Lab 3. Adding Forces with a Force Table

Goals

- To describe the effect of three balanced forces acting on a ring or disk using vector addition.
- To practice adding force vectors graphically and mathematically in a simple geometry where two of the vectors are perpendicular.
- To describe the effect of three forces acting in arbitrary directions using the method of vector components.

Introduction

Vectors are defined as quantities that behave like displacements (distances with specified directions) when added together. All true vectors have the same mathematical properties as displacements. We know that if we walk a path due east for 1 km and then walk due north for 1 km, we end up 1.41 km from our starting point ("as the crow flies"). We are now northeast of our starting point (an angle of 45° north of east). If forces can be represented by vectors, they must have these same properties. A force of 1 N to the east added to a force of 1 N to the north should add together to give a net force of 1.41 N directed to the northeast at an angle of 45°. We want to demonstrate this property for the case of force vectors.

Balancing the force table

In this experiment the Pasco force table is used to apply three forces to a central ring or disk so that the central object is in so-called "static equilibrium," that is, the object has zero acceleration. The net force on the object is therefore also zero. These forces are exerted by the earth (gravity) acting on masses suspended from strings that run over pulleys. Each string is attached to the ring at the center of the table. When the ring is centered over the middle of the force table, the directions of the applied forces can easily be determined using the angle markings on the table itself.

Make sure that all of the strings lie along radial lines directed outward from the center of the force table. This usually requires sliding the knots that attach the strings to the ring at the center of the table. If the readings obtained on the inner indicator and on the outer indicator match, then the string is in a straight line.

When the forces acting on the ring are in static equilibrium, their vector sum is zero. To check that equilibrium has actually been attained, pull the ring slightly to one side. Then release the ring and check to see that the ring returns to the center. If not, adjust the size of the hanging masses until the ring always returns to the center after being moved in any direction. Careful attention will ensure good results.

A relatively easy-to-analyze set of three forces is produced when two of these forces are perpendicular. With this in mind, set up the force table as follows:

1. Position one pulley to apply a force at an angle of 270°. Call the force exerted by the string that runs through the this pulley $\mathbf{F_1}$.
2. Position a second pulley to apply a force at an angle of 180°. Call the force exerted by the string that runs through the second pulley $\mathbf{F_2}$.
3. Position the third pulley to apply a force at an angle of your choice between 10° and 80°. Hang a total mass of 105 g on the string going over this pulley. (Remember that the mass of the mass hanger is 5 g.) Call the force exerted by the string that runs through the third pulley $\mathbf{F_3}$.

Now use trial and error to find the correct masses to place on the first and second strings to center the ring exactly at the center point of the force table. When the central ring is centered and stationary, the sum of the three applied forces is zero. You can compute the magnitude of each force from the value of the hanging masses. ($F = mg$. In Pullman, the magnitude of g equals 9.80 m/s^2.) The direction of each force can be read from the angle marking on the force table.

A diagram showing how the three force vectors sum to zero is shown in Figure 3.1. Each vector is oriented along one of the force table strings. The length of each vector is drawn proportional to the magnitude of the force supported by the corresponding string.

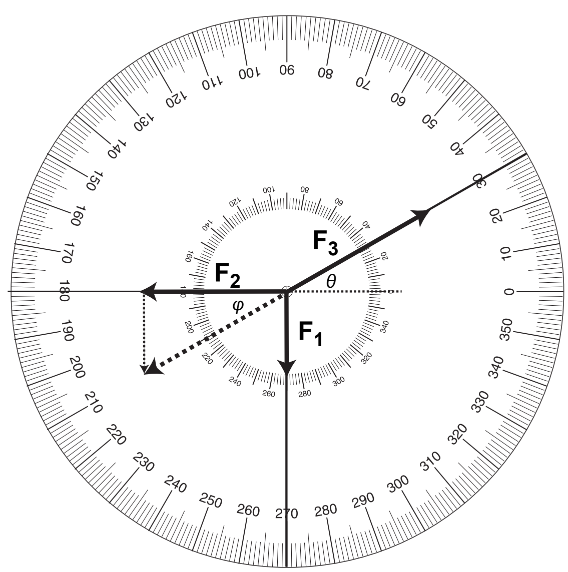

Figure 3.1. Diagram of three force vectors that add to zero, superimposed on an image of the Pasco Force Table.

Adding perpendicular forces graphically

Add the forces graphically using the same techniques that you would use for displacements. To do this, draw a full page set of x- and y-axes in your lab notebook and label them with suitable force units—here, newtons. (The origin will generally need to be near the center of the page.) Draw the three force vectors on your plot using the measured angles and force magnitudes. Now add the three force vectors graphically to find $\mathbf{F_1} + \mathbf{F_2} + \mathbf{F_3}$.

Is the measured sum of these three forces consistent with zero? To estimate the uncertainty in your sum, consider that the 0.5 g mass is the lightest mass at your disposal. Ideally, much smaller masses would be required to exactly balance the forces. In practice, you can expect to get within 0.25 g of this ideal mass on the end each string. Using this observation, make some reasonable estimates of the uncertainties in your experiment and your graph, and compare these with the magnitude of the vector sum of the three forces. If the magnitude of the sum of forces from your graph is less than the sum of the uncertainties, the sum is consistent with zero.

The sketch you have drawn is essentially a two-dimensional "free body diagram" of the forces on the ring of the force table. (The ring is the free body, and the diagram shows the forces on it.) In most of your physics work, graphs will not be used for quantitative calculations of the sum of forces. A free-hand sketch of these forces, however, is almost always necessary to make sure that you use the correct trig functions (sine versus cosine) for their vector components.

Adding perpendicular forces mathematically

Perpendicular vectors are used extensively in physics—especially in the context of vector components. Using an (x, y) coordinate system, any two-dimensional vector can be expressed as the sum of two other, perpendicular vectors: one parallel to the x-axis and the other parallel to the y-axis. One cannot add vectors with different directions by summing their magnitudes. But since the x-component of one vector is parallel to the x-component of any other vector (and ditto for their y-components), one can find the sum of two vectors by summing their x-components and y-components separately, and then reconstructing a new vector with the new x- and y-components. This "reconstruction" involves using the Pythagorean theorem to find the length of the new vector and simple trigonometry to find its direction.

If three forces sum to zero, the sum of the first and second forces is a force with the same magnitude as the third force, but with the opposite direction ($\mathbf{F_1} + \mathbf{F_2} = -\mathbf{F_3}$). If the positive x-axis points in the 0° direction and the y-axis points in the 90° direction, then $\mathbf{F_2}$ corresponds to the x-component of $-\mathbf{F_3}$ and $\mathbf{F_1}$ corresponds to the y-component of $-\mathbf{F_3}$. If forces add like vectors, the vector "reconstructed" from the vector components should be equal to $-\mathbf{F_3}$. Test this quantitatively by calculating the magnitude of the sum of $\mathbf{F_1}$ and $\mathbf{F_2}$ using the Pythagorean theorem. Then use trigonometry to calculate the angle along which this sum is directed. Compare the magnitude of the vector sum to the magnitude of $\mathbf{F_3}$. Is the direction of the sum of the first two forces opposite to that of $\mathbf{F_3}$? (The directions of two forces are opposite if their angles differ by 180°.) Include sample calculations in your lab notes.

Repeat your measurements at 10° intervals from 10° to 80°, using the Pythagorean theorem and trigonometry to calculate the magnitude and direction of the sum of the two forces at 270° and 180°. Make a suitable table (or tables) to record your data and calculations. Compare these magnitudes and directions with the magnitudes and directions of the forces applied to the third pulley. Do your results support the idea that forces are vectors, like displacements?

Finding the components of a vector

Use your data to calculate the x- and y-components of $\mathbf{F_3}$. If forces add like vectors, the x-component of $\mathbf{F_3}$ will equal $-\mathbf{F_2}$, and the y-component of $\mathbf{F_3}$ should equal $-\mathbf{F_1}$. Test this quantitatively for the forces measured at each angle above. Again, include sample calculations in your lab notes. Do your experimental results support the idea that a single force can be equivalently represented by components? (Compare any differences with the uncertainty you expect from the precision of your mass adjustments.)

Qualitative observations

One unintuitive aspect of vector addition is that adding two large vectors (two vectors with large magnitudes) often yields a much smaller vector. Other times, adding two vectors yields a larger vector, as one might expect. Look over your data and find a three pairs of relatively large vectors whose sum is smaller than either vector in the sum. Then find three pairs of vectors whose sum is larger than either vector in the sum. Note any patterns that you observe.

Adding forces with arbitrary directions

In this exercise, your TA will assign your lab group three arbitrary force directions. Find a combination of hanging masses that center the ring on the force table. Make sure that you do not exceed the 200 g limit on any one string as stated on the force table. Add the three forces vectorially by calculating and adding their components. (These forces will not be perpendicular, so the Pythagorean Theorem will not be much help.) As always, include sample calculations in your lab notes. Compare the calculated value of your net force to the expected value in this case. Can you account for the difference between the calculated net force and the expected net force on the basis of reasonable estimates of your experimental uncertainty? Be specific and quantitative in your explanation.

Although the Pythagorean theorem does not apply to triangles without a right angle, two other laws of trigonometry do: the Law of Sines and the Law of Cosines. Occasionally you will find that vector addition goes faster using one of these relations. For instance, you should be able to quickly solve the vector addition problem given to you by your teaching assistant using the Law of Sines.

Summary

Summarize the pertinent findings of your investigation. Cite specific results based on your experimental data.

Grading Rubric

	No Effort	Progressing	Expectation	Exemplary
AA Is able to extract the information from representation correctly Labs: 1-12	No visible attempt is made to extract information from the experimental setup.	Information that is extracted contains errors such as labeling quantities incorrectly, mixing up initial and final states, choosing a wrong system, etc. Physical quantities have no subscripts (when those are needed).	Most of the information is extracted correctly, but not all of the information. For example physical quantities are represented with numbers and there are no units. Or directions are missing. Subscripts for physical quantities are either missing or inconsistent.	All necessary information has been extracted correctly, and written in a comprehensible way. Objects, systems, physical quantities, initial and final states, etc. are identified correctly and units are correct. Physical quantities have consistent and informative subscripts.
AB Is able to construct new representations from previous representations Labs: 1-12	No attempt is made to construct a different representation.	Representations are attempted, but omits or uses incorrect information (i.e. labels, variables) or the representation does not agree with the information used.	Representations are constructed with all given (or understood) information and contain no major flaws.	Representations are constructed with all given (or understood) information and offer deeper insight due to choices made in how to represent the information.
AC Is able to evaluate the consistency of different representations and modify them when necessary Labs: 2-10, 12	No representation is made to evaluate the consistency.	At least one representation is made but there are major discrepancies between the constructed representation and the given experimental setup. There is no attempt to explain consistency.	Representations created agree with each other but may have slight discrepancies with the given experimental representation. Or there is inadequate explanation of the consistency.	All representations, both created and given, are in agreement with each other and the explanations of the consistency are provided.
AE Force Diagram Labs: 1-12	No representation is constructed.	Force Diagram is constructed but contains major errors such as mislabeled or not labeled force vectors, length of vectors, wrong direction, extra incorrect vectors are added, or vectors are missing.	Force Diagram contains no errors in vectors but lacks a key feature such as labels of forces with two subscripts vectors are not drawn from single point, or axes are missing.	The diagram contains no errors and each force is labeled so that it is clearly understood what each force represents. Vectors are scaled precisely.

	No Effort	Progressing	Expectation	Exemplary
AG Mathematical Labs: 2-5, 7-12	No representation is constructed.	Mathematical representation lacks the algebraic part (the student plugged the numbers right away) has the wrong concepts being applied, signs are incorrect, or progression is unclear.	No error is found in the reasoning, however they may not have fully completed steps to solve problem or one needs effort to comprehend the progression.	Mathematical representation contains no errors and it is easy to see progression of the first step to the last step in solving the equation. The solver evaluated the mathematical representation with comparison to physical reality.
BA Is able to identify the phenomenon to be investigated Labs: 1-3, 6, 10-12	No phenomenon is mentioned.	The description of the phenomenon to be investigated is confusing, or it is not the phenomena of interest.	The description of the phenomenon is vague or incomplete but can be understood in broader context.	The phenomenon to be investigated is clearly stated.
BE Is able to describe what is observed concisely, both in words and by means of a picture of the experimental setup. Labs: 1-3, 6, 11, 12	No description is mentioned.	A description is incomplete. No labeled sketch is present. Or, observations are adjusted to fit expectations.	A description is complete, but mixed up with explanations or pattern. OR The sketch is present but relies upon description to understand.	Clearly describes what happens in the experiments both verbally and with a sketch. Provides other representations when necessary (tables and graphs).
BG Is able to identify a pattern in the data Labs: 1-3, 6, 10-12	No attempt is made to search for a pattern	The pattern described is irrelevant or inconsistent with the data.	The pattern has minor errors or omissions. OR Terms labelled as proportional lack clarity- is the proportionality linear, quadratic, etc.	The patterns represents the relevant trend in the data. When possible, the trend is described in words.
BI Is able to devise an explanation for an observed pattern Labs: 1-3, 6, 10-12	No attempt is made to explain the observed pattern.	An explanation is vague, not testable, or contradicts the pattern.	An explanation contradicts previous knowledge or the reasoning is flawed.	A reasonable explanation is made. It is testable and it explains the observed pattern.
GD Is able to record and represent data in a meaningful way Labs: 1-12	Data are either absent or incomprehensible.	Some important data are absent or incomprehensible. They are not organized in tables or the tables are not labeled properly.	All important data are present, but recorded in a way that requires some effort to comprehend. The tables are labeled but labels are confusing.	All important data are present, organized, and recorded clearly. The tables are labeled and placed in a logical order.

	No Effort	Progressing	Expectation	Exemplary
GE Is able to analyze data appropriately Labs: 1-12	No attempt is made to analyze the data.	An attempt is made to analyze the data, but it is either seriously flawed or inappropriate.	The analysis is appropriate but it contains minor errors or omissions.	The analysis is appropriate, complete, and correct.
IA Is able to conduct a unit analysis to test the self-consistency of an equation Labs: 1-12	No meaningful attempt is made to identify the units of each quantity in an equation.	An attempt is made to identify the units of each quantity, but the student does not compare the units of each term to test for self-consistency of the equation.	An attempt is made to check the units of each term in the equation, but the student either mis-remembered a quantity's unit, and/or made an algebraic error in the analysis.	The student correctly conducts a unit analysis to test the self-consistency of the equation.

EXIT TICKET:

- ☐ Quit any software you have been using.
- ☐ Straighten up your lab station. Put all equipment where it was at start of lab.
- ☐ Return the masses to the appropriate bins on the table in the center of the room.
- ☐ Report any problems or suggest improvements to your TA.
- ☐ Have TA validate Exit Ticket Complete.