Lab 9. RC Circuits

Goals

- To appreciate the capacitor as a charge storage device.
- To measure the voltage across a capacitor as it discharges through a resistor, and to compare the result with the expected, theoretical behavior.
- To use a semilogarithmic graph to verify that experimental data is well described by an exponential decay and to determine the decay parameters.
- To determine the apparent internal resistance of a digital multimeter.

Introduction

A diagram of a simple resistor-capacitor (RC) circuit appears in Figure 9.1. A power supply is used to charge the capacitor. During this process, charge is transferred from one side of the capacitor to the other. A digital multimeter set on a voltage scale behaves in a circuit like a large (in ohms) resistor. When the power supply is disconnected from the capacitor, charge "leaks" from one side of the capacitor, through this resistor, back to the other side of the capacitor, until no voltage appears across the terminals of the capacitor.

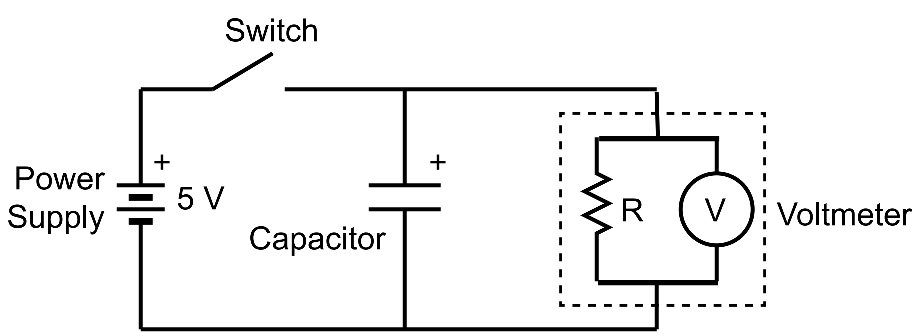

Figure 9.1. Diagram of RC circuit and power supply.

The power supply in Figure 9.1 is represented by a battery. Note that the positive output of the power supply is connected to the plate of the capacitor marked with a plus sign. The capacitors used in this experiment make use of a thin layer of dielectric material that forms by a chemical (electrolytic) reaction when the appropriate voltage is placed across the capacitor. These layers can be quite thin and uniform. Electrolytic capacitors can be made inexpensively and are widely used in power supplies. As you may remember from chemistry, the sign of the voltage is critical in electrolytic reactions. Make sure that the plus end of the capacitor is connected to the plus output of the power supply in your circuit.

The voltmeter in Figure 9.1 is enclosed by a dashed line. The voltage sensing circuit is represented by a circle with a "V" inside. All voltmeters have resistance, and this resistance is represented by the resistor symbol inside the box. Our goal is to measure the value of this resistance, R.

Theory

We plan to monitor the voltage across the capacitor as a function of time after the switch is opened. The functional form of this dependence can be derived by circuit analysis using Kirchhoff's loop law. A simplified diagram of the circuit after the switch is opened is shown in Figure 9.2. For the purposes of analysis, we indicate the positive direction of current by an arrow. This choice defines the sign of positive charge, Q on the capacitor. (Q is positive when the arrow points toward the plate with positive charge.) It also defines the positive direction of ΔV. (ΔV is positive when the arrow points in the direction of *increasing* potential.)

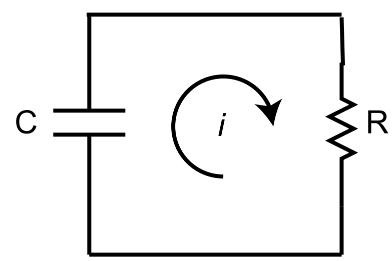

Figure 9.2. Diagram of RC circuit and power supply.

Because our circuit contains no source of emf, the only potential differences in the circuit appear across the capacitor and across the resistor. By Kirchhoff's loop rule, the total potential change as you go all the way around the loop (ΔV_{loop}) must be zero. Let the potential difference across the capacitor be ΔV_C and the potential difference across the resistor be ΔV_R. Then

$$\Delta V_{loop} = \Delta V_C + \Delta V_R = 0 \quad . \tag{9.1}$$

In the presence of a positive charge Q on the capacitor, ΔV_C must be negative, as the potential drops as one moves from a positively charged plate to a negatively charged plate. The capacitance, C, of a capacitor is defined so that the magnitude of ΔV_C is Q/C. Therefore $\Delta V_C = -Q/C$. Likewise, potential drops as charge passes through a resistor in the direction of positive current, I. The magnitude of this drop is given by Ohm's law, so that $\Delta V_R = -IR$. Substituting these relations into Kirchhoff's loop rule yields

$$\Delta V_C + \Delta V_R = -\frac{Q}{C} - IR = 0 \quad \text{or} \quad I = -\frac{Q}{RC} \tag{9.2}$$

When the switch in Figure 9.1 is closed, a positive charge $Q = \Delta V \times C$, where $\Delta V = 5$ V, is on the top plate of the capacitor. According to our choice of positive direction, both Q and ΔV_C are initially negative. While negative charges and potential differences may appear to be inconvenient, they make no difference as far as the math is concerned. It is safe to choose the direction of positive current arbitrarily and work from there. With a positive negative charge on the top plate of the capacitor, a positive current I will flow through the resistor. In this case, a positive current will *decrease* the magnitude of Q, but since Q is initially negative, the corresponding dQ/dt is positive. Therefore $I = dQ/dt$. One of the handy features of Kirchhoff's loop rule is that I always equals dQ/dt if you set it up correctly. This is not always true for other approaches to circuit analysis. This relation allows us to reduce Kirchhoff's loop rule to a simple equation with one derivative.

$$\frac{Q}{C} = -R\frac{dQ}{dt} \quad \text{or} \quad \frac{1}{Q}\frac{dQ}{dt} = -\frac{1}{RC} \tag{9.3}$$

Equation 9.3 is a simple differential equation. The expression on the right hand side of Equation 9.3 is easily integrated, but its solution depends on the initial charge across the capacitor. If we start with an initial charge Q_0 on the capacitor, the charge as a function of time, $Q(t)$ is given by

$$Q(t) = Q_0 \exp\left(-\frac{t}{RC}\right) \tag{9.4}$$

Since the voltage across the capacitor is directly proportional to the charge stored on it at any instant of time, the voltage difference ΔV_C can be written as

$$\Delta V_C(t) = \frac{Q_0}{C}\exp\left(-\frac{t}{RC}\right) = \Delta V_0 \exp\left(-\frac{t}{RC}\right) \tag{9.5}$$

where ΔV_0 is the initial voltage across the capacitor. The voltmeter measures this voltage directly. When t/RC equals one (that is, when $t = RC$), the voltage has decayed to $1/e$ of its original value. The quantity RC is called the time constant of the decay process. When R and C are expressed in the SI units of ohms and farads, respectively, the RC time constant has units of seconds.

Before proceeding, verify that the expression for $Q(t)$ given above is really a solution to the differential equation preceding it. Include this verification in your lab notes.

Experiment

Set up the circuit shown in the diagram using the Fluke voltmeter. Have your TA check it before continuing. With the power supply set to 5.0 V, close the switch and charge the capacitor. When the switch is opened, the voltage begins to decrease. Try it! Now read the initial voltage, then open the switch and read the voltmeter at 10-second intervals until the voltage is less than 10% of its original value. Repeat this process two more times, making sure that the initial voltage is the same for all three trials.

Analysis

To determine the resistance of the voltmeter, make a table in Excel listing the observed voltages and times for your three data sets. (Enter the time data only once.) Add an additional column in the spreadsheet to average the three voltage readings corresponding to each time. Then calculate the standard deviation of the mean of these three values for each time. (Refer to the Uncertainty/Graphical Analysis Supplement at the back of the lab manual for additional details.) The standard deviation of the mean gives an estimate of the uncertainty in the individual voltage measurements.

Plot the average voltage values as a function of time with error bars. Your error bars should look like the one in Figure 9.3. The circle in Figure 2 marks the calculated average value for one data point, y_{avg}. The top and bottom bars mark the maximum and minimum values (y_{max} and y_{min}) on either end of the range of y-values within one standard deviation (σ) of y_{avg}. Thus the top bar is located at $y_{max} = y_{avg} + \sigma$, while the bottom bar is located at $y_{min} = y_{avg} - \sigma$. Get help from your TA if you aren't sure how to plot error bars in Excel.

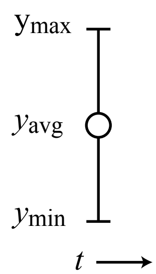

Figure 9.3. Diagram of a data point with upper and lower error bars.

Clearly this is not a linear graph. To determine the value of RC, one could perform an exponential curve fit using Excel. However, Excel's curve fit function does not provide the uncertainty estimate we need. Excel's Regression function will provide an uncertainty, but it requires a linear function. Taking the natural logarithm of both sides of Equation 9.5 will produce the linear equation we need.

$$\ln \Delta V_C(t) = ln(\Delta V_0) - \frac{t}{RC} \tag{9.6}$$

A plot of $\ln[\Delta V_C(t)]$ vs t should produce a line of slope $-t/RC$ and that equals $\ln(\Delta V_0)$ at time $t = 0$. The intercept is not very useful this case, except to confirm our knowledge of ΔV_0. The slope, however, gives us $1/RC$. Assuming that the value of C marked on the capacitor is reasonably accurate, we can calculate R, the internal resistance of the meter. In practice, the uncertainty in the marked value of C is ±20%. With this and the uncertainty (standard error) in the slope given by Excel's Regression feature, we can estimate the uncertainty in R.

A graph with the logarithm of one quantity on one axis versus a non-logarithmic quantity on the other axis is called a semilog graph. (The logarithm appears on only one of the two axes.) Plot a semilog graph of your data. Again include the error bars with each plotted point. Does your graph support the hypothesis that the relationship between the voltage and time is an exponential function? Using the value of C marked on your capacitor, compute the value of the R of the voltmeter and compare it to the value from the manufacturer's specification.

Internal resistance of an inexpensive voltmeter

Repeat your measurements of $\Delta V_C(t)$ versus time using the relatively inexpensive (smaller, red or black) digital voltmeter at your lab station. Repeat the analysis above to determine its internal resistance. How does it compare with the internal resistance of the relatively expensive Fluke digital voltmeter?

The internal resistance of a voltmeter is one measure of its quality. To measure the potential difference across a component with a high resistance, the internal resistance of your voltmeter should be much higher than the resistance of the component. A voltmeter with a high internal resistance can be used in applications where the measurement error of a meter with a low internal resistance would be unacceptably high.

Summary

Begin by "filling in the blanks" of the argument for a simple exponential function being a straight line when plotted semi-logarithmically. Then state your findings clearly, succinctly, and completely.

Before you leave the lab please:
- Turn off the power to all the equipment, including the battery-powered digital voltmeters.
- Please put all leads and small components in the plastic tray provided.
- Report any problems or suggest improvements to your TA.