Lab 4. Newton's Second Law

Goals

- To determine the acceleration of a mass when acted on by a net force using a Smart Pulley sensor to acquire the data. Two cases are of interest: (a) the mass of the system is fixed and the net force is varied, and (b) the net force is fixed and the mass of the system is varied.
- To make and analyze appropriate graphs of the resulting data that test the validity of your application of Newton's Second Law of Motion to this system.

Introduction

Newton's First Law states that no change in the motion of an object takes place in the absence of a net force. In other words, the acceleration (change in velocity) of an object is zero unless there is a net force. But how is the acceleration related to the force? Newton's Second Law deals with this relationship. Experimentally we will explore the relationship between the net force on an object, the mass of the object, and the acceleration of the object due to the force. Newton's Second Law of Motion makes some definite predictions that you can test.

Be sure to level the track carefully before you take any data.

Accelerating a fixed mass with a variable force

Behind Newton's Second Law is the assumption that an object (or group of objects) can be modeled as a point with a definite mass and location, that moves along a well defined trajectory through space with a definite velocity and acceleration. A group of objects can often be modeled as a point if it moves together rigidly, without rotation or stretching—and if its mass does not change. When we apply Newton's Second Law to a suitable group of objects, we call the group "the system."

In this experiment, a small mass is connected to the cart by a string that hangs down over a pulley. To apply Newton's Second Law to this situation, the system mass (the m in $F = ma$) and the net force on the object (the F in $F = ma$) must be clearly specified. Draw a free body diagram of the cart, the pulley, and the slotted masses, and use it to express the net force and the system mass in terms of the masses of each part of the system. Ignore friction for the time being. Check that each part of the system has the same acceleration (the a in $F = ma$). This simplification is possible because both the cart and the hanging mass move as if they were glued together (assuming that the string is not stretchy). Your free body diagram and any text or mathematics needed to support your expressions for F and m must appear in your lab notes.

Using the Pasco Smart Pulley, the computer can take the time and displacement measurements required to compute the displacement, velocity, and acceleration of the system as functions of time. The Smart Pulley section of the Computer Tools Supplement at the back of the lab manual has specific instructions on connecting the pulley to the computer. You may wish to plot all three of these quantities to determine the best method of determining the acceleration. If the acceleration is constant, the slope of the velocity-time graph will be a straight line whose slope equals the acceleration. The slope of a straight line is easily found along with a useful uncertainty estimate using the ÒFitÓ tool in the toolbar at the top of the graph window. Does the slope of the velocity-time graph yield a more precise acceleration value than simply taking an average acceleration value directly from the acceleration-time graph? Compare the two methods for a single data run. Explain your findings in your lab notes.

Although the computer fitting routine gives an uncertainty estimate for the slope of a graph based on one data run, relying on only one data run for each configuration of the system is risky. Some experimental conditions may remain constant for a single run, but vary from run to run. In this case, the uncertainty for a single run (reported by DataStudio) can grossly underestimate the uncertainty in the measured quantity. Incorrect uncertainties can result in false conclusions about the consistency of two sets of data or calculated values. The preferred approach is to make at least three separate runs for each configuration, and to use the standard deviation of the mean for these acceleration measurements as the measure of uncertainty in acceleration.

For each run, the acceleration is simply the slope of the velocity-time graph. Now calculate the average, or mean value, and the standard deviation of the average acceleration for each configuration. (Refer to the Uncertainty-Graphical Analysis supplement in the back of your lab manual for a discussion and definition of standard deviation.) The experimental standard deviation for repeated runs is typically about ten times larger than the standard deviation determined from a single run. This is strong evidence for the presence of uncontrolled experimental factors that vary from run to run. Since reproducibility is the key consideration here, the larger value of standard deviation is by far more realistic.

Vary the total hanging mass from 10 g to 60 g in 10 g increments. To keep the mass of the system as a whole fixed, make sure that any unused hanging masses ride in the cart. It is important that the hanging mass not exceed 60 g. The velocities achieved using larger masses can be sufficient to damage the equipment when the cart strikes the end of the track.

Now make a graph of the force (vertical axis) as a function of the acceleration (horizontal axis). Include error bars on your graph that correspond to the uncertainties in your measured acceleration values.

Newton's Second Law states that the acceleration of a system with constant mass is directly proportional to the net force and that the acceleration of an object under a constant net force is inversely proportional to its mass. If Newton's Second Law is correct, then you should be able to compute the mass of the system from the slope of your graphical analysis. Ask your TA for assistance if it is not clear how to proceed.

Compare the value of the system mass determined from your data, $m_{F/a}$ with the total system mass measured using an electronic balance at the back of the room, m_{bal}. To make this comparison quantitative, compare the difference between the two mass estimates (the "error", $m_{dif} = |m_{F/a} - m_{bal}|$) to the expected uncertainty in the error, $u(m_{dif})$. From the Uncertainty and Graphical Analysis appendix to the lab manual,

$$u(m_{dif}) = \sqrt{u(m_{F/a})^2 + u(m_{bal})^2} \tag{4.1}$$

where $u(m_{F/a})$ is the uncertainty of the acceleration from your acceleration data and $u(m_{bal})$ is the uncertainty in your balance reading, which you estimate from the least significant digit of your reading.

If the slope of your acceleration versus force plot is less than two times $u(m_{dif})$, it is fair to say that your acceleration data are consistent with Newton's Second Law and your mass measurement. If the ratio is greater than three times $u(m_{dif})$, carefully examine your procedures and analysis for sources of systematic error. Include the results of your examination in your notes.

Accelerating a variable mass with a fixed force

By measuring the acceleration of the system as a function of net force, while holding the system mass constant, we can study the affect of net force on acceleration. Similarly, by keeping a fixed value of mass hanging on the string, while keeping the net force constant, we can also observed the effect of system mass on acceleration. Hang 50 or 60 g on the end of the string for this experiment. Additional rectangular steel bars (two maximum!) can be placed on top of the cart to increase the mass of the whole system. Plot the acceleration (vertical axis) as a function of the system mass (horizontal axis). Qualitatively, what happens to the acceleration when the mass increases?

Newton's Second Law states that the acceleration of a system with constant net force is inversely proportional to its mass. In mathematical form this looks like:

$$F = ma \quad \text{or} \quad a = \frac{F}{m} = F\frac{1}{m} \tag{4.2}$$

The relationship between acceleration and mass with a constant force is trickier than the relationship between acceleration and force, with constant system mass. The mathematical relation between the variables is hard to guess when the graph curves. On the other hand, straight-line graphs are simple to identify and analyze. Is there a way of plotting the accelerations and mass data that is consistent with Newton's second law, but so that the graph is a straight line? You may need help from your TA. When you have sorted this out, you should be able to compute the value of the net force applied to the system.

Quantitatively compare the value of applied net force calculated from your graph to the gravitational force acting on the hanging mass, using your experimental uncertainties. That is, compare F_{dif}, the magnitude of the difference between the force values, to $u(F_{dif})$, the uncertainty in the difference between two force measurements.

Real world effects

Are there reasons that your results might not be totally consistent with the predictions based on Newton's Second Law? Have we included all the forces acting on the system? What effect does the ubiquitous force of friction have? Examine your graph of acceleration versus force carefully. Newton's Second Law says that the net force and the resulting acceleration are directly proportional, meaning that zero net force produces zero acceleration. Does your graph show this to be the case? Look carefully! Explain how and why the graph might deviate slightly from this ideal. Does the presence of friction invalidate your graphical determination of the system mass? Ask your TA whether your explanation is reasonable.

Your acceleration versus system mass measurements are also affected by friction; however, you do not have enough data to justify pursuing the details.

Conclusion

Is your data consistent with the predictions of Newton's Second Law? To support your conclusion, you must compare any observed discrepancy with your experimental uncertainty. Now a question to just think about. How was Newton able to formulate the Second Law of motion? Did he have access to equipment comparable to what you used today? Was he compelled to formulate the Second Law based on his experimental results?

Before you leave the lab please: Quit all computer applications that you may have open. Collect the slotted masses that were hung on the string. Please make sure that all of them are there! Report any problems, equipment or otherwise, to your TA.