experiment with your TA present to observe. Were you able to hit the target? If you have trouble, check your calculations. Is your calculator in radian or degree mode? Get assistance from your TA, if necessary. Again, compare your experimental range value to the range value assigned by your TA. If not, check your calculations and your procedure.

Conclusion

Summarize all your results, preferably in a table showing the measured and calculated quantities with their uncertainties. Clearly display your comparisons between predicted values and experimental values. Are you convinced that the theoretical predictions made by separating the horizontal and vertical motions agree with experiment, at least within the calculated uncertainties of the experiment? Your answers must be based on your experimental results and the calculated uncertainties of the quantities you are comparing. Do not make vague statements that are not directly supported by your calculations and measurements.

Grading Rubric

	No Effort	Progressing	Expectation	Exemplary
AA Is able to extract the information from representation correctly Labs: 1-12	No visible attempt is made to extract information from the experimental setup.	Information that is extracted contains errors such as labeling quantities incorrectly, mixing up initial and final states, choosing a wrong system, etc. Physical quantities have no subscripts (when those are needed).	Most of the information is extracted correctly, but not all of the information. For example physical quantities are represented with numbers and there are no units. Or directions are missing. Subscripts for physical quantities are either missing or inconsistent.	All necessary information has been extracted correctly, and written in a comprehensible way. Objects, systems, physical quantities, initial and final states, etc. are identified correctly and units are correct. Physical quantities have consistent and informative subscripts.
AB Is able to construct new representations from previous representations Labs: 1-12	No attempt is made to construct a different representation.	Representations are attempted, but omits or uses incorrect information (i.e. labels, variables) or the representation does not agree with the information used.	Representations are constructed with all given (or understood) information and contain no major flaws.	Representations are constructed with all given (or understood) information and offer deeper insight due to choices made in how to represent the information.
AC Is able to evaluate the consistency of different representations and modify them when necessary Labs: 2-10, 12	No representation is made to evaluate the consistency.	At least one representation is made but there are major discrepancies between the constructed representation and the given experimental setup. There is no attempt to explain consistency.	Representations created agree with each other but may have slight discrepancies with the given experimental representation. Or there is inadequate explanation of the consistency.	All representations, both created and given, are in agreement with each other and the explanations of the consistency are provided.
AE Force Diagram Labs: 1-12	No representation is constructed.	Force Diagram is constructed but contains major errors such as mislabeled or not labeled force vectors, length of vectors, wrong direction, extra incorrect vectors are added, or vectors are missing.	Force Diagram contains no errors in vectors but lacks a key feature such as labels of forces with two subscripts vectors are not drawn from single point, or axes are missing.	The diagram contains no errors and each force is labeled so that it is clearly understood what each force represents. Vectors are scaled precisely.
AF Sketch Labs: 1, 4, 5, 7-9, 11, 12	No representation is constructed.	Sketch is drawn but it is incomplete with no physical quantities labeled, or important information is missing, or it contains wrong information, or coordinate axes are missing.	Sketch has no incorrect information but has either a few missing labels of given quantities. Subscripts are missing or inconsistent. Majority of key items are drawn.	Sketch contains all key items with correct labeling of all physical quantities have consistent subscripts; axes are drawn and labeled correctly.

	No Effort	Progressing	Expectation	Exemplary
AG Mathematical Labs: 2-5, 7-12	No representation is constructed.	Mathematical representation lacks the algebraic part (the student plugged the numbers right away) has the wrong concepts being applied, signs are incorrect, or progression is unclear.	No error is found in the reasoning, however they may not have fully completed steps to solve problem or one needs effort to comprehend the progression.	Mathematical representation contains no errors and it is easy to see progression of the first step to the last step in solving the equation. The solver evaluated the mathematical representation with comparison to physical reality.
CA Is able to identify the hypothesis to be tested Labs: 4, 5, 7-9, 12	No mention is made of a hypothesis.	An attempt is made to identify the hypothesis to be tested but is described in a confusing manner.	The hypothesis to be tested is described but there are minor omissions or vague details.	The hypothesis is clearly stated.
CC Is able to make a reasonable prediction based on a hypothesis Labs: 4, 5, 7-9, 12	No prediction is made. The experiment is not treated as a testing experiment.	• A prediction is made but it is identical to the hypothesis OR • Prediction is made based on a source unrelated to hypothesis being tested OR • is completely inconsistent with hypothesis being tested OR • Prediction is unrelated to the context of the designed experiment.	Prediction follows from hypothesis but is flawed because • relevant experimental assumptions are not considered and/or • prediction is incomplete or somewhat inconsistent with hypothesis and/or • prediction is somewhat inconsistent with the experiment.	A prediction is made that • follows from hypothesis, • is distinct from the hypothesis, • accurately describes the expected outcome of the designed experiment, • incorporates relevant assumptions if needed.
CD Is able to identify the assumptions made in making the prediction Labs: 4, 5, 7-9, 12	No attempt is made to identify any assumptions.	An attempt is made to identify assumptions, but the assumptions are irrelevant or are confused with the hypothesis.	Relevant assumptions are identified but are not properly evaluated for significance in making the prediction.	Sufficient assumptions are correctly identified, and are noted to indicate significance to the prediction that is made.
CE Is able to determine specifically the way in which assumptions might affect the prediction Labs: 4, 5, 7-9, 12	No attempt is made to determine the effects of assumptions.	The effects of assumptions are mentioned but are described vaguely.	The effects of assumptions are determined, but no attempt is made to validate them.	The effects of the assumptions are determined and the assumptions are validated.

	No Effort	Progressing	Expectation	Exemplary
CF Is able to decide whether the prediction and the outcome agree/disagree Labs: 4, 5, 7-9, 124, 5, 7-9, 12	No mention of whether the prediction and outcome agree/disagree.	A decision about the agreement/disagreement is made but is not consistent with the outcome of the experiment.	A reasonable decision about the agreement/disagreement is made but experimental uncertainty is not properly taken into account.	A reasonable decision about the agreement/disagreement is made and experimental uncertainty is taken into account.
CG Is able to make a reasonable judgment about the hypothesis Labs: 4, 5, 7-9, 12	No judgment is made about the hypothesis.	A judgment is made but is not consistent with the outcome of the experiment.	A judgment is made, is consistent with the outcome of the experiment, but assumptions are not taken into account.	A judgment is made, consistent with the experimental outcome, and assumptions are taken into account.
GA Is able to identify sources of experimental uncertainty Labs: 4, 5, 8, 10, 11	No attempt is made to identify experimental uncertainties.	An attempt is made to identify experimental uncertainties, but most are missing, described vaguely, or incorrect.	Most experimental uncertainties are correctly identified. But there is no distinction between random and experimental uncertainty.	All experimental uncertainties are correctly identified. There is a distinction between experimental uncertainty and random uncertainty.
GB Is able to evaluate specifically how identified experimental uncertainties may affect the data Labs: 4, 5, 8, 10, 11	No attempt is made to evaluate experimental uncertainties.	An attempt is made to evaluate experimental uncertainties, but most are missing, described vaguely, or incorrect. Or only absolute uncertainties are mentioned. Or the final result does not take the uncertainty into account.	The final result does take the identified uncertainties into account but is not correctly evaluated. The weakest link rule is not used or is used incorrectly.	The experimental uncertainty of the final result is correctly evaluated. The weakest link rule is used appropriately and the choice of the biggest source of uncertainty is justified.
GC Is able to describe how to minimize experimental uncertainty and actually do it Labs: 4, 5, 8, 10, 11	No attempt is made to describe how to minimize experimental uncertainty and no attempt to minimize is present.	A description of how to minimize experimental uncertainty is present, but there is no attempt to actually minimize it.	An attempt is made to minimize the uncertainty in the final result is made but the method is not the most effective.	The uncertainty is minimized in an effective way.
GD Is able to record and represent data in a meaningful way Labs: 1-12	Data are either absent or incomprehensible.	Some important data are absent or incomprehensible. They are not organized in tables or the tables are not labeled properly.	All important data are present, but recorded in a way that requires some effort to comprehend. The tables are labeled but labels are confusing.	All important data are present, organized, and recorded clearly. The tables are labeled and placed in a logical order.

	No Effort	Progressing	Expectation	Exemplary
GE Is able to analyze data appropriately Labs: 1-12	No attempt is made to analyze the data.	An attempt is made to analyze the data, but it is either seriously flawed or inappropriate.	The analysis is appropriate but it contains minor errors or omissions.	The analysis is appropriate, complete, and correct.
IA Is able to conduct a unit analysis to test the self-consistency of an equation Labs: 1-12	No meaningful attempt is made to identify the units of each quantity in an equation.	An attempt is made to identify the units of each quantity, but the student does not compare the units of each term to test for self-consistency of the equation.	An attempt is made to check the units of each term in the equation, but the student either mis-remembered a quantity's unit, and/or made an algebraic error in the analysis.	The student correctly conducts a unit analysis to test the self-consistency of the equation.

EXIT TICKET:

- ☐ Return the projectile and the carbon paper to the TA Table.
- ☐ Remove all tape from the floor.
- ☐ Wrap the plumb bob string around the cardboard spool.
- ☐ Store the plumb bob and string in its plastic bag.
- ☐ Return the goggles to their plastic bags.
- ☐ Place the plumb bob, tape measure, goggles and rulers in your equipment basket.
- ☐ Loosen the thumbscrew on the launcher so it can move freely.
- ☐ Quit any software you have been using.
- ☐ Straighten up your lab station. Put all equipment where it was at start of lab.
- ☐ Report any problems or suggest improvements to your TA.
- ☐ Have TA validate Exit Ticket Complete.