Physics 201

Lab Manual

Summer 2018

Manual Owner ________________
Lab Section Number __________
TA Name _____________________
TA e-mail ____________________
Lab Group Rotation Number _____

Version: Summer ’18, 201, May 31, 2018
# Lab Schedule

## Class 201

<table>
<thead>
<tr>
<th>No.</th>
<th>Date</th>
<th>Lab Title</th>
<th>Knight Chapter</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>June 5</td>
<td>Tutorial: Intro to Lab work</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>Experiments, Documentation, Uncertainty, Excel</td>
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</tr>
<tr>
<td>1</td>
<td>June 7</td>
<td>Free Fall</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>June 12</td>
<td>Projectile Motion</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>June 14</td>
<td>Newton’s Second Law</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>June 19</td>
<td>Friction</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>June 21</td>
<td>Simple Pendulum</td>
<td>15</td>
</tr>
<tr>
<td>6</td>
<td>June 26</td>
<td>Newton’s Third Law, Impulse and Momentum</td>
<td>7 &amp; 11</td>
</tr>
<tr>
<td>7</td>
<td>June 28</td>
<td>Work and Energy</td>
<td>9 &amp; 10</td>
</tr>
<tr>
<td>8</td>
<td>July 3</td>
<td><strong>No Labs (Fourth of July)</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td>July 5</td>
<td>Ballistic Pendulum</td>
<td>10 &amp; 11</td>
</tr>
<tr>
<td>9</td>
<td>July 10</td>
<td>Buoyancy</td>
<td>14</td>
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<tr>
<td>10</td>
<td>July 12</td>
<td>Rotational Dynamics</td>
<td>12</td>
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<tr>
<td>11</td>
<td>July 17</td>
<td>Spring-Mass Oscillations</td>
<td>15</td>
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<tr>
<td>12</td>
<td>July 19</td>
<td>Vibrating Strings</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>July 24</td>
<td><strong>Lab Exam</strong></td>
<td></td>
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<tr>
<td></td>
<td>July 26</td>
<td><strong>No Labs (Finals)</strong></td>
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**NOTE:** Lab rooms change frequently. Consult the bulletin boards across the halls from the elevators on the second, third, or fourth floors of Webster Physical Sciences Building for the latest lab room assignments.

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Lab Syllabus

Point of contact:

Lab Director  Dr. Marc Weber
Office       Webster Hall Room 527
Office hours M,W,F 10:00 am to 11:00 am
             (walk-in are always welcome)
Phone        335-7872
Email        physics.labs@wsu.edu

Prerequisites: MATH 107 or 108 (trigonometry) with a grade of C or better, a minimum ALEKS math placement score 75%, or passing MATH 140, 171, 202, or 206. Algebra/trigonometry-based physics; topics in mechanics, wave phenomena, temperature, and heat; oriented toward non-physical science majors.

Learning goals: To experimentally probe concepts learned from the classroom and related materials; to reduce settings from the natural world to basic testable configurations; to carry out the tests, analyze the observations and conclude by comparing results to initial hypotheses; to document the activities in lab-notes, sketches, and diagrams; to acquire data with various sensors and computer support; to analyze results with Excel and other software and fit them with linear regression tools; to record results and findings; to compose formal reports; to work with a lab partner; to manage limited time allotted for each experiment.

Disability: Reasonable accommodations are available for students with documented disabilities. If you have a disability and need accommodations to fully participate in the lab or the lecture, call or visit the Access Center (Washington Building Room 217, Phone: 335-3417, e-mail: access.center@wsu.edu, URL: [http://accesscenter.wsu.edu](http://accesscenter.wsu.edu)). All accommodations are to schedule an appointment with an Access Advisor. All accommodations must be approved through the Access Center. Notify both your lab director and the lecture instructor during the first week of lecture concerning any approved accommodations. Late notification may cause the requested accommodations to be unavailable.

Campus Safety: Classroom and campus safety are of paramount importance at Washington State University, and are the shared responsibility of the entire campus population. WSU urges students to follow the “Alert, Assess, Act.” protocol for all types of emergencies and the “Run, Hide, Fight” URL: [https://oem.wsu.edu/emergency-procedures/active-shooter](https://oem.wsu.edu/emergency-procedures/active-shooter) response for an ac-
tive shooter incident. Remain ALERT (through direct observation or emergency notification), ASSESS your specific situation, and ACT in the most appropriate way to assure your own safety (and the safety of others if you are able). Please sign up for emergency alerts on your account at MyWSU. For more information on this subject, campus safety, and related topics, please view the FBI’s Run, Hide, Fight video (URL: https://oem.wsu.edu/emergency-procedures/active-shooter/) and visit WSU Safety: URL https://oem.wsu.edu/about-us/.

Academic Integrity: Academic dishonesty, including all forms of cheating, plagiarism, and fabrication, is prohibited as defined in the Standards of Conduct for Students, WAC 504-26-010(3) (URL: http://apps.leg.wa.gov/WAC/default.aspx?cite=504-26-010). The instructor reserves the right to take appropriate action. A failing grade in the class may result. Incidents of academic dishonesty will be referred to the Office of Student Conduct. If you have any questions about what is and is not allowed in this course, you should ask the course instructors before proceeding.

A partial list of prohibited conduct appears in Washington Administrative Code (WAC) Section 504-26 (http://apps.leg.wa.gov/wac/default.aspx?cite=504-26). Of special importance to the laboratories is the false reporting of data, experiment results, information, or procedures. The data and results in your lab notebook and reports must result from your own work in the current semester. Reporting data acquired by others (including your lab partner if you did not contribute) or in previous semesters is academically dishonest. Fabrication of results, information, or procedures, and sabotaging other students’ work is also prohibited. Likewise, sharing information about the end-of-semester lab exam with students yet to take the exam is prohibited. Violations of this policy will affect your lab grade and may be reported to the Student Conduct Committee as instances of academic dishonesty.

Attendance, conduct, exams, evaluation, grading

Attendance: Attendance is mandatory. Summer classes are conducted in a very condensed time frame. Missing a lab results in a zero score for that lab. Make-up labs are not granted. In lieu, the two lowest scoring lab grades will be dropped.

Student conduct: Academic integrity is expected of all students (see above). During the experiments students are expected to work in teams of two. The lab partners will jointly carry out all lab activities and at times may even share data among all stations in the room. Data and graphs should be printed out in duplicate or saved for each of the team. However, data analysis and evaluation and reporting in lab-notes or formal reports will be an individual activity. For more information regarding lab notes and reports, refer to the “Lab Work, Notes and Reports” section immediately following the syllabus.

The lab work is conducted in small teams. We expect students to be on time to labs and lab exams and to mute their cell phones for the duration. Being late may drag your lab-partner’s grade down. Many physics concepts are subtle, and even the most intelligent students make mistakes. In this environment, it is important that students be willing to ask questions if they don’t understand what their lab partners say or do. To this end, we require that students and teaching assistants alike avoid behavior that discourages communication. This includes threats and insults. Students who
repeatedly disrupt lab may be directed to leave the room and may receive a zero grade for that week’s lab.

Once the lab is complete, students are expected to tidy up and leave the station as they found it.

**Submission of work for evaluation:** Records (i.e. the lab-notes) of work completed during lab, including tutorials, must be turned in to your teaching assistant before you leave the room. At times students may be permitted to complete assignments after the lab session. Work completed after lab must be turned in to the TA no later than at the start of the next session. Failure to turn in lab-notes will result in low scores.

**Exam:** An exam is administered during the last week before the lecture final exam within your regularly scheduled laboratory section. Do not skip the exam. This is the time to demonstrate your record keeping skills. The exam may include any experimental techniques, methods of data analysis, and/or concepts covered during the semester. You may need to refer to your graded lab work for the current semester and the lab manual during the exam. You may not refer to the textbooks or other references. Work on the exam is individual (no lab partners). Bring your calculator.

**Evaluation:** The class grade is composed of 75% from the lecture and 25% laboratory section. See your instructor regarding the lecture component. The experiments include

- 1 introductory tutorial with up to 20 points,

- lab notes from 12 experiments with up to 100 points each with options for bonus points such as homework

- 1 final laboratory exam with up to 100 points.

Each will be scored on a 0 to 100 scale. The lowest two scoring lab notes from experiments will be dropped. Not attending a laboratory experiment or not submitting lab-notes results in a zero score for that lab. Sum up the 10 highest scoring labs and add all bonus points. Cap this at 1000, the maximum lab score. There will be no curving. The teaching TA may not be the person grading your lab.

**Grading:** The grades are compiled as follows: For each component item (first column) a maximum number of points are given (second column). They contribute to the lab grade according to column 3 (lab only) and the overall grade in the class (column 4).
item | max. points | lab grade % | class grade % | comment
--- | --- | --- | --- | ---
tutorial | 50 | 4 | 1 | 1st session
10 of 12 labs | 1000 | 76 | 19 | 10 highest scoring labs of 12 plus bonus
tutorial & labs | 1050 | 80 | 20 |
lab exam | 100 | 20 | 5 |
laboratories | 100 | 25 | see lab director |
lecture | n/a | n/a | 75 | see instructor |
total | | | 100 | total grade |

By Physics and Astronomy Department policy, should the lab grade fall below 50% lab grade, the student will receive a failing F grade as the class grade independent of lecture performance. In the other end of the scale, should the lab grade be 80% lab grade and higher but the student fails the lecture, he/she may choose to "carry over" the lab grade when the course is retaken. In addition a high laboratory grade in excess of 80% lab grade. To take advantage of this option, the student must notify the lab director no later than the first week of the semester that you are repeating the course. 100-level labs cannot substitute for 200-level labs.

Questions regarding grades on lab assignments need to be discussed with your teaching assistant within two weeks of receiving the graded material (earlier at the end of the semester). Final lab grades will be posted on the bulletin board on the 3rd floor of Webster Hall. To affect the lab grade submitted to your instructor, changes must be made by Friday morning of Final Exam week. Errors that affect your physics course grade will be corrected after final grades are submitted to the Registrar, if necessary.

### Safety resources

General information on campus safety is posted at http://safetyplan.wsu.edu—the Campus Safety Plan. Information on how to prepare for potential emergencies is posted on the Office of Emergency Management web site (http://oem.wsu.edu/). Safety alerts and weather warnings are posted promptly at the WSU Alerts site (http://alert.wsu.edu/). Urgent warnings that apply to the entire University community will also be broadcast using the Campus Outdoor Warning System (speakers mounted on Holland Library and other buildings) and the Crisis Communication System (e-mail, phone, cell phone). For this purpose, it is important to keep your emergency contact information up-to-date in MyWSU. To enter or update this information, click the “Update Now!” link in the “Pullman Emergency Information” box on your MyWSU home page (https://my.wsu.edu/).

Safety information that applies to the laboratories appears in the Lab Manual. Your teaching assistant will also present any safety information that applies to the current laboratory at the beginning of the laboratory. Students are expected to conduct themselves responsibly and take no unnecessary risks. Students who disobey the safety instructions will be directed to report to the lab director. All accidents and injuries must be reported promptly to your teaching assistant.

An Emergency Guide is posted by one door of each lab room. If faced with an emergency, follow the “Alert, Assess, Act,” protocol: Remain ALERT (through direct observation or emergency notification), ASSESS your specific situation, and ACT to ensure your own safety and the safety
of those around you. In case the fire alarm sounds, leave the building promptly in an orderly fashion. If you are not on a ground floor, use the stairs. Do not use the elevators. After exiting the building, gather across from the basketball court behind Waller Hall (down the hill, south of Webster Hall, see Figure 1) with the other members of your lab. A representative of the Department of Physics and Astronomy will tell you when it is safe to re-enter the building. If this does not happen before the end of the lab period, you are free to leave for your next class. If the emergency involves an active shooter, your options are to RUN, HIDE, or FIGHT (https://www.youtube.com/watch?v=5VcSwejU2D0). Each lab room door can be locked from inside in case of a lock down.

Figure 1. Physics and Astronomy assembly point. In case of a fire alarm, exit the building and gather at the basketball court behind Waller Hall. Use the stairs. Do not use the elevators in case of fire. A department representative will tell us when it is safe to re-enter the building.

Changes

The lab director reserves the right to correct errors in the syllabus and to modify lab schedules and room assignments. The lab director has delegated some authority to modify assignments and due dates to your teaching assistant. This helps ensure that you are graded according the criteria stated during your lab meeting.
“It is very necessary that those who are trying to learn from books the facts of physical science should be enabled by the help of a few illustrative experiments to recognize these facts when they meet with them out of doors.” James Clerk Maxwell “Introductory lecture on experimental physics” in “The Scientific Papers of James Clerk Maxwell”, W.D. Niven editor, Volume II, pp 242 to 243, Cambridge University Press (1890).

Just like when learning to drive a car, to perform open heart surgery or to acquire pretty much any skills, book knowledge is insufficient. Hands on practice is makes the driver, surgeon, skier, scientist or engineer. To deepen the understanding of what you learn in the lecture, you will carry out some experiments. “An experiment is a question which science poses to Nature, and a measurement is the recording of Nature’s answer.” Max Planck in “The Meaning and Limits of Exact Science”, Science (30 Sep 1949), 110, No. 2857, 325. You will develop some skills and concepts of this interaction with Nature. They are best learned in the laboratory. These skills include posing questions, build models and devise experiments, collect and analyze data, and critically comparing results to predictions or theory. Keeping good laboratory and composing formal reports of results helps communicating with peers. You will need some background on statistics to perform quantitative testing of hypothesis. These skills apply to quantitative work in many fields, including health- and life-sciences, mathematics, and engineering and chemistry. Many students in introductory physics courses have had lab experience in chemistry and other disciplines. We build on that experience. Your teaching assistants will not be as specific about their requirements as your chemistry teaching assistants were. You will often be expected to figure things out on your own in consultation with your lab partner, and will be graded on the quality of those decisions. Since you will be working more independently, you will be required to document your work more carefully, with less input from your teaching assistant.

To accomplish these goals, you will be expected to:

- Pose a question to Nature.
- Build simple physical models that incorporate lecture material.
- Design and perform simple experiments to test or improve these models.
- Employ representative software packages to collect and analyze data.
• Document your experimental methods, results, and data analysis in a lab notebook.
• Evaluate and compare results using uncertainties.
• Communicate your work in writing (short and long formal assignments).

**Student responsibilities**

You should be prepared for the laboratory activities. At times, the laboratory material may not have been covered in class. You should

• Read the syllabus. The regulations/guidelines in this syllabus take precedence over any oral commitments that may be made. The lab director is responsible for the final interpretation of these policies.

• Before each lab, read the relevant chapter of the lab manual, particularly if the material has not already been covered in lecture. Review related course material

• Arrive at your lab on time. Note that the lab rooms change from week to week. The room schedules are posted on the bulletin boards across from the elevators on the second, third and fourth floors of Webster Hall.

• Bring your lab manual, calculator, pen and pencil, a lab notebook with carbonless copies, and scratch paper to lab each week.

• Come prepared to perform mathematical calculations based on the level of math appropriate for the course. This includes algebra, geometry, and trigonometry. For Physics 201 and 202, calculus is also required.

• Do not bring food, tobacco, or beverages into a lab room.

• If you miss or expect to miss a lab due to sickness or another valid reason, arrange for a make-up laboratory as described in the Requests for Make-Up Laboratories section of this syllabus.

During the laboratory session, your TA will provide introductory material. She/he is there to guide and nudge towards sound experimental practice. The TA will not provide plain answers to you but will respond with counter questions. If specific equipment must be set up or malfunctions, your TA will help or call for further assistance. You should

• Note down the date, class and section, the laboratory experiment name, your lab partner.

• Don’t panic, be creative, trust your reasoning skills. Interact with your lab partner; bonus credit may lurk around.

• Use only carbonless copy laboratory notebooks with page numbers.

• Complete all labs and the lab exam.

• Computers have crashed. If at all possible, record all measurement data and results in your lab notes! You and your lab partner should each have all data.
• Make sure that all submitted work is your own. Academic dishonesty is not tolerated and is grounds for failing the course.

• Submit the original of your lab notes to your TA. This will be part of your grade. Retain the copy to complete any take home assignments.

After the laboratory session

• Complete all writing assignment and any formal reports as requested.

• Submit your work in the mail slot of your section on the 3rd floor of Webster.

• Do so on time! Do so in the correct mail slot. Failure may result in loss of credit.

Written communication of laboratory work

Records of laboratory work take at least two forms. The lab notebook is a protocol of all activities in the lab. Formal reports communicate key findings and results to a larger audience.

**Lab notes**: For reference and legal purposes, the primary record of lab work is the lab notebook. In virtually every work environment, be the research lab at universities or in industry or in a medical practice or repair shop, detailed records of activities are maintained. These are the lab notes. They function as memory aides, means to collect thoughts and to lay out upcoming steps in work and research. The notes are used as a workspace for new ideas and the efforts towards their validation, or to prove them wrong. They are a chronological and legal record. We require that you use a commercial notebook with index pages at the front, and numbered carbonless copy pages for notes. Many introductory chemistry laboratories use suitable notebooks. If your chemistry notebook is otherwise suitable and has blank pages left, you are free to use it for this course. At the end of each laboratory, you will submit the copy pages from your notebook to your teaching assistant. You will submit the copies for any work you do outside of class with the rest of the lab assignment. You will retain the original copies for your record and study. When you fill up one notebook, you are expected to obtain another.

**Formal report**: For communication within a broader technical community, lab work is summarized in technical reports. These reports communicate results and omit many details recorded in your lab notebook. Because the preparation of proper lab reports require considerable time and effort, we will not require a complete report for each laboratory. However, to satisfy UCORE requirements, some formal writing is necessary. For some labs, we will ask that you submit a well written, formal report, where you focus on communication tasks.

These two forms of communication employ different standards that can be only partially implemented in an instructional lab. What we require is described below.
Lab notes—official record of attendance and work performed

In general: The contents of your lab notes are the basis for grading the labs and for you to succeed in the exam. Neatness is not essential, but lab notes must be legible. If a TA cannot read it, you will not get credit. Your notes must include a full record of activities in the lab section. The details will be discussed during the first “introductory session” of your lab. Essential components are:

- **Identify yourself** Your name, WSU ID, your partner’s name, the date, the class and lab section, the Teaching Assistant’s name (TA). Is this a makeup lab?

- **What do you want to know?** The objective of the lab. What concept of physics is up for testing.

- **What do you know?** A collection of knowledge to help with the answer. Key components of the teaching assistant’s (TA) introduction belong here.

- **What equations are useful?** Write down equations that are to be tested. These may come from your TA or the classroom lecture or are derived here in the lab.

- **Sketches and free body diagrams** of the experimental setup with definitions. Make large drawings. Do not clutter with irrelevant details. Define components (i.e. cart, motion sensor, track, “The track is level). Add physics parameters and define them (i.e. momentum $p$, initial position $x_0$).

- **Make predictions:** What results do you expect to observe? For example: “The cart will roll along the track with constant speed”, or “The cart will accelerate until it hits the end of the track”. Illustrate with drawings. Specify, if something is constant, increases linear or “comes to a full stop”.

- **Timestamps:** What was done or observed when? Note the time on the right margin! At least once on each page.

- **Write down any activities chronologically.** Note the values of setup parameters. “Tracking the fall of a basketball with a motion sensor”. “The track is set horizontally as checked with a level.”

- **Data and results** Raw data, analysis results and units must be recorded. Tables are very useful. Values that you enter into Excel spreadsheets must appear in your handwritten notes.

- **Error analysis** as essential to lab work as the measured values and their units. Uncertainties and standard deviations quantify the reliability of a measurement.

- **Graphs** Large format graphs of recorded data (landscape format full page printouts; use zoom and pan). Label them and note where they belong in the lab notes. Clearly mark which part of the graph is used for any analysis such as curve fitting.

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1 A detailed introduction to the lab notebook is found in: Howard M. Kanare, *Writing the Lab Notebook*, (American Chemical Society, Washington, DC, 1985).
• **Math derivations** Details of mathematical derivations or algebraic steps. The grader should be able to follow your algebraic steps.

• **Comparisons** Experimental findings should be compared to predictions. A quantitative method to do this will be introduced in the labs. If there is disagreement, point that out and explain. Disagreements will not result in loss of grade credit. Failure to point out disagreements or not to discuss them will lead to points subtractions.

**Lab notes and exams** At the end of the semester, you will take a lab exam in which parts of a few selected experiments are to be reproduced — usually with small changes. Parts of the exam will be impossible to resolve if you did not keep good records in your lab notebook. The exam is much easier if your notes are complete.

With the exception of computer-generated graphs and tables printed during lab, lab notes must be handwritten in pen. Although lab notes are not formal documents, they are legal records. Any attempt to remove information from the record after the fact destroys this value and is considered scientific misconduct. *If you decide that any original data or notes are in error, put a single “X” through it, make short note in the margin explaining why it is in error, then record the new information in a new entry.* Both sets of data must be legible in your lab notes. Your grade will not be lowered due to properly marked errors. This practice conforms to standard scientific and engineering practice. You are free to work through any derivations that should appear in your lab notes on scratch paper before entering them in your lab notebook.

In case of a dispute over lab attendance or what you did in lab, pages torn from your lab notebook will not be accepted as evidence. Likewise, notes on regular notebook paper will not be accepted as evidence. A computer printout is evidence only if it is permanently attached (taped, not stapled) to an original page in your notebook or shows the signature of the supervising teaching assistant. Missing original pages are evidence for suspicious activity and carry a “presumption of guilt”: we will assume you are guilty of something—the only question is what.

If you rewrite or type your notes, understand that your original notes are the official record, not the rewritten notes. Notes made after the fact are not valid records and will not be treated as such. The copy pages with your notes must be submitted in order to receive a grade for laboratory work.

Each entry in your lab notebook should start with the current date and time in the right margin. If you work on your lab notes at home after lab, the entries made at home must also begin with the current date and time (the time of writing, not the time of the lab). Each entry must be recorded at the same time the work is performed. Entries must be sequential. Leaving one or more blank pages or part of a page in your notebook for later work is not acceptable. When you move on to a new page, draw a diagonal line through any large blank areas of the previous page. To work on an earlier lab after you have started work on a later lab, start your addition on first blank page in sequence. Mark the top of the new page, “Continued from page . . . ” and another note at the bottom of the old page, “Continued on page . . . ”. Many lab notebooks provide spaces for these notes. You lab notebook should also have an index for this information.

Unlike formal lab reports, texts in lab notes should be brief. These are not novels. Think of headlines and bullet lists. It is appropriate to write out questions you have about the lab and one or
two sentences of introductory material in your notebook before coming to lab; these entries must be
dated at the time or writing. Each step of your procedure must be recorded as you actually perform
it. Do not copy procedures from the manual into your lab notes before coming to lab. (When pre-
recorded procedures are absolutely necessary, draw a vertical line down the center of the notebook
page, with your intended procedure on the left and your record of what you actually did on the
right.) Likewise you should record your data as you take the data. There is no data section. To
help you avoid missing important points, the lab manual includes some questions about each lab;
these questions should be answered in your lab notes where the questions arise in the lab. If you
print a graph or data table in lab, attach it to your other notes as close as possible to the handwritten
notes that describe the data and how it was collected. Do not collect your computer printouts at the
end. Submit your notes in chronological order.

Your lab notes must be sufficiently detailed that you or another student with your background
can reproduce your work. The reader must be able to “trace” your work from the original data,
through your analysis, to your conclusions. Your notes should leave no doubt about how the data
were collected, what sensors and sensor settings were used (if any), and which equations were
used to calculate the quantities you report. Define any symbols used in your equations and include
appropriate units for numerical data. Sample calculations are often necessary.

Each graph printed during lab should fill a full sheet of paper to allow room for notes. To provide
this room, computer-generated graphs should normally be printed in the “landscape” (rather than
the “portrait”) mode. Landscape mode will print the x-axis along the longer dimension of the paper
and thus makes most graphs about 50% larger. In some cases it is useful to display computer-

generated graphs, for example, showing position, velocity, and acceleration as functions of time,
on the same page to facilitate comparison. These graphs should be printed in the mode that most
completely fills the page. All graphs must have a descriptive title that indicates what is being
graphed. (“Graph 1” or “Exercise 1” is not sufficient.) Labels and units are required for both the
x- and y-axes. If you are asked to draw a “curve” through your data points, this should always be
a best-fit curve (for example, a straight line if appropriate) that best represents your data. Best-fit
lines can be drawn by eyeball and a ruler, or with the help of the computer. If you are asked to
calculate the slope (or perform other analysis) of the graph by hand, show the results of this analysis
directly on the graph, clearly identifying which points are being used to calculate the desired
quantities. When a computer-generated best fit curve is displayed on a graph, the resulting equation
(with parameters and uncertainties) should also be displayed on the graph. This allows the reader to
evaluate the curve fit results without referring back to the text. Refer to the “Uncertainty/Graphical
Analysis Supplement” near the back of your lab manual for more information about using graphs
to find mathematical relationships between graphed quantities.

Keeping good records during lab takes time, and it is virtually impossible using formal English,
with complete sentences and paragraphs. Record your actions and data in the most clear, efficient
way possible. Use phrases instead of sentences. Annotated diagrams—simple sketches with the
parts labeled and notes—save time and are easier to understand (i.e. grade). Descriptive titles for
graphs and table columns also help. If an equation is used to describe the data in a graph, write the
equation on the graph. Putting it elsewhere usually requires additional text.
**Lab reports—formal communication with peers**

Although lab notebooks are the primary records of lab work, they are poor communication devices. Experimental results are communicated in technical reports. Unlike lab notes, these reports omit most “historical” aspects of the work: false starts are omitted. While one often reports the manufacturer and model number for important pieces of equipment, operational details are usually omitted. (The operational details must be recorded in your lab notes.) While lab notes often include derivations, technical reports normally include only the result. As communication devices, we expect lab reports to conform to the standards of formal written English, with appropriate word choice, grammar, and structure.

Because writing formal lab reports is time consuming, an entire report will not be required for each lab. Some labs will require short writing assignments that focus on one element of an entire report—perhaps an introduction or an experiment section. If the teaching assistant believes a submission is inadequate, the teaching assistant may require that it be rewritten and resubmitted for partial credit. As time permits, we will require complete, formal reports for one or two labs. The deadline for the submission of complete reports will be specified at the time of assignment. Typically a week or two are granted after the lab is performed. Your teaching assistant will inform you of the report requirements.

Lab reports (partial or complete) must be typewritten or printed from a text editor, using the format specified in the “Formal Lab Report Instructions” supplement near the back of the lab manual. You will have the original copies of your lab notes to use in preparing your report. Carbon copies of all relevant lab notes must be submitted to your teaching assistant for credit. The statements and conclusions in your formal report must be supported by the data and analysis in your lab notes. Omissions and gaps in logic, when observed, will lower your grade.

**Special requirements for lab assignments**

**Cover Page**

A cover page is required for every submission. It must include:

- The title of the experiment
- Your name and student ID number
- The name of your lab partner
- The date that the lab was performed
- The name of your teaching assistant
- The course and lab section numbers (for example, Physics 101, Lab Section 5)

Nothing else should appear on this page. Lab reports that are submitted in the wrong slot or are otherwise misplaced take much longer to reach your teaching assistant if the information on the cover page is incorrect or incomplete. Work submitted during lab might not require a cover page.
Uncertainty analysis

Many experiments involve a quantitative comparison between values of the same quantity determined by two or more distinct methods. When you compare two values, you must address the question of whether or not they agree within the limits of the expected or measured uncertainties. The Uncertainty/Graphical Analysis Supplement near the back of your lab manual defines important quantities, such as the standard deviation, and supplies details about determining uncertainties. As the semester progresses, you will need to make decisions by yourself on appropriate methods for calculating the uncertainties in your various measured and calculated quantities. Physics 102 and 202 students are expected to be aware of the uncertainty methods learned in Physics 101 and 201, respectively, and to use them appropriately.
Lab 0. Intro to Work in Laboratory

Goals

• To understand the purpose of experiments.
• To understand the key ingredients of experiments.
• To learn how to prepare lab-notes.
• To understand data acquisition with Capstone.
• To analyze data with Capstone and Excel.
• To compare results and predictions.

Experiments in Physics

“An experiment is a question which science poses to Nature, and a measurement is the recording of Nature’s answer.” Max Planck

All objects fall the same. Not so! Or, yes, indeed! Before Galileo objects were considered to fall differently. In 1590 Galileo figured out that it’s the air resistance that causes differences, not the falling. Their position changes proportional to the square of elapsed time. Isaac Newton figured out the concepts of forces and gravitation. He generalized his thoughts and managed to expand the understanding of falling objects on the surface of Earth to the motion of planets, asteroids, and comets. In 2012 Felix Baumgartner make daredevil history by free falling from 39 km altitude (3 times the cruising altitude of airliners) in 260 seconds. He broke the sound barrier and reached a speed of 373 m/s (1345 km/hr) in excess of the sound barrier. If all had gone according to Galileo and Newton and Earths gravitational pull, Felix would have fallen 330 km, almost 10 times more. Air resistance is not to be ignored.

Science evolved from describing moving objects, one at a time to one equation or explanation for all. If, that is, air resistance and friction etc. are considered separately.

Experiments are a method to check and verify or disqualify these generalizations. They are used to probe the limits of generalization. Experiments are also meant to be reproducible. Given sufficient information, anybody should be able to repeat the experiment and observe the same results.
Returning to Felix, in the first lab the motion will be simplified to that of a basketball falling from a few meters. Circumstances will be specified in more detail and tested qualitatively and quantitatively. This introductory tutorial is to convey the tools of performing such experiments.

**Ingredients of experiments**

An experiment is basically a series of answers to a list of questions. In the undergraduate labs a number of them are either covered in the lecture or pre-configured in the laboratory:

- **What do you want to know?**
- **What do you know about this?**
- **How to simplify the problem as much as possible?**
- **What will happen?**
- **How much time is available?**
- **What’s the plan?**
- **Execute the plan**
- **Analyze the data**
- **Compare to prediction**
- **Draw conclusions**
- **What can/should be done in further experiments?**

“What do you want to know?” in this tutorial and the upcoming first lab, it is the motion of falling objects. Will the motion be proportional to the square of elapsed time? What is the acceleration? The lecture introduced concepts of position, velocity and acceleration. They are known components regarding free fall.

To simplify the situation, a 1-dimensional motion on a straight line is considered only. Attempts are made to reduce the effect of air resistance.

Predictions can be made based on the time elapsed from start of the fall. That’s it. No other factor.

Each lab lasts just under 3 hours. The teaching assistant will give some instructions at the onset. Then, about 2.5 hours are left. It is useful to manage the available time. Each lab experiment consists of several mini-experiments or runs. Each run may take a minute. For example, sets of about 5 runs are to be carried out each for a number of different starting parameters. 5 runs of 1 minute each for 5 different parameters result in 25 minutes. After 15 to 20 minutes of an introduction that leaves the experimenters nearly 2 hours to analyze data and record the activities and results.

The plan for the experiment is established in the lab manual as well as the introduction by the teaching assistant. However, at times planning may be left to the experimenters. The equipment is installed, sensors are selected and mounted. What remains is to initialize the sensors and data acquisition and to execute the plan.

Typically users are left to execution of the plan and to analyze the data, i.e. to finalizing the experimental setup, collect data and to extract results from an analysis.
Finally, the results are **compared to predictions** or previously existing data. Based on the comparisons conclusions are drawn to either confirm the original hypothesis and predictions. Or, they have to be augmented or even discarded.

**Lab-notes**

A big component of the power of science in general and physics in particular is the emphasis on reproducibility. One can perform a calculation or an experiment over and over and obtain the same result. For that it is crucial to understand how to compare results, a topic that will be covered further down. The other component are detailed and precise notes.

Lab notes and lab notebooks are legal documents. They are used in patent cases and ownership disputes. Nobel prizes and honors are handed out based on records from lab notes. Important findings are witnessed and countersigned and dated. In undergraduate labs this is practiced to get the hang of it.

While conducting the experiments, the lab-notes are a running log of all activities undertaken in chronological order. They commence with the objective of the experiment at hand. They include the setup of the experiment, the recorded data and the methods of analysis and the results. These notes may be cryptic and a form of short-hand for the experimenter. A more formal report is more like a communication with the community that includes more details and combines data from series of related experiments. The formal notes tend to omit deviations or dead ends in order to streamline the notes. They also include more on the background and motivation for the experiments. Details of a formal lab report are discussed elsewhere.

Providing some structure in lab notes is useful not only for grading but also when it comes to their use as a memory aide. This is tested during the final lab exam. Items that are required for lab notes and make them easy to use include:

- **A cover page**: Start with the number and title of the laboratory. List your name, your WSU ID, your lab partner’s name, the class and section number (for example PHYS 101 lab 03) and the date. Note if this is a makeup lab.

- **Timestamps**: List the current time on the right margin of your notes at least once per page. Many labs have different components. List the times at their start points. Timestamps (and dates for longer efforts) help in the organization. External events may contribute to outcomes but are discovered only later. Correlation becomes possible with good time keeping.

- **An introduction**: This part should include the objective of the experiment. All relevant physics, such as equations should be included here. They may come from the lecture material and textbooks as well as information from the teaching assistant. A bullet list may be advisable.

- **The setup and sketches**: A description of the experimental setup. Following the proverb “A picture is worth a thousand words.” Sketches, free-body diagrams and drawings are worth more. Sketches are not images or photographs. They leave out all but the essential parts. For example, the position of a sensor is more important than the actual shape. If a line is meant to be straight, add a note. Labels for physics parameters such as $v_{cart}$ for velocity of
a cart are useful definitions. Make the sketch **big**. Less than 1/2 page sketches lead to lower credit. Add labels like “cart”, “motion sensor”, and “pulley”. Arrows that indicate “this way in time and positive direction” are essential. Most of us cannot draw straight lines freehand. If a specific slope is important, note that down. If it is important that certain conditions are met, record them. Examples are “leveling the track” and “the sensor is 21 cm from the cart at start time”.

- **Commentary for math**: Don’t just write down equations. Say “$p_{\text{cart}}$” is the momentum of the cart. After all, it could be “power”. “F” may be a force but could denote friction. In algebra, include steps. Basically, imagine you physics book math without any annotation. Would that be comprehensible?

- **Run configuration**: For each mini-experiment or run, one or at times more parameters may be changed. Note the changes. One may want to label the data-set or resulting printout accordingly for easier identification.

- **Taking data**: When recording the motion of a cart along a track, write in you notes what is done: “First run: give cart a push to let it travel from the right towards the sensor”. If the data are printed, add labels to the printout for easy correlation.

- **The actual experiment**: Not all of the collected data may be equally relevant. Only the parts that help the question to nature are of interest. Highlight this part and say so in your notes. For example “the experiment takes place from time = 5 sec to time = 21 sec, as highlighted by the shaded box”. If you start the recording of “the basketball falling over time” and then position and let go of the ball, the first part until the ball drops is not part of the actual experiment. It’s part of the setup. Once the ball bounces out of the field of view of the sensor the actual experiment is over. The rest until the stop button is hit is not part of the actual experiment. When the cart stops mid-track or hits the end of the track, that may be the end of the actual experiment.

- **Taking data**: When recording the motion of a cart along a track, write in you notes what is done: “First run: give cart a push to let it travel from the right towards the sensor”. If the data are printed, add labels to the printout for easy correlation.

- **Printouts**: must be labeled. One may note in the lab notes to refer to printout number 5 at this location. Tables of analyzed results for example from Excel should be printed out.

- **Graphs of data**: Make them **big**. Full page landscape is essential for full credit. Maximize the view of the data such that possibly important trends are in plain sight.

- **Relevant in a graph**: Axes must be labeled and have correct units. If you are graphing inverse mass, the units are “1/kg”. Label the graph so you and your grader can correlate the graph with specific notes in the report. The actual experiment part should cover at least 50% of the graph area and be pointed out. At times, a separate overview graph may show a full set of data once.

- **Fits and other analysis in graphs**: Fits will be made to find best matches of functions with datasets. Again, highlighting the region that is the basis for a fit. Results should also be in the lab notes and not just on the graph.
• **Results:** Record the results. Draw a box around them, to highlight. Be mindful of significant digits. Digits beyond the uncertainty are irrelevant and add confusion. Every result has a value, an uncertainty and units. You may measure the length of the track and write down to the nearest millimeter. Nobody would even think of using micro or nanometers. The tape measure and the viewing angle play a role. So the nearest precision is a millimeter. That’s the uncertainty. “The track is $1.234 \pm 0.001\text{m}$ long” is the example. More digits are meaningless.

• **Summary and conclusions:** Statements like “we measured a lot” summarize nothing. A summary pulls together key results and findings that answer the initial question. “The basketball falls with constant acceleration. The acceleration is $9.8 \pm 0.1 \text{m/s}^2$.” “The cart does not travel with constant velocity along a horizontal track. Friction, even though small, is the cause. It is equivalent of a deceleration of $0.015 \pm 0.004 \text{m/s}^2$.” “The equation of motion in 1 dimension predicts the motion within uncertainty of xxx.”

Other practices are useful, help when the notes function as memory aides and make grading easier, i.e. result in higher scores.

• **Be brief:** These are not novels. Lab-note are memory aides for you (and the exam) and recipes and procedures to reproduce the experiment.

• **Do not leave blank space to fill in later:** These are chronological notes.

• **Mistakes** are simply crossed out. Add a reference to where the corrected or updated information is found. There, add why the change was made.

• **I am going to write it up neatly later:** is not an option. These are not memoirs. These are the life tapes of what is going on in the experiment. Do not leave space for filling in the blanks later.

### To understand data acquisition with Capstone

The program Capstone is used to control a variety of sensors used in the coming labs. It is of great advantage to master essential parts of the program’s capabilities. A separate more detailed file will be available online during the labs. An overview of them will be given in order to:

• **Start the program and layout:** Double click on the little blue and white brick icon and the maximize the display to full screen. Around a mostly blank central white page on all four sides are your main control options. In order of use, start on the **left** with hardware configuration and **data parameters**; on the **right** select **display options**; on the **bottom** row buttons for start/stop recordings and related parameters. Finally, on the **top** icons are located for data highlighting, analysis, fitting, and output printing and saving. There is also an option or keeping a **journal** of the session for later printing or saving. All data can be saved or exported for safekeeping on flash-drives or importing into Excel.

• **Select and configure a sensor:** At the top on the left, the **hardware** button opens sub-screens to select the sensor(s) of the day. Little **gear wheel** icons offer options for fine tuning.
• Adjust significant digits: Next down on the left is a triangle rainbow button where significant digits can be set up. Similar adjustments can be made elsewhere as well.

• Set up display options: On the right click, hold and drag your choices for display onto the central page. There are graphs, tables, histograms, and more to choose from. Details depend on the requirements of the lab. Up front, simple graphs will do. The axes labels are buttons where you can choose what to display as the x- and y-axis values. Starting with the y-axis automatically sets the x-axis to time in seconds. Buttons at the top of each graph or table appear when clicking on a graph. These let you manipulate the display or add further graphs.

• Analyze datasets: Some of the top of the graph options allow to highlight (select) regions with the actual experiment data as defined earlier. Then you may perform statistics or fitting operations. On the graph you may pan and zoom to optimize the display as required for the lab notes and reports.

• Prepare the display for printing: On the very top list of tabs, file lets you set up the print format (must be landscape) and print graphs. Before printing, add labels. Several areas on the display allow for that. You may also drag a textbox onto the graph. This lets you correlate the printout with a location in your notes (and makes your grader happy).

• Multiple measurements: Capstone lets you take multiple runs. Just restart the recording and a new display starts. The older data are still present. A little rainbow triangle on the top bar lets you toggle through older runs.

• Keeping a log: Capstone offers the option to maintain a Journal. The button is at the top. It takes snapshots of the central display and maintains all in chronological order.

• Record and saving activities: Computer crashes and power outages happen. Note your findings in the lab notes. Print graphs. Save your data temporarily in the thaw-space on the computer’s hard drive (or permanently on your flash-drive). Do not depend on the computer to keep your data. Crashes happen. You do not want to start all over.

• What if the computer crashes? Did you save your work and log it in your notes? No? You just learned the hard way why you keep records on paper. Restart the lab with.

To analyze data with Capstone and Excel

Capstone offers a large range of options to fit datasets on display in your graphs. It lets you select subsets of the full dataset and analyze them exclusively. Little boxes appear and show the results. Thin lines graphically represent the analysis outcome. Excel — originally developed to help with tasks in business — offers some additional capabilities for data analysis. Results from multiple measurement runs can be combined on spreadsheets to be graphed together as a function of your controlling parameters. Simple linear regression analysis can be performed on these results. More complex math can be performed on your columns of data. You may have some experience in using Excel. Some less common operations are covered here.

• Math: Excel lets you do all kinds of math on any worksheet item or column. Each little box has a column character (A, B, C, ...) and row (1, 2, 3, ...). Click on a box where you want
a calculation done and then start typing “$= 3 \times B5 + D2$” to multiply the value in box “B5" with 3 and add the value of box “D2”. Clicking on the box, will give it a fat black outline with a small separate dot in the lower right corner. Clicking on that dot and dragging the mouse down over multiple boxes will apply the same math to all the new boxes. The “B5" and “D2" box addresses will also move along. This can be prevented by changing “B5" to “$B$5" for example. Now the same value will be used.

- **Graphs**: On the top bar of tabs go to **insert** and find **scatter**.
- **Axes labels**: Excel is not as convenient as Capstone for that. But, all the options are available.
- **Error bars**: Excel has powerful graphing options. You can add error bars to your graphs.
- **Linear regression**: This is the important one! **Do NOT use trendline.** On the top bar of tabs go to **data**. At the very right side under **Data analysis** a window pops up. Scroll down to look for **Regression**. This version will let you select x- and y-datasets, and fit a linear function. The results for intercept and slope will also carry an uncertainty. This is crucial. You must have uncertainties to finish your notes. Also, choose the options to display the results on a separate sheet to avoid overwriting existing data.
- **Record results**: Do not depend on Excel sheet print outs. Record the results in your lab notes. Annotate them so your grader understands what you did!
- **Graphs**: The same rules as for Capstone graphs apply. Excel requires more legwork to maintain units and suitable axes ranges.

For many users Capstone as well as Excel are big mysterious programs. They are very powerful and hence the learning curve may be steep. Hang in there. Most tasks in these labs are relatively simple. Learning how to calculate functions in Excel rapidly leads to lots of time saved. Using Excel reduces the chance for errors and makes it possible to find and fix errors that do occur.

**To compare results and predictions**

In many cases the experiment is to demonstrate that a concept introduced in class is true and applies at least in the lab, if not in the real world where more out of control variables are at play. Coming back to the skater on the ice rink, friction of the skates on ice was replaced by friction of the wheel bearings of the cart. The ice is not perfectly smooth. The zamboni may be broken. The track on the other hand may have scratches or is bowed. And so on.

In the end, the prediction is that the cart does not slow down without friction or accelerate constantly up or down a ramp. The experimental results provide a value for acceleration with uncertainty and units. The value likely is not zero.

Is the prediction of zero acceleration proven wrong? Is the difference to the prediction of zero significant? The comparison or ratio of this difference to the uncertainty offers a clear cut and reproducible method of deciding.

The difference of a measurement $m$ to a prediction $p$ of a theorist or the measurement of a com-
petitor is $\Delta$:

$$\Delta = |m - p|$$

(1)

This is then compared to the uncertainty of the measurement $m$ called $u(m)$ combined with the uncertainty of the prediction $p$ called $u(p)$. From mathematics and statistics it is known that the combined uncertainty $u(\Delta)$ is by adding in quadrature:

$$u(\Delta) = \sqrt{u(m)^2 + u(p)^2}$$

(2)

The ratio of these is called the $t'$-score.

$$t' = \frac{\Delta}{u(\Delta)} = \frac{\Delta}{\sqrt{u(m_{F/a})^2 + u(m_{bal})^2}}$$

(3)

Nominator and denominator have the same units (from the measurement). Consequently the $t'$-score has no units. It is also always positive. The higher the $t'$-score is, the more likely the compared values are not in agreement. In the undergraduate laboratories the cut off value is $t' > 3$ for $m$ and $p$ being different and $t' < 3$ for them to be in agreement. In the latter case, if $p$ is the prediction, then the prediction is confirmed by the experiment.

In modern groundbreaking experiments where standing theories are overturned, a $t'$-score of more than 5 or even 7 is essential to convince the community of physicists.

The nature of how uncertainties are evaluated based on statistics make it harder and harder to reduce the value of the combined uncertainty in order to drive up $t'$ for a given difference. In general the uncertainty $u(m)$ says that in 68 out of 100 repeats of an experiment the result of any two will agree within $t' < 1$. A $t' > 2$ says that in 5 of 100 cases there is agreement. The higher the $t'$-score the more likely something is different or the prediction was wrong or something unforeseen in the measurement deviates from the plan.

Let’s try this out

Together with the teaching assistant walk through the first experiment. In the next session, you will be left to do more on this by yourself.

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<table>
<thead>
<tr>
<th>#</th>
<th>Task or Activity</th>
<th>Labnotes</th>
<th>Data</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Start</td>
<td>Heading and date/time Name, ID, TA, class, section</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>What do you want to know?</td>
<td>is the free fall displacement proportional to $t^2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>What is known?</td>
<td>time = $t$, position = $x$ change $\Delta$ in time and position velocity $= v = \frac{\Delta x}{\Delta t}$ acceleration $= a = \frac{\Delta v}{\Delta t}$ $x(t) = \frac{1}{2}at^2 + v_0t + x_0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>setup</td>
<td>sketch</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Predict</td>
<td>$x(t) - x_o \propto t^2$ $a = g = 9.8 \pm 0.01 m/s^2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Finish plan</td>
<td>connect sensor calibration</td>
<td>Motion Sensor II $x_{min} = ?$, units?</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Execute</td>
<td>Start record</td>
<td>graphs of $t$ and $x$, $v$, $a$</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Analyze</td>
<td>Excel: linear regression transfer data linear regression sensor position agrees with ruler Capstone: fit quadratic extract a from position Capstone: fit linear extract a from velocity Capstone: acceleration mean of data $a = const \pm u(a)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Compare</td>
<td>predicted: $x(t) - x_o \propto t^2$ agree? predicted: $a = g$ data $a_m \pm u(a_m)$ $t' = \frac{\Delta a}{u_{combined}}$ agree? $t' &lt; 3$?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Conclusions</td>
<td></td>
<td>$g = 9.80 m/s^2$</td>
<td></td>
</tr>
</tbody>
</table>
Summary

This is not an actual lab but more a tutorial of what to expect to encounter in the lab. Together with the teaching assistant an experiment was carried out. The experiment was preconfigured and set up. Data were acquired and analyzed, the resulting values for acceleration were compared to the prediction of zero acceleration. It was found that the prediction was .... (to be continued).

Keep these instructions and guides in mind for your upcoming labs and note taking activities.

Before you leave the lab please:
   - Save what you would like to keep on a thumb drive.
   - Quit Capstone and straighten up your lab station.
Lab 1. Free Fall

Goals

- To determine the effect of mass on the motion of a falling object.
- To review the relationship between position, velocity, and acceleration.
- To determine whether the acceleration experienced by a freely falling object is constant and, if so, to calculate the magnitude of the acceleration.
- To calculate the appropriate uncertainties and to understand their significance when analyzing data.

Introduction

When an object is dropped from rest, its speed increases as it falls—that is, it accelerates. In this experiment you will characterize the motion of freely falling objects using an ultrasonic motion sensor. A significant part of the experiment entails understanding the relationship between the acceleration, velocity, and position of an object. You will also employ the concepts of mean (or average) value, standard deviation, and standard deviation of the mean of a measured quantity. An introduction to these concepts is given in the Uncertainty/Graphical Analysis Supplement at the back of the lab manual.

Effect of mass on the motion of a falling object

At your lab station locate the small yellow plastic ball and a steel ball of the same diameter. After recording the masses of the balls, hold the balls at the same height and drop them together. Note which ball (if either) reaches the floor first. Use the padded catch box to minimize damage to the floor by the steel ball. If the balls strike the floor at different times, consider how accurately you can release the balls at the same time. An experiment with two identical balls can indicate how small differences in release time affect your results.

If you change the height at which the balls are released, does the result change?

Record the conditions and the observed results for each trial that you do. Based on your
findings summarize your observations. What can you conclude regarding the effect of the mass on the motion of the falling balls?

Try dropping another object such as a pen or pencil along with the steel ball. How do the motions compare now? What conclusions can you draw from your observations?

Be sure that the notes you make in lab are sufficient for you to repeat the experiment later in the semester if asked to do so.

**Characterizing the operation of the motion sensor**

3. Consult the Computer Tools Supplement at the back of the lab manual to learn how to connect the “**Motion Sensor II**” to the computer interface box at your lab station. Once the sensor is connected, set up the Capstone software to simultaneously display graphs of position, velocity, and acceleration as functions of time. You may need to change the “sample rate” displayed along the bottom of the Display Area from its default value of 20 Hz in order to obtain enough data for analysis.

Due to electronic limitations, the motion sensor only measures distances greater than 0.25 m (newer version) or 0.40 m (older version). In all your experiments with the motion sensor, make sure that the object of interest is never closer than the minimum value for the sensor you are using. The sensor has two settings. A slider switch on the side can be moved to select a narrow angle (cart pictogram) to a wide angle (stick figure pictogram) operation. For the falling balls, the wide angle version should be used. Note all settings in your lab-notes.

4. Hold a flat object directly beneath the motion sensor and measure its position using a meter stick. Now take some motion sensor data with the flat object at the same position. How are the two position values related? Take another set of motion sensor data with the flat object at another position and compare it with its new position as measured with a meter stick. Analyze the position data to determine the location of the origin \((x = 0)\) point and the \(+x\)-direction of the coordinate system used by Capstone. Report the results of your measurements, your reasoning, and your conclusions in your lab notebook.

Now take some data while moving the flat object up and down under the motion sensor. Display this position, velocity, and acceleration data in a table. Starting from the position data in this table, determine how Capstone computes the velocity as a function of time. Which position values are used to calculate the velocity at a given time? Repeat this process for the acceleration calculation. Which velocity values are used to calculate the acceleration at a given time? Use this to determine which position values are used to calculate the acceleration at a given time. Show sample velocity and acceleration calculations in your lab notebook, taking care to label the time at which each position value used in the calculation was recorded.

5. Use this information to determine how fast the motion sensor can respond to changes in velocity and acceleration. If the velocity of an object suddenly (instantaneously) changes from one value to another, how long will it take the velocity reported by Capstone to reach the new value? How is this change reflected in the value of acceleration reported by Capstone?
If the acceleration of an object instantaneously changes from one value to another, how long will it take the acceleration reported by Capstone to reach the new value?

**Characterizing free fall with a motion sensor**

**Data acquisition**

6. Hold the basketball under the motion sensor such that the top of the ball is greater than the minimum distance for your sensor. Make sure that hands, feet, stools, backpacks, and such are removed from the target area so the motion sensor “sees” only the basketball. Click on the “Start” button of Capstone to start the data taking process. Wait a few seconds before quickly removing your hand(s) and releasing the ball. Allow it to fall to the floor and bounce twice. Then click the “Stop” button to terminate data acquisition. Expand the graphs to display only the motion during the fall and through the second bounce. Check with your TA to make sure that you have a good set of data. If necessary, repeat the data taking process until satisfactory data is obtained. Print out a copy of the three graphs on a single page in the “landscape” format to include in your notes.

**Qualitative analysis**

7. Observe the acceleration-time graph. Expand the graph vertically so that the acceleration during free fall occupies most of the graph. Ignore the noisy regions during each bounce, when the ball contacts the floor. What conclusions can you make regarding the acceleration of the basketball during the initial fall and between the first and second bounces?

After the first bounce, the ball is moving upward toward the motion sensor and slowing down before it speeds up again and bounces the second time. Explain the sign of the acceleration (negative or positive) during this interval both as the ball slows down while moving upward and speeds up while moving downward.

Is the velocity-time graph consistent with the observed acceleration during each segment of the ball’s motion? Compare them using the definition of acceleration in terms of velocity.

**Quantitative analysis**

8. The value of the basketball’s acceleration can be found from the position data, from the velocity data, or from the acceleration data. If the kinematic equations describe the path of the basketball, each data set should give the same acceleration.

1. Use the **position vs. time** graph to determine the average acceleration between the first and second bounce with Capstone’s curve fit function. Select the position data between the first and second bounces using the “Highlight range of points in active data” (pencil) tool from the tools along the top of the graph. From the kinematic equation, we expect the position of basketball to be described by an equation of the form \( y = At^2 + Bt + C \): the quadratic equation. With the data selected, choose “Quadratic” from the Curve Fit
(2) Use the velocity vs. time graph to determine the average acceleration and its uncertainty between the first and second bounces. From the kinematic equations, we expect the velocity of the basketball during free fall to be described by an equation of the form $v = mt + b$: a linear equation. Select the velocity data between the first and second bounces and choose “Linear” from the curve fit menu to obtain the slope of the velocity-time graph (the constant $m$). On your printout, identify the data points used to determine the acceleration. The uncertainty in this acceleration measurement is equal to the “standard error” reported by Capstone in the curve fit window.

(3) The acceleration vs. time shows the acceleration value direction. One can simply select the data between the first and second bounce and check the mean and standard deviation buttons under the $\Sigma$ button along the top of the graph. Again, identify the data points used to determine the mean acceleration. The uncertainty in the mean value is calculated by dividing the standard deviation by the square root of $N$, where $N$ is the number of data points used to calculate the mean. You will have to count the points by hand. Capstone will compute the uncertainty for you if you use the “User Defined Fit” function in the Curve Fit function, then enter an equation of the form $y = A$ into the Curve Fit in the Tools Palette on the left side bar.

(4) Compare: Did the acceleration values determined in this experiment agree with your expectations? Do they agree with each other? Use the quantitative test for consistency described in the “Uncertainty and Graphical Analysis: Using uncertainties to compare measurements or calculations” section of the lab manual. Briefly, we conclude that two measurements, $a_1$ and $a_2$, with uncertainties $u(a_1)$ and $u(a_2)$, are consistent if $t' = |a_1 - a_2|/\sqrt{u(a_1)^2 + u(a_2)^2} < 3$. You will need to compare the three accelerations measurements you made from the position, velocity, and acceleration data, respectively. You should also compare these measurements with the “expected” value of $a = g = 9.80 \pm 0.01 \text{m/s}^2$.

(5) Decide and note: Are some acceleration values “better” (more precise or more accurate) than others? Explain your reasoning.
Conclusion

Discuss what you have discovered about objects in free-fall. What did you expect to find? Did your experiment agree with your expectations? Did the various methods of determining the acceleration of falling objects give the same values? Discuss and explain, using your results for the experiment. You may wish to summarize all your experimental numerical results in a small table here, making it easier to refer to them in this part of your notes. How does the concept of uncertainty assist us in making logical conclusions?

Before you leave the lab please:
- Quit all computer applications that you may have open.
- Place equipment back in the plastic tray as you found it.
- Report any problems or suggest improvements to your TA.
Lab 2. Projectile Motion

Goals

- To determine the launch speed of a projectile and its uncertainty by measuring how far it travels horizontally before landing on the floor (called the range) when launched horizontally from a known height.

- To predict and measure the range of a projectile when the projectile is fired at an arbitrary angle with respect to the horizontal.

- To predict the initial firing angle of the launcher for a prescribed range value.

- To determine quantitatively whether the measured ranges in (2) and (3) are consistent with the desired range values.

Introduction

When objects undergo motion in two (or even three) dimensions rather than in just one, the overall motion can be analyzed by looking at the motion in any two (or three) mutually perpendicular directions and then putting the motions “back together,” so to speak. In the case of projectiles, the horizontal and vertical directions are usually chosen. Why is this choice made? Ignoring the effects of air resistance, an object moving vertically near the surface of Earth experiences a constant acceleration. We know this by experiment. Likewise an object moving horizontally experiences zero acceleration. Any other choice of perpendicular directions would have nonzero, constant values of acceleration in both directions. When we write the descriptions of the motion in mathematical terms, the horizontal/vertical choice of directions results in the simplest description.

Under what conditions can the effects of air resistance be ignored? One condition is that the object’s speed is not too high, since the effect of the air resistance increases with speed. If two objects are the same size and shape, the lighter one of the two will experience the larger effect on its motion due to the air. (Imagine a ping-pong ball and a steel ball bearing of the same size.) In designing this lab, care has been taken to ensure that air resistance has a negligible effect on the trajectory of the projectile. When conditions are such that air resistance cannot be ignored, the motion is more complicated.
Mathematical preliminaries—Equation for range

To accomplish the first two of our stated goals, we need a general mathematical relationship between the horizontal range of the projectile and the initial height, initial velocity, and launch angle. See Figure 2.1. You will need to solve the appropriate kinematics equations for motion with constant acceleration in the horizontal and vertical directions simultaneously. Rather than writing the equations in terms of the angle, $\theta$, it is suggested that you use the symbols $v_{0x}$ and $v_{0y}$, where $v_{0x} = v_0 \cos \theta$ and $v_{0y} = v_0 \sin \theta$, to simplify the algebra. You need to solve for the range, $R$, in terms of $v_{0x}$, $v_{0y}$, $h$, and $g$. The details of this derivation must be included in your lab notes.

Instructions and precautions for using the ball launcher

Warning: Never look down the barrel of a launcher. Wear eye protection until all the groups have finished launching projectiles.

1. Make sure that the launcher is attached securely to the table so it does not move when the launcher is fired. Make sure the launcher is at the proper angle by using the built-in plumb bob on the side of the launcher. Note that the angle measured by this plumb bob is the angle between the “barrel” of the launcher and the horizontal.

2. Since the projectiles will be hitting the floor, use a second plumb bob to locate and mark the position on the floor (blue tape works) directly below the launch point of the projectile. This indicates the initial horizontal position of the ball at floor level so the range (horizontal distance traveled by the ball) can be measured later. You will have to measure the height to get the vertical distance. Clearly indicate in a diagram how you measured the height (from where to where). If you are not sure how the height should be measured, please discuss it with your TA.

3. To launch the projectile, load the ball into the projectile launcher. Use the rod to push the ball into the launch tube to one of the three preset launch positions (short, medium, or long
range). You will hear a click as you reach each position. Notify others nearby and across the room before firing the ball. Stand out of the way and fire the launcher by pulling on the string attached to its trigger on the top. To minimize the force applied by the string to the launch tube, pull the string at right angles to the launch tube.

4. To record the position where the projectile strikes the floor, tape a white paper target to the thin hard-board sheet (about 0.3 m × 0.5 m in size) at your lab station. Place the sheet and target at the approximate place where the ball lands. When you are ready to record some landing points, lay a piece of carbon paper (carbon side down) on top of the target. Do not put tape on the carbon paper. The ball will leave a dark smudge on the white paper where it lands. If necessary you can tape the hard-board sheet to the floor to keep it from moving, but avoid the indiscriminate use of tape on the floors.

**Determining the initial speed of the projectile**

1. Simplify your general equation for the range for the case when \( \theta = 0 \) (horizontal launch). Then solve for \( v_0 \) in terms of \( R \), \( h \), and \( g \).

2. Set the launcher to fire horizontally, that is, to launch at an angle of zero degrees. Care with this angle setting can significantly improve your results later in the lab.

3. Starting with the medium range launch setting, fire the projectile (using the four steps in the previous section) a couple times noting where the projectile lands. Center the paper target as best you can where the ball will land. Now use the carbon paper to record the landing position of four or five launches using the same initial conditions.

4. From your data determine the average range, \( R \), of the ball. Use this average distance to calculate the average initial speed of the ball as it was launched.

5. Repeat the same procedure for the short and long range settings on the launcher.

**Range for nonzero launch angles**

1. Choose a launch angle between 30° and 40°. Using the values of the initial speed of the ball measured above and your general equation for the range, calculate the horizontal distance (range) from the launch point to where the ball should land for the short and medium range settings using the initial launch angle that you have chosen. (Do not use the long range setting.)

2. For the short and medium range settings, place a paper target on the floor at the calculated position and fire the projectile. If the projectile misses the target completely, check your calculations and/or discuss it with your TA. If the projectile does hit the target, then repeat several times to get a good average experimental range value and its corresponding standard deviation to compare with your calculated range.

3. Compare your predicted range values with the experimental range values using your experimental standard deviations. Assume that your predicted range, \( R_{predicted} \) has zero un-
certainty. Then your measurement is consistent with your prediction if 
\[ t' = \left| \frac{R_{\text{measured}} - R_{\text{predicted}}}{\sigma(R_{\text{measured}})} \right| < 3 \]. If you find that 
\( t' > 3 \), check your calculations and consider carefully what systematic errors may be present in your experiment.

**Launch angle to achieve a given range**

1. Ask your TA to assign a value of horizontal distance (range) for your group.

2. Calculate a suitable angle at one of the range settings for launching the projectile to the target set at the assigned distance. The relationship giving the initial launch angle in terms of the other parameters is:

\[
\tan \theta = \frac{v_0^2}{gR} \pm \left[ \left( \frac{v_0^2}{gR} \right)^2 - 1 + \frac{2v_0^2h}{gR^2} \right]^{1/2} \quad (2.1)
\]

3. Now set the target and do the experiment with your TA present to observe. Were you able to hit the target? If you have trouble, check your calculations. Is your calculator in radian or degree mode? Get assistance from your TA, if necessary. Again, compare your experimental range value to the range value assigned by your TA. If not, check your calculations and your procedure.

**Conclusion**

Summarize all your results, preferably in a table showing the measured and calculated quantities with their uncertainties. Clearly display your comparisons between predicted values and experimental values. Are you convinced that the theoretical predictions made by separating the horizontal and vertical motions agree with experiment, at least within the calculated uncertainties of the experiment? Your answers must be based on your experimental results and the calculated uncertainties of the quantities you are comparing. Do not make vague statements that are not directly supported by your calculations and measurements.

**Before you leave the lab please:**

- Return the projectile and the carbon paper to the TA Table.
- Remove all tape from the floor.
- Wrap the plumb bob string around the cardboard spool.
- Store the plumb bob and string in its plastic bag.
- Return the goggles to their plastic bags.
- Place the plumb bob, tape measure, goggles and rulers in your equipment basket.
- Straighten up your lab station.
- Report any problems or suggest improvements to your TA.
Lab 3. Newton’s Second Law

Goals

- To determine the acceleration of a mass when acted on by a net force using data acquired using a pulley and a photogate. Two cases are of interest: (a) the mass of the system is fixed and the net force is varied, and (b) the net force is fixed and the mass of the system is varied.
- To make and analyze appropriate graphs of the resulting data that test the validity of your application of Newton’s Second Law of Motion to this system.

Introduction

Newton’s First Law states that no change in the motion of an object takes place in the absence of a net force. In other words, the acceleration (change in velocity) of an object is zero unless there is a net force. But how is the acceleration related to the force? Newton’s Second Law deals with this relationship. Experimentally we will explore the relationship between the net force on an object, the mass of the object, and the acceleration of the object due to the force. Newton’s Second Law of Motion makes some definite predictions that you can test.

Be sure to level the track carefully before you take any data.

Accelerating a fixed mass with a variable force

Behind Newton’s Second Law is the assumption that an object (or group of objects) can be modeled as a point with a definite mass and location, that moves along a well defined trajectory through space with a definite velocity and acceleration. A group of objects can often be modeled as a point if it moves together rigidly, without rotation or stretching—and if its mass does not change. When we apply Newton’s Second Law to a suitable group of objects, we call the group “the system.”

In this experiment, a small mass is connected to the cart by a string that hangs down over a pulley. To apply Newton’s Second Law to this situation, the system mass (the \( m \) in \( F = ma \)) and the net force on the object (the \( F \) in \( F = ma \)) must be clearly specified. Draw a free body diagram of the cart, the pulley, and the slotted masses, and use it to express the net force and the system mass in terms of the masses of each part of the system. Ignore friction for the time being. Check that each part of the system has the same acceleration (the \( a \) in \( F = ma \)). This simplification is possible
because both the cart and the hanging mass move as if they were glued together (assuming that the string is not stretchy). Your free body diagram and any text or mathematics needed to support your expressions for $F$ and $m$ must appear in your lab notes.

Using a photogate to measure how fast the pulley rotates, the computer can take the time and displacement measurements required to compute the displacement, velocity, and acceleration of the system as functions of time. The “Photogate with Pulley” section of the Computer Tools Supplement at the back of the lab manual has specific instructions on connecting the pulley to the computer. You may wish to plot all three of these quantities to determine the best method of determining the acceleration. If the acceleration is constant, the slope of the velocity-time graph will be a straight line whose slope equals the acceleration. The slope of a straight line is easily found along with a useful uncertainty estimate using the “Fit” tool in the toolbar at the top of the graph window. Does the slope of the velocity-time graph yield a more precise acceleration value than simply taking an average acceleration value directly from the acceleration-time graph? Compare the two methods for a single data run. Explain your findings in your lab notes.

Although the computer fitting routine gives an uncertainty estimate for the slope of a graph based on one data run, the experimental conditions are not exactly the same each time the cart moves down the track. Therefore, the uncertainty estimate given by the curve fitting routine is a gross underestimate (often by a factor of ten in this exercise). Below you will compare the mass of the cart calculated from your acceleration measurements with the mass of the cart determined by an electronic balance. Incorrect uncertainties can result in false conclusions about the consistency of these two mass values. To measure the uncertainty due to variations in the condition of the cart/track, repeat the acceleration measurement for at least three trips down the track. The average acceleration value is the best estimate for the “true” acceleration, and the standard deviation of the mean is the best estimate for its uncertainty. The Uncertainty-Graphical Analysis supplement at the back of your lab manual defines and discusses the standard deviation of the mean.

Vary the total hanging mass from 10 g to 60 g in 10 g increments. To keep the mass of the system as a whole fixed, make sure that any unused hanging masses ride in the cart. It is important that the hanging mass not exceed 60 g. The velocities achieved using larger masses can be sufficient to damage the equipment when the cart strikes the end of the track.

Now make a graph of the force (vertical axis) as a function of the acceleration (horizontal axis). Include error bars on your graph that correspond to the uncertainties in your measured acceleration values.

Newton’s Second Law states that the acceleration of a system with constant mass is directly proportional to the net force and that the acceleration of an object under a constant net force is inversely proportional to its mass. If Newton’s Second Law is correct, then you should be able to compute the mass of the system from the slope of your graphical analysis. Ask your TA for assistance if it is not clear how to proceed.

Compare the value of the system mass determined from your data, $m_F/a$, with the total system mass measured using an electronic balance at the back of the room, $m_{bal}$. To make this comparison quantitative, compute the $t'$-score for the discrepancy (the “error”, $\Delta = |m_F/a - m_{bal}|$) and the uncertainty of this difference, $u(\Delta)$. The $t'$-score is described in the Uncertainty and Graphical Analysis appendix to the lab manual. If $t' < 3$, it is fair to say that your acceleration data are consistent.
sistent with Newton’s Second Law and your mass measurement. If \( t' > 3 \), carefully examine your procedures and analysis for sources of systematic error. Include the results of your examination in your notes.

**Accelerating a variable mass with a fixed force**

By measuring the acceleration of the system as a function of net force, while holding the system mass constant, we can study the effect of net force on acceleration. Similarly, by keeping a fixed value of mass hanging on the string, while keeping the net force constant, we can also observed the effect of system mass on acceleration. Hang 50 or 60 g on the end of the string for this experiment. Additional rectangular aluminum bars (4 maximum!) can be placed on top of the cart to increase the mass of the whole system. Plot the acceleration (vertical axis) as a function of the system mass (horizontal axis). Qualitatively, what happens to the acceleration when the mass increases?

Newton’s Second Law states that the acceleration of a system with constant net force is inversely proportional to its mass. In mathematical form this looks like:

\[
F = ma \quad \text{or} \quad a = \frac{F}{m} = \frac{1}{m} \tag{3.1}
\]

The relationship between acceleration and mass with a constant force is trickier than the relationship between acceleration and force, with constant system mass. The mathematical relation between the variables is hard to guess when the graph curves. On the other hand, straight-line graphs are simple to identify and analyze. Is there a way of plotting the accelerations and mass data that is consistent with Newton’s Second Law, but so that the graph is a straight line? You may need help from your TA. When you have sorted this out, you should be able to compute the value of the net force applied to the system.

Use the \( t' \)-score to compare the value of applied net force calculated from your graph to the gravitational force acting on the hanging mass.

**Real world effects**

Are there reasons that your results might not be totally consistent with the predictions based on Newton’s Second Law? Have we included all the forces acting on the system? What effect does the ubiquitous force of friction have? Examine your graph of acceleration versus force carefully. Newton’s Second Law says that the net force and the resulting acceleration are directly proportional, meaning that zero net force produces zero acceleration. Does your graph show this to be the case? Look carefully! Explain how and why the graph might deviate slightly from this ideal. Does the presence of friction invalidate your graphical determination of the system mass? Ask your TA whether your explanation is reasonable.

Your acceleration versus system mass measurements are also affected by friction; however, you do not have enough data to justify pursuing the details.
Conclusion

Is your data consistent with the predictions of Newton’s Second Law? To support your conclusion, you must compare any observed discrepancy with your experimental uncertainty. Now a question to just think about. How was Newton able to formulate the Second Law of motion? Did he have access to equipment comparable to what you used today? Was he compelled to formulate the Second Law based on his experimental results?

Before you leave the lab please:
- Quit all computer applications that you may have open.
- Collect the slotted masses that were hung on the string.
- Please make sure that all of them are there!
- Report any problems, equipment or otherwise, to your TA.
Lab 4. Friction

Goals

• To determine whether the simple model for the frictional force presented in the text, where friction is proportional to the product of a constant coefficient of friction, $\mu K$, and the magnitude of the normal force between the surfaces, $n$, applies to the cases of sliding aluminum-wood and aluminum-felt surfaces.

• If appropriate, to determine the kinetic coefficients of friction between wood and aluminum and between felt and aluminum.

• To determine whether the “constant” coefficients of friction are independent of the speed that one surface slides over the other for the two cases previously characterized. This is accomplished by letting the wooden block accelerate on the surface of the aluminum track.

Introduction

From a fundamental point of view one can say that all friction is due in one way or another to electromagnetic forces. Inter-atomic forces (also known as chemical bonds) are electromagnetic forces that act through distances that are on the order of the spacing between atoms, that is $10^{-10} - 10^{-9}$ m. Most surfaces that appear smooth are rough when viewed microscopically. When viewed using visible light (with wavelengths between 400 and 700 nm), the surface will appear smooth and shiny if the roughness is smaller than the wavelengths of the light. Consequently the actual surface roughness can be as large as $10^{-7}$ m, equivalent to a hundred or more atomic spacings, before the roughness becomes apparent to the eye.

What we call friction arises from adhesion (atomic attraction) between the atoms of two surfaces in close proximity, even when roughness limits physical contact to the “peaks” on each surface. In some instances pieces of material can be torn off in the process of sliding across another surface, thus breaking some of the chemical bonds. For example, material is removed from a skidding rubber tire or a piece of wood as it is smoothed with sandpaper. Because chemical forces are ultimately electromagnetic in nature, friction can be attributed to electromagnetic forces.

The details are not yet well enough understood to make meaningful calculations and predictions. This is unfortunate, since about one-third of the world’s energy resources are ultimately consumed by friction in one form or another. The alternative is to characterize friction empirically, in other

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words by experiment. Since static friction (the friction between two surfaces which do not move with respect to one another) is even more difficult to reproduce consistently, we will limit our study to kinetic friction, when two surfaces slide with respect to each other. Your textbook has made some simple claims for kinetic friction, namely that (1) the frictional force for a particular interface is directly proportional to the normal force exerted by one surface on the other, and (2) the frictional force is independent of the speed with which the surfaces are sliding with respect to each other.

Both of these claims are tested in this experiment, so you can begin to “get a feel” for the concept of friction. We will be studying the friction of a wooden block on a smooth aluminum surface and the friction of a felt covering on the block relative to the same aluminum surface.

**Equipment set up**

A 1.2 m long aluminum track acts as the supporting surface as a wooden block is dragged over it. To supply a constant force to the system in the direction of motion, a hanging mass is suspended by a string passing over a pulley at the end of the track and attached to the wooden block. By equipping the pulley with a photogate, the rotation rate of the pulley can be measured. From this, Capstone computes the speed of the sliding block. This will allow you to keep the speed relatively constant as we do the measurements. Naturally, it will be impossible to keep the speed exactly constant.

**Friction between wood and aluminum**

**Constant velocity measurements**

If the wooden block and the hanging mass are moving at constant velocity, then we know that the net force acting on the system of the block and the mass must be zero (since the acceleration of the system is zero). Draw a free-body diagram of this system and show that the magnitude of the frictional force is equal to the weight of the hanging mass if the acceleration is zero. Your task is to test the hypothesis that $f_K = \mu_K n$, where $f_K$ is the frictional force between the wooden block and the aluminum track, $n$ is the normal force at the block-track interface, and $\mu_K$ is the constant coefficient of kinetic friction between the block and the aluminum track. Remember that $\mu_K$ is a dimensionless quantity; it has no units.

It is easier to achieve constant velocity at higher normal forces. Starting with the highest normal force and working down is less frustrating than starting with a low normal force and working up. Start with all four 100 g masses on top of the block for a total of 400 g of added mass, then remove one 100 g mass at a time. For each value of normal force, find the weight of the hanging mass which will keep the velocity of the system constant at a value between 0.2 and 0.3 m/s. You must give the system an initial push at the desired final velocity, since the acceleration after you quit pushing should be essentially zero when the weight of the hanging mass and the friction force are equal. From your text, you know that the force of friction on objects at rest can exceed that for objects in motion. Therefore the force of friction must change at some low velocity. It is important
to make sure that the velocity is approximately constant and within the correct range. Keeping the variation in speed small is better, but don’t spend too much time on it.

Make an appropriate graph to determine whether the frictional force is directly proportional to the normal force in this case. If it is directly proportional, then determine the value for the coefficient of kinetic friction from your graph.

**Checking for velocity effects**

If we make the wooden block accelerate from rest rather than moving at a constant velocity, we can check whether the frictional force is really constant over a range of velocities from zero to the final velocity of the block at the end of the track. This is easily achieved by adding mass to the hanger. To evaluate the results, you will need to derive a relation between the acceleration of the wooden block, the hanging mass, and the frictional force. Include the details of this derivation in your report.

Draw free body force diagrams for both the hanging mass and the wooden block. For each of these objects the net force is equal to the mass times the acceleration according to Newton’s second law of motion. (Assume that the pulley has no mass and has frictionless bearings.) The tension of the string connecting the two objects together should appear in the equations for both objects. You should have two equations with two unknowns, namely, the string tension, $T$, and the acceleration, $a$. Eliminate $T$ and solve for $a$ in terms of various known quantities, including the $\mu_K$ value you found above. If the frictional force is indeed a constant, you should observe that the acceleration depends only on quantities with constant values. (The mass values can be varied, of course, but they are constant during a given run; in the equation, therefore, they are constants.) The acceleration during a given run should be constant if the frictional force is constant for a given combination of mass values.

Place 300 g of mass on top of the block and add enough mass to the hanger (start with 120 g) so that the final velocity of the block when the hanger hits the floor is 1.0–1.2 m/s. A significant range of velocities is necessary if we are to test the hypothesis that the frictional coefficient is a constant, independent of velocity. Release the block from rest and let it accelerate down the track. Avoid looking at an acceleration graph directly because it is quite noisy; averages of noisy data are not so precise. If the acceleration is constant, the velocity-time graph should be linear. If so, use the curve-fitting capability of the Capstone software to get the numerical values for the slope and its standard deviation. (If you need help from your TA to do this, ask.) Taking three or more runs with same mass values gives a more reliable average acceleration value than simply a single run. You can also see how consistent the acceleration values are from run to run.

Using the equation that you derived earlier for acceleration versus mass, predict the magnitude of the acceleration for the block using the numerical value of $\mu_K$ determined from your constant velocity measurements. Compare this prediction with the actual acceleration obtained from the slope of the velocity plot in this exercise. (If the velocity-time graph is significantly curved, compare your predicted acceleration with the slope of the graph over the velocity range employed for the constant velocity measurements.) Use the standard error of the slope in this region as the uncertainty estimate for acceleration.
Based on your data, does the frictional force between the wood and aluminum surfaces depend on the relative velocity of the two surfaces? If so, does the frictional force increase or decrease as the relative velocity at the interface increases?

**Friction between felt and aluminum**

**Constant velocity measurements**

Turn the wooden block over so the larger felt side slides along the aluminum track. Use the same procedure you used for constant velocity measurements of $\mu_K$ on the wood-aluminum interface, keeping the velocity in the range 0.2–0.3 m/s. Make an appropriate graph to determine whether the frictional force is directly proportional to the normal force in this case. If it is directly proportional, then determine the value for the coefficient of kinetic friction from your graph.

**Checking for velocity effects**

Examine the effect of velocity on friction at the felt-aluminum interface using the same procedure you used to study the wood-aluminum interface. Place 200 g on top of the wooden block and about 100 grams on the mass hanger. Use enough mass to achieve a maximum velocity of 1.0–1.2 m/s. Do several runs, checking for consistency and for constancy of the acceleration. Predict the acceleration, using your value of $\mu_K$ from the constant velocity measurements (valid for velocities in the 0.2–0.3 m/s range, at least) and the equation you derived above. Compare this predicted value with the average acceleration over the same velocity range you used to determine $\mu_K$. Again take three or more runs with the computer to get a better average and a meaningful uncertainty.

Based on your data, is the frictional force between the felt and aluminum surfaces independent of the relative velocity of the two surfaces? Does the frictional force increase, decrease, or remain constant as the relative velocity at the interface increases?

**Summary**

Summarize your findings clearly and concisely. Do your results support the hypothesis that friction can always be described by the simple equation given in your textbook? Cite numerical values from elsewhere in your report as you address this question.

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Before you leave the lab please:

- Quit the Capstone software.
- Straighten up your lab station.
- Report any problems or suggest improvements to your TA.
Lab 5. Simple Pendulum

Goals

• To design and perform experiments that show what factors, or parameters, affect the time required for one oscillation of a compact mass attached to a light string (a simple pendulum).

• To use a simple pendulum in an appropriate manner to determine the local acceleration of gravity.

Introduction

A simple pendulum consists of a relatively small (in dimension) mass on the end of a string, so the motion may be analyzed as if the mass were a point mass. (For masses with larger dimensions the rotational motion of the mass must also be included in the analysis for good agreement with experiment.) The parameters that potentially affect the period of a simple pendulum are relatively easy to study. Therefore the simple pendulum provides a good “test case” for the application of the scientific method. Although you may ask your TA for help, each lab group is responsible to decide how to measure the relevant quantities on their own. It is especially important in this lab for you to record how you make these measurements. If you are asked to repeat some portion of this laboratory on the lab exam, your lab notes will be your only source of information. Your choice of method also affects how you interpret your results.

Preliminary observations

Although theory is helpful, it is unwise to design an experiment on the basis of theory alone. A few preliminary observations can dramatically improve your experiment. First, set up Capstone to measure the elapsed time for one complete oscillation, or period, of the pendulum. Be sure that it is measuring what you think it is. Then let the pendulum oscillate 40–50 times and display your data in a table. Use care in releasing the pendulum at large amplitudes so that the photogate is not damaged. Practice releasing the pendulum so that it swings in a single plane. The mean value and the standard deviation of the period can then be determined. Do this for at least two initial amplitudes (initial angles). One amplitude should be about as large as you can reasonably manage. The other should be as small as you can reasonably manage, where the mass swings through a distance of only three or four mass-diameters.
Consider carefully whether the period varies randomly from swing to swing, or whether the period changes in a systematic fashion. When you examine the data on screen or on a printout, the scales may hide the information. If all period values are identical, maybe the number of displayed significant digits are too small. Make appropriate adjustments. If in trouble, ask your TA for assistance. If the period varies randomly, the standard deviation of the mean for the data in your table reflects random variations and is a good measure of uncertainty. In this case, averaging the period over ten or more back-and-forth swings can improve the precision of your period measurement. If the period varies systematically, the standard deviation is more related to (possibly unknown) changing conditions than it is to random variations. If at all possible, your experiment should be designed so that any systematic variations are smaller than the random variations.

One way to reduce the systematic error in this case is to make five separate measurements of the period for single back and forth swings; then calculate the average and standard deviation of the mean of these measurements. It is important to have a reliable value for the uncertainty in your measurements, as they are needed to determine which parameters affect the period in this experiment. Real differences must be larger than these uncertainties.

What makes a pendulum tick?

For the simple pendulum determine which parameters affect the period (defined as the time for one complete back and forth swing) of the oscillation. Consider such things as the amplitude (the angle of the swing) of the oscillation, the mass of the bob, and the string length. Vary each parameter over as wide a range as is feasible with the equipment at hand. You will need to support your findings with adequate data in order to be convincing. Explain the effect of each parameter on the period.

Determine the acceleration of gravity

When a pendulum is displaced from the vertical position, it is the gravitational force that is ultimately responsible for bringing it back to the vertical position. Thus it is not surprising that there is a relationship between the oscillation period and the acceleration of gravity. We can imagine that on the Moon, where the acceleration due to gravity is less than here on Earth, the force bringing the pendulum back to vertical would be smaller; thus the acceleration would be smaller, and the time for an oscillation would be larger. It appears then that the period of the pendulum and the acceleration of gravity are related by an inverse relation; that is, when one parameter gets larger the other gets smaller.

You may need to look in a textbook to find an expression for the relationship between the acceleration of gravity and the period of oscillation. (A shortcut to a PDF copy of the open source OpenStax College Physics is on the desktop of your lab computers.) Be sure to note under what conditions the relationship is valid and plan your experiment accordingly. Then take data to determine the acceleration of gravity. Some of the data from the previous exercise may be useful, but you will need to supplement it in order to make a good determination of $g$.

Use a graphical technique to find $g$. (Hint: Find a way to graph the measured parameters in such a way...
fashion that $g$ may be calculated from the slope of a straight-line graph. Use the uncertainty of the slope (the standard error) to determine whether your measurement is consistent with the accepted value of $g$ for Pullman, Washington. Your TA will have some suggestions here, if necessary.

**General reminders**

Carefully describe your measurement procedures in your notes. Be sure that any conclusions you make are justified by your data. When can differences in measured values be attributed to random variations, and when do they represent real differences? How do you decide? Show representative calculations for each step in your analysis.

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<td>Straighten up your lab station.</td>
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<td>Report any problems or suggest improvements to your TA.</td>
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Lab 6. Newton’s Third Law and Momentum

Goals

• To explore the behavior of forces acting between two objects when they touch one another or interact with one another by some other means, such as a light string.

• To compare the magnitudes of the forces exerted by one object on another object and vice versa during collisions.

• To experimentally explore the relationship between an impulse and a change in motion.

• To understand the relationship between impulse and momentum.

• To explore and understand conservation of momentum.

Introduction

You have already explored how the motion of an object is affected by applied forces, such as gravity. We can extend our understanding of forces by recognizing that all forces are exerted by some object on some other object. This raises the question of whether forces operate in a reciprocal fashion. Suppose Object A exerts a force on Object B. Does Object B exert any kind of force on Object A? If so, what can we say about this second force? Most of our work so far has involved the force of gravity, where Earth is the actor or agent of force. The great size and mass of Earth make it difficult to measure changes in its motion due to any forces we might apply to it. In this experiment we focus on forces acting between two smaller objects.

We also wish to explore the relationship between the impulse, defined as the area under the force-versus-time graph during a brief interaction, to changes in the momentum, \(mv\), of an object with mass \(m\) and velocity \(v\). (For 201 students, the impulse \(J\) can also be defined as \(J = \int F\,dt\), where the integral is calculated over the time during which the force, \(F\), is applied.) Finally we will look at the momentum of a two-body system of masses immediately before and after they interact with each other.

Forces of interaction—Connected objects

Two force sensors are attached individually to the tops of two carts that can roll on an aluminum track. Before beginning any measurements, make sure that both force sensors are “zeroed” by
pressing the “tare” button on the side of each sensor. Check by means of a quick force-time graph that both sensors really read very close to zero. Also make certain that the data sampling rate that you have set is sufficiently high to record the force variations that take place. If the graphs look “jagged,” with straight line segments connecting the data points, then increase the sampling rate until the lines connecting the data points form a smooth curve (even when you expand the time scale). Note that the force sensor mounted on Cart 1 measures the force exerted on Cart 1 by Cart 2, and the force sensor mounted on Cart 2 measures the force exerted on Cart 2 by Cart 1. Since the force sensors are oriented “back-to-back,” one sensor will measure positive forces to the right and the other will measure positive forces to the left. This difference is critical because forces are vector quantities.

**Carts with equal masses**

Place both carts on the track and connect the hooks on the force sensors together. Place one hand on each cart (you can use both hands or use one of your’s and one of your lab partner’s) and push them together or pull them apart (not literally; getting too violent here can damage the force sensors!) all the while recording both forces as a function of time. Sketch a representative time segment of your data in your lab notes. From your data what can you conclude about the magnitudes of the forces exerted by Cart 1 on Cart 2 and by Cart 2 on Cart 1? Do your conclusions change if the carts experience a nonzero acceleration? What can you conclude about the direction of the force on each cart due to the other cart? Remember to zero (tare) the force sensors before each set of measurements!

**Carts with unequal masses**

Add two steel bars (approximately 0.5 kg each) to the cart on the right. With your hand touching only the cart on the left, pull and push the cart on the right while recording a graph displaying both forces as a function of time. (Of course the force sensor hooks must still be connected.) From your data what can you conclude about the magnitudes of the forces exerted by Cart 1 on Cart 2 and by Cart 2 on Cart 1? Do your conclusions change if the carts experience a nonzero acceleration? What can you conclude about the direction of the force on each cart due to the other cart?

**Carts connected by string**

Now connect the carts with a short length of string. (Only “pulls” are possible with a string, because the string goes slack if you push the carts together.) Do your conclusions change if the carts experience a nonzero acceleration? How does the presence of the string between the carts affect your answers regarding the forces you observed previously?

**Summary of forces of interaction—connected objects**

Summarize your conclusions for connected objects clearly and concisely before continuing. Compare your results with what you would predict on the basis of Newton’s Third Law. Several common mistakes in homework and on exams relate to the observations you have just made. Ask your teaching assistant if you have any doubts.
Force of interaction—Colliding objects

“Bouncy” collisions with equal cart masses

Mount the springs in place of the hooks on the ends of the force sensors. Starting with one cart stationary, give the other cart a push so it collides with the stationary cart. Don’t push the cart so hard that the springs totally compress during the collision. Display both forces as a function of time in a graph. Focus on the time interval from just before the collision to just after the collision and rescale the graph to show this region clearly. You may also need to adjust the sampling rate of the force sensors to get sufficient data during the collision itself. From your data what can you conclude about the magnitudes of the forces exerted by Cart 1 on Cart 2 and by Cart 2 on Cart 1? What can you conclude about the direction of the force on each cart due to the other cart?

What difference does it make in the force relationships if both carts are moving prior to the collision? Support your answer with additional data.

“Bouncy” collisions with unequal cart masses

Add two steel bars (approximately 1 kg) to one of the carts. Starting with one cart stationary, give the other cart a push so it collides with the stationary cart. Perform two trials, first with the high-mass cart stationary and second with the low-mass cart stationary. Again avoid pushing the cart so hard that the springs totally compress during the collision. Display both forces as a function of time in a graph. Focus on the time interval from just before the collision to just after the collision and rescale the graph to show this region clearly. From your data what can you conclude about the magnitudes of the forces exerted by Cart 1 on Cart 2 and by Cart 2 on Cart 1? What can you conclude about the direction of the force on each cart due to the other cart?

What difference does it make in the force relationships if both carts are moving prior to the collision? Support your answer with additional data.

“Sticky” collisions with equal cart masses

Remove the springs from the ends of the force sensors and replace them with small metal “cups” holding pieces of clay. Starting with one cart stationary, give the other cart a push so it collides with the stationary cart. Disregard any trial in which the carts don’t remain stuck together after the collision. Display both forces as a function of time in a graph. Focus on the time interval from just before the collision to just after the collision and rescale the graph to show this region clearly. You may also need to adjust the sampling rate of the force sensors to get sufficient data during the collision itself. From your data what can you conclude about the magnitudes of the forces exerted by Cart 1 on Cart 2 and by Cart 2 on Cart 1? What can you conclude about the direction of the force on each cart due to the other cart?

What difference does it make in the force relationships if both carts are moving prior to the collision? Support your answer with additional data.
“Sticky” collisions with unequal cart masses

Add two steel bars (approximately 1 kg) to one of the carts. Starting with one cart stationary, give the other cart a push so it collides with the stationary cart. Perform two trials, first with the high-mass cart stationary and second with the low-mass stationary. Disregard any trial in which the carts don’t remain stuck together after the collision. Display both forces as a function of time in a graph. Focus on the time interval from just before the collision to just after the collision. Rescale the graph to show this region clearly. From your data what can you conclude about the magnitudes of the forces exerted by Cart 1 on Cart 2 and by Cart 2 on Cart 1? What can you conclude about the direction of the force on each cart due to the other cart?

What difference does it make in the force relationships if both carts are moving prior to the collision? Support your answer with additional data.

Summary for forces of interaction—colliding objects

Summarize your conclusions for “bouncy” and “sticky” collisions clearly and concisely before proceeding. Based on your observations how are the results changed by the different collision conditions? You will eventually learn that mechanical energy is lost in sticky collisions but is mostly conserved in bouncy collisions. Something that is true in both kinds of collisions can help you when the concept of conservation of energy is not useful.

Impulse and momentum during collisions

In this experiment, you will measure the impulse delivered to a cart as it strikes the end of the track. The bracket mounted at one end of the Pasco track has a small hole at just the right height for mounting a spring or clay cup to meet the end of the force sensor on the cart. The impulse is calculated by finding the “area” under the force-time curve during the collision using Capstone. This area has the units of force \( \times \) time, or N-s. The impulse will be compared to the change in momentum of your cart. Momentum has units of kg-m/s. Show that units of N-s are equivalent to momentum units in your notes.

The change in momentum experienced by the cart can be calculated from the velocities of the cart just before and after the collision. An ultrasonic motion sensor is used to measure the velocity of the cart. Please refer to the Computer Tools Supplement at the end of your lab manual for more information on using the motion sensor.

Impulse and momentum in “sticky” collisions

Screw the clay cup from the unused force sensor into the bracket at the end of the track, and set up the motion sensor to measure the velocity of the cart as it moves down the track. The force sensor on the cart you are using should still have the clay cup attached to it. Give the cart a quick push down the leveled track so that it sticks to the clay on the end bracket. Disregard any trial when it doesn’t stick securely. Display graphs of the force and the velocity as functions of time as the cart travels down the track and sticks to the end.
To find the impulse (the area under the force-time plot), use the “Highlight range of points in active data” tool (icon with yellow pencil and red points) along the top of the graph to select the force data that correspond to the collision. Then click on the “Display area under active data” icon (red line shaded below in gray). The area under the selected force data will appear in a box on the graph in units of N-s. Now compare the values of the impulse of the contact force during the collision with the change in momentum of the cart. What conclusion can you draw from your data? Several trials may be necessary to discover a pattern. Remember that impulse and momentum are vector quantities, so the positive x-directions for the force sensor and the motion sensor need to be considered carefully.

**Impulse and momentum in “bouncy” collisions**

Replace the clay cups with the springs to produce a bouncy collision at the end of the track. Again compare the values of the impulse of the contact force during the collision with the change in momentum of the cart. What conclusion can you draw from your data? Several trials may be necessary.

**Summary for impulse and momentum in collisions**

Summarize your conclusions for “sticky” and “bouncy” collisions clearly and concisely before proceeding. Based on your observations, how do the results compare when the collision conditions change?

**Conservation of momentum**

Using the motion sensor with clay cups on both carts, explore whether the sum of the momenta of the two carts is the same before and after a “sticky” collision. We investigate only the “sticky” collision, because we can determine the total momentum with only one velocity measurement before the collision and one after. That is all the motion sensor can do.

**Equal cart masses**

Push the cart closest to the motion sensor toward the second stationary cart so the carts stick together and move off together after the collision. Compare the momentum of the system of the two carts just prior to the collision with the combined momentum just after the collision. Several trials may be necessary to get a good measurement of the velocity after collision, where the carts stick together.

**Unequal cart masses**

Add two steel bars to the cart closest to the motion sensor and repeat the experiment. Compare once again the momentum of the two-cart system before and after the collision. Several trials are in order.
Summary for conservation of momentum

Summarize your conclusions for equal and unequal cart masses clearly and concisely before going to the Synthesis section. Based on your observations, how are the results changed by varying the cart masses? What predictions can be made regarding the total momentum of both carts just prior to a collision compared to the total momentum of both carts just after the collision? Do your experimental momentum measurements results agree with your predictions about the momentum of the two-cart system before and after the collision?

Synthesis

Using your observations on forces between interacting objects, discuss how the impulses given to the carts in a two-cart collision are (should be) related. (Refer to Newton’s Third Law.) From your observations of impulse and momentum, how do the momenta of the two carts change during a collision if the contact forces during the collision represent for all practical purposes the net forces acting on the carts? Remember that the change in any quantity is defined as the final value minus the initial value. Impulse and momentum are vector quantities, so you need to pay close attention to directions as well as magnitudes.

Before you leave the lab please:
- Quit all computer applications that you may have open.
- Put the hooks back on the force sensors.
- Place equipment back in the plastic tray as you found it.
- Report any problems or suggest improvements to your TA.
Lab 7. Work and Energy

Goals

- To apply the concept of work to each of the forces acting on an object pulled up an incline at constant speed.
- To compare the total work on an object to the change in its kinetic energy as a first step in the application of the so-called Work-Energy Theorem.
- To relate the work done by conservative forces to the concept of potential energy.
- To apply the concept of conservation of mechanical energy, where mechanical energy is defined as the sum of kinetic and potential energy, to a system where the work done by nonconservative forces is zero or cancelled out, as in this experiment.

Introduction

The notion of “work” has a special meaning in physics. When the applied force is constant in magnitude and direction, and the motion is along a straight line, the formula for work reduces to:

\[ W = F d \cos \theta = (F \cos \theta) d = F (d \cos \theta) \quad (7.1) \]

where \( F \) is the magnitude of the force, \( d \) is the magnitude of the displacement, and \( \theta \) is the angle between the force vector and the displacement vector. Since magnitudes are always positive, \( F \) and \( d \) are always positive, and the sign of the work is determined by the factor of \( \cos \theta \).

If the force is not constant, then one must sum the work done over each of a series of very small displacements, where the force is approximately constant over each small displacement. In calculus, this process is described in terms of integration.

The concept of work is most useful for point particles in the presence of conservative forces (no friction). Because work is a scalar and forces are vectors, problems that can be solved using the work concept are usually easier to solve by using work than by using Newton’s Second Law.
CHAPTER 7. WORK AND ENERGY

Work done on a cart moving at constant velocity

Carefully place the wooden block on edge under the end of the track opposite the pulley so that the track is inclined at an angle of 4–5° to the horizontal. “Hooking” the small rubber feet on the bottom of the track over the edge of the block will keep the track from slipping and changing the angle if the track is bumped. Determine the angle of the ramp to within about one-tenth of a degree. A protractor can’t be read accurately enough; use trigonometry! Measure the mass of the cart and the mass of one of the aluminum bars. Two aluminum bars will be placed in the cart for this experiment.

Work done by you on the cart with spring scale parallel to track

Using the small spring scale held parallel to the ramp, pull the cart with the aluminum bars on board at a slow constant velocity up the ramp a distance of 0.5 m. From the definition of work in your textbook or the Introduction above, calculate the work done by you on the cart as you pull the cart up the ramp. Be careful of units! The gram readings of the spring scale must be converted to newtons.

Repeat the measurement as you carefully lower the cart down the ramp at constant velocity.

Work done by you on the cart with spring scale inclined 60° to track

Pull the cart up the track through the 0.5 m distance at constant velocity while holding the spring scale at an angle of 60° with respect to the ramp. Again calculate the work done by you on the cart as you pull the cart up the ramp. Compare these values and comment on the results.

Repeat the measurement as you carefully lower the cart down the ramp at constant velocity.

Work done by gravity on the cart as it moves up and down the ramp

Draw a free-body force diagram of the cart being pulled up the ramp. (Ignore friction.) You have already computed the work you did as the cart was pulled up the ramp. Now calculate the work done by each of the other forces. Show the steps of your analysis carefully and be careful of signs.

Repeat the free-body diagram and work calculations for the cart as it moves down the ramp. Use a table to show the values of the work done by each force acting on the cart for the 0° and 60° orientations of the spring balance. Sum the values of the work to find the total work done by all the forces acting on the cart for each of the two cases.

When a net force begins to act on an object at rest, the object begins to move. One can argue mathematically (see your textbook for the details) that the work done on the object (neglecting friction) is equal to the change in its kinetic energy if we define the kinetic energy to be \( \frac{mv^2}{2} \), where \( m \) is the mass of the object and \( v \) is its speed. Remember that the net force on the cart is zero when it moves with constant velocity.
Based on the Work-Energy Theorem, what total work would you expect for each case? Did your calculated total work values behave in accordance with these expectations? Make quantitative comparisons between your expected and experimental results. Explain.

**Applying the Work-Energy theorem to an accelerating cart**

Before beginning this investigation, level the track and take the aluminum bars off the cart. Then add or subtract paper clips on the end of the string as necessary to cancel the frictional forces acting on the pulley and cart as the cart moves toward the pulley end of the track. When you have achieved the correct balance between the weight of the paper clips and the friction, the net force on the cart will be very close to zero. Then the acceleration of the cart should also be very close to zero. (What will the velocity-time graph look like if the acceleration is zero?) Give the cart a gentle push toward the pulley and use Capstone with the “Photogate with Pulley” sensor to measure the acceleration. Adjust the number of paper clips as necessary to “fine-tune” the apparatus, so that the average acceleration is as close to zero as possible.

Place a 20-g mass on the end of the string in addition to the paper clips. The net force on the system (cart plus hanging mass) is now the weight of the 20-g mass. As the cart moves and the 20-g hanging mass descends, work is done by the gravitational force on the cart-hanging mass system. Is work done by any other forces acting on the system? Remember that we have “cancelled out” the frictional force with the paper clips, so friction need not be considered here.

Move the cart to the end of the track opposite the pulley and release it from rest. Click the “Start” button in Capstone a couple seconds before releasing the cart. This ensures that you get some data before the cart is released. With the “Photogate with Pulley” sensor, Capstone defines the position of the cart at the beginning of data collection to be zero. This will be helpful below.

Take appropriate data to address the question of whether the total work done on the system is equal to the change in kinetic energy. Note that the computer can calculate the total work done since the system was released from rest and the instantaneous value of the kinetic energy in real time as the cart moves. Use the Capstone “Calculator” tool from the Tools Palette to define expressions for the work done on the system and for the kinetic energy. Your TA can assist if necessary. Be sure to show the reasoning used to get these expressions in your lab notes. These defined quantities can then be displayed on a graph just like other measured quantities. Displaying the work and the kinetic energy on the same graph provides the simplest method for comparing the two as a function of time.

Print out the results and discuss your findings. Be sure to address the issue of “change in kinetic energy” versus just “kinetic energy.” Are they ever the same?

**Kinetic and potential energy**

A slightly more complicated arrangement is produced by raising the end of the track opposite the pulley to make an angle of about 2° with the horizontal. (Laying the block of wood flat on the table under the feet of the track should be adequate.) With the 20-g mass still hanging from the end of the string, the net force on the system of the cart and hanging mass now involves more than one
force. (Note: The assumption that the friction force is not changed much by raising the ramp to a small angle should be very good.) Again address the question, does the total work done by the forces equal the change in the kinetic energy of the system? Collect appropriate data to determine the validity of this hypothesis.

**Potential energy and the conservation of energy**

Where does the idea of “potential energy” come from? In many ways potential energy is an intuitive concept from everyday experience. For example, if you are hit by a falling apple, you know instinctively (or by experience?) that the damage it does depends on the height from which it falls. We might even be tempted to think about the notion of “conservation of energy.” While the apple is falling and losing energy of position (potential energy), is it possible that the energy of motion (kinetic energy) increases so that the sum of the two energies remains constant? One approach to answering this question is to assume that the sum of the kinetic and potential energies remains the same, and then try to discover how the potential energy would have to be defined to obey such a conservation law.

In the previous exercises you have hopefully shown that the work \( W \) done on an object by a net force is equal to the change in kinetic energy \( KE \) of that object. Mathematically we say:

\[
W = \Delta KE = KE_{\text{final}} - KE_{\text{initial}}
\]  
(7.2)

If our energy conservation idea is to be correct, then something like potential energy (denoted \( PE \)) must exist so that the sum of \( KE \) and \( PE \) is constant:

\[
KE_{\text{final}} + PE_{\text{final}} = KE_{\text{initial}} + PE_{\text{initial}} \quad \text{or}
\]  
(7.3)

\[
KE_{\text{final}} - KE_{\text{initial}} = PE_{\text{initial}} - PE_{\text{final}} \quad \Rightarrow \quad \Delta KE = -\Delta PE
\]  
(7.4)

The energy conservation idea from Equation 7.4 can be reconciled with the work-energy relationship from Equation 7.2 only if the change in \( PE \) is equal to the negative of the work done. That is,

\[
\Delta PE = -W
\]  
(7.5)

This relationship cannot be used in the presence of forces like friction since the position of an object in space does not uniquely specify the work done by friction in the process of moving the object to that position; thus the potential energy cannot be uniquely defined either. Fortunately for us, the work done by the gravitational force and the electric force, two of the most common forces in nature, are both defined uniquely as objects move from one place to another. (Refer to your textbook for a discussion of conservative forces, such as gravity and electric forces, and non-conservative forces, such as friction.) Let’s apply Equation 7.5 to a simple case of an object of mass, \( m \), near the earth’s surface falling vertically from a position, \( y + h \), to a lower position, \( y \).
Since the gravitational force is downward, the work done by gravity is positive and equal to $mgh$. Therefore

$$\Delta PE = PE_y - PE_{y+h} = -mgh \quad \text{or} \quad PE_{y+h} - PE_y = mgh \quad (7.6)$$

Interestingly, Equation (7.6) only specifies that the difference in the potential energy from the initial to the final state must be $mgh$. This can be satisfied by setting $PE_{y+h}$ to $mgh$ and setting $PE_y$ to zero. An equally valid solution would be to choose $PE_{y+h} = mg(y + h)$ and $PE_y = mg y$. In this case the zero of potential energy will occur when $y = 0$. There is a certain amount of latitude in choosing the “zero” of potential energy. This is always the case!

If an object moves horizontally near the earth’s surface, the gravity force has no component along the direction of the displacement. Thus no work is done, and the potential energy of the object does not change. In cases where both horizontal and vertical displacements occur, only the vertical displacement leads to a change in potential energy of the body.

**Plotting the cart’s total mechanical energy as a function of time**

Use Capstone’s Calculator Tool to calculate and plot the sum of the kinetic and potential energies of the cart/mass system as the cart moves down the track. Is the total mechanical energy of the system conserved?

**Conclusion**

Summarize all your findings carefully and succinctly. Where possible, discuss your results in terms of the measurement uncertainties.

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Before you leave the lab please:

- Quit all computer applications you may have open.
- Place equipment back in the plastic tray as you found it.
- Report any problems or suggest improvements to your TA.
Lab 8. Ballistic Pendulum

Goals

• To determine the launch speed of a steel ball for the short, medium, and long range settings on the projectile launcher apparatus using the equations for projectile motion.

• To use the concepts of gravitational potential energy and conservation of mechanical energy to determine the speed of the ball plus pendulum as it first begins to swing away from the vertical position after the “collision.”

• To explore the relationships between the momentum and kinetic energy of the ball as launched and the momentum and kinetic energy of the ball plus pendulum immediately after the ball is caught by the pendulum apparatus.

Introduction

The “ballistic pendulum” carries this name because it provides a simple method of determining the speed of a bullet shot from a gun. To determine the speed of the bullet, a relatively large block of wood is suspended as a pendulum. The bullet is shot into the wooden block so that it does not penetrate clear through it. This is a type of “sticky” collision, where the two masses (bullet and block) stick to one another and move together after the collision. By noting the angle to which the block and bullet swing after the collision, the initial speed can be determined by using conservation of momentum. This observation incorporates some predictions that we can check. In this experiment, the ballistic pendulum apparatus will be used to compare the momentum of the steel ball before the “collision” to the momentum of the ball and pendulum apparatus, equivalent to the wooden block plus the bullet, after the collision. A comparison of the kinetic energy of the ball before the collision with the kinetic energy of the system afterward will also be made.

Figure 8.1 shows a diagram of the ballistic pendulum apparatus. For the ballistic pendulum experiment, the projectile launcher from the projectile motion laboratory is mounted horizontally so that the pendulum can catch the emerging steel ball. The angle indicator can be used to measure the maximum angle reached by the pendulum as it swings after the collision. The angle indicator should read close to zero when the pendulum is hanging in the vertical position. If the reading is measurably different from zero, then take the difference in the angle readings (maximum angle reading minus initial angle reading).
Warning: Never look down the launcher barrel. Wear eye protection until everyone is finished launching projectiles.

Figure 8.1. Ballistic pendulum apparatus.

Momentum of steel ball before collision

For this part of the experiment, remove the pendulum by gently unscrewing the rod that supports its upper end. Now determine the muzzle velocity of the steel ball by firing it horizontally and measuring the distance traveled horizontally before striking the ground. Do this for the short, medium, and long range settings of the launcher. The momentum of the ball is found by multiplying its mass times its velocity. Quantitatively estimate the uncertainties in these momentum values based on the uncertainties of the measured horizontal distance traveled and the measured vertical height. The momentum of the ball-pendulum system before the ball collides with the pendulum is now known.

Momentum of ball and pendulum after collision

The speed (and from it the momentum) of the ball and pendulum just after the collision is computed by assuming that the kinetic energy of the ball and pendulum just after the collision is totally converted into gravitational potential energy at the top of its swing. This requires that the frictional forces on the ball and pendulum system during the swing are small (negligible). The increase in gravitational potential energy is just the weight of the pendulum times the change in height, and the change in height can be computed from the maximum angle of the pendulum swing and some straightforward trigonometry. Since the pendulum is not a point mass, the change in potential energy is given by the change in height of its center of gravity. The center of gravity can be located...
by removing it from its support screw at the top and then balancing it on a “knife edge”. (A thin ruler works.) While you have the pendulum disassembled, be sure to measure the mass of the pendulum and the distance from the pivot point at the top to the center of gravity.

Mount the pendulum so that it will catch and trap the steel ball before proceeding. Be gentle as you screw in the pendulum support rod; it does not need to be tight. Now launch the ball into the pendulum using the short, medium, and long range settings of the projectile launcher. Repeat each measurement several times and take appropriate averages. (Remember to check the initial angle of the pendulum at rest.)

From your data calculate the speed of the pendulum and ball together just after the collision. Multiply by the appropriate mass to get the momentum. The momentum of the ball-pendulum system just after the collision is now known.

Is momentum conserved?

Compare the initial momentum of the ball and pendulum system before the collision with the final momentum of the same system just after the collision using your calculated velocities and measured masses just before and just after the collision. Is momentum conserved? You cannot answer this question without comparing the difference between the two momenta with the uncertainty of this same difference. That is the motivation of the $t'$-score. The score is the ratio of two values. The nominator the ratio of the difference of the momentum before and after the collision (the absolute value). The denominator is the combined uncertainty of both momenta (remember that they add in quadrature). The $t'$-score is large when the difference in momenta before and after the collision is much larger than the combined uncertainty. In these labs the cut-off threshold is 3. $t' > 3$ means that momentum is not conserved. $t' < 3$ means that momentum is conserved during the collision.

If your $t'$-score exceeds 3, maybe there is an error in your math. Check your procedures and calculations.

Is kinetic energy conserved?

Since you know the masses and speeds of the objects before and after the collision, you can calculate the kinetic energies of the system before and after the collision. Is kinetic energy conserved? To answer this question, you will need to estimate your experimental uncertainties and compare them with any observed differences, as you did to test conservation of momentum. Assuming that momentum is conserved before and after the collision, find a general symbolic mathematical expression for the ratio of the final kinetic energy over the initial kinetic energy. You may need some help from your TA here. Using the data from your earlier calculations, compare your experimental kinetic energy ratio to that predicted by assuming momentum is conserved. Is it the same ratio? Is overall energy conserved in this collision? If so, what forms of energy would need to be included to satisfy the general energy conservation principle?

Note: A simplification has been made by assuming that the pendulum consists of a point mass on the end of a string whose length is equal to the distance from the pivot point to its center of mass.
When the pendulum swings, it necessarily rotates about its center of mass. This suggests that some rotational kinetic energy is imparted to the ball and pendulum system along with its translational kinetic energy ($mv^2/2$). If significant, this would produce a systematic error in the calculated speed of the ball and pendulum system after the collision. Would it make the calculated speed too high or too low? Can you detect any systematic error in your calculated values? Discuss.

**Summary**

Mechanical energy and momentum are conserved only when certain conditions are met. Qualitatively summarize your results, explaining why the collision between the ball and the pendulum conserves momentum but not mechanical energy. Similarly, explain why the motion of the pendulum during its swing conserves mechanical energy but (apparently) not momentum.

<table>
<thead>
<tr>
<th>Before you leave the lab please:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quit all computer applications that you may have open.</td>
</tr>
<tr>
<td>Place equipment back in the plastic tray as you found it.</td>
</tr>
<tr>
<td>Report any problems or suggest improvements to your TA.</td>
</tr>
</tbody>
</table>
Lab 9. Buoyancy

Goals

• To experimentally determine the relationship between the buoyant force on an object that displaces a known weight of water.

• To compare the buoyant behavior of an object more dense than water with that of an object less dense than water.

• To calculate the densities of aluminum and wood cylinders from your data and to compare these densities to handbook values.

Introduction

Buoyancy is the name given to the force that arises when an object displaces a fluid (either a gas or a liquid) in a force field (usually gravity). The buoyant force is responsible for keeping ships from sinking and for keeping hot air balloons in flight. If an object is at rest in or on a fluid, it experiences a net force of zero. The forces acting on a partially or completely submerged object at rest generally include the gravitational force, \( F_g \), the buoyant force, \( F_{\text{Buoyant}} \), and whatever additional force (if any) is required to hold the object in position. In the work below, this holding force is supplied by and measured with a force sensor. When the object is at rest,

\[
F_{\text{Sensor}} + F_g + F_{\text{Buoyant}} = 0
\]  

(9.1)

In this lab, you will measure the buoyant forces acting on two cylinders as they are submerged in water, and relate these forces to the weight of the water they displace. You will also calculate the average densities of each cylinder. The cylinders are marked at centimeter intervals along their lengths. The cylinders are lowered into a beaker with their long axes perpendicular to the water surface, so the volume submerged, \( V_s \), is

\[
V_s = \pi R^2 L_s = AL_s
\]  

(9.2)

where \( L_s \) is the length of the submerged portion of cylinder, \( R \) is the cylinder radius, and \( A \) is the cylinder’s cross-sectional area. The weight of the water displaced by the object is given by
\[ W_s = \rho_w V_s g \]  

(9.3)

where \( \rho_w \) is the density of water \((1.000 \times 10^3 \, \text{kg/m}^3)\).

**Buoyant force on an object more dense than water**

Before beginning, draw a free-body diagram showing the forces on an object (more dense than water) partially submerged in water.

If you have questions on how to set up the force sensor, refer to the Force Sensor section of the Computer Tools Supplement at the back of the lab manual. The force sensor uses one of the analog input channels (A, B, or C) on the interface unit.

Carefully hang the aluminum cylinder on the force sensor. With the lab jack near its maximum height, adjust the position of the force sensor so that the cylinder sits completely inside the tall beaker without bumping the sides. Then add water to the beaker until the aluminum cylinder is completely submerged. This ensures that the beaker has enough water to cover the cylinder, but is not so full that water will spill when the cylinder is immersed. Use water from the sink at the back of your lab.

To begin the experiment, lower the lab jack so that the aluminum cylinder is completely out of the water. Lift the aluminum cylinder slightly so no weight force pulls on the force sensor. While lifting the cylinder tare the force sensor. This sets the force sensor reading to zero force. Gently release the cylinder. The force sensor should now read the weight force of the cylinder. The force sensor reading as the cylinder is immersed will then decrease by an amount equal to the upward buoyant force by the water. The force sensor reads positive for pushing into the sensor. Hence, the weight force is pulling on the sensor and negative; the buoyant force is pushing up; it changes the reading in the positive direction.

Now vary \( L_s \), the length of the cylinder that is submerged (in 1-cm increments), measuring the buoyant force for each increment, until the cylinder is completely submerged. Try to complete these measurements within about five minutes. This minimizes the effect of sensor drift on the force readings. At the end of the measurements completely lower the labjack to remove the cylinder from the water and make a final reading. Any drift in the sensor will show up as a somewhat different weight force due to the cylinder.

Make a graph that compares the buoyant force (on the \( y \)-axis) with the weight of the water displaced (on the \( x \)-axis) for each value of \( L_s \). Since the measured force is the combination of weight force of the cylinder and the buoyant force, some math is required to extract the buoyant force. From the graph, can you deduce a simple mathematical relationship between the two? Discuss/Explain. Look up Archimedes’ Principle in a textbook. Do your results support the validity of Archimedes’ Principle?
Buoyant force on an object less dense than water

Before beginning, draw a free-body diagram showing the forces on an object (less dense than water) partially submerged in water. Do the directions of the forces shown depend on the density of the object, or just their magnitudes?

Replace the aluminum cylinder with the wood cylinder, which attaches directly to the force sensor by a screw. Do not remove the threaded screw in the top of the wood cylinder! Its mass is negligible relative to the mass of the cylinder itself. Tare the sensor once again, before screwing the wood cylinder to the sensor. Do not screw the cylinder more than 5 mm (about 1/4 inch) into the force sensor. The force sensor can be damaged by screwing it in too far.

Again, measure the buoyant force as the submerged portion of the cylinder is increased in 1-cm increments. Do the buoyant force and the weight of the displaced water obey the same relation in this case? Discuss/explain.

Relative density of aluminum and wood

In your notebook, show that the density of any solid unknown material $\rho_{\text{Unknown}}$ obeys the following relationship:

$$\frac{\rho_{\text{Unknown}}}{\rho_w} = \frac{W_{\text{Unknown}}}{F_{\text{MaxBuoyant}}}$$

(9.4)

where $W_{\text{Unknown}}$ is the weight of the unknown object, $F_{\text{MaxBuoyant}}$ is the maximum buoyant force on the object (that is, the buoyant force when it is totally submerged in water), and $\rho_w$ is the density of water.

Use Equation 9.4 to calculate the density of the aluminum and the wood. Compare your calculated densities with accepted values from a textbook or handbook. A copy of the Handbook of Chemistry and Physics should be in your lab room. The density of natural wood can vary considerably, but the handbook should list some kinds of wood with densities that are consistent with your measurement. List some examples in your notebook. In contrast, the density of aluminum should not vary more than 0.05%. Does your measured density for aluminum agree with the handbook value within the limits of the expected error for this experiment? If your measurement and the handbook value disagree by more than three standard deviations, carefully examine your calculations and procedures for sources of error.

Some questions for extra credit

For extra credit, you may address the following questions in your notes:

1. How can an aircraft carrier (or a concrete canoe) float when the density of the material used to build it is greater than the density of water?
2. Does air create a buoyant force? If so, estimate the magnitudes of the buoyant forces due to the atmosphere on your wood and aluminum cylinders?

3. Is the direction of the buoyant force always opposite the direction of the gravitational force?

4. Does the buoyant force on a totally submerged object change as the object moves farther and farther below the surface of the fluid?

Before you leave the lab please:

- Quit Capstone.
- Replace the small hook in the force sensor.
- Straighten up your lab station.
- Report any problems or suggest improvements to your TA.
Lab 10. Rotational Dynamics

Goals

• To calculate the moment of inertia of two metal cylindrical masses from their measured dimensions and their distance from the axis of rotation.

• To use the principle of conservation of mechanical energy (the sum of kinetic and potential energies) to derive an expression for the moment of inertia in terms of other measurable quantities for a system involving both translational and rotational motion.

• To determine the moment of inertia of two brass masses from the motion of the rotating system using the Rotary Motion Sensor to determine the angular speed of rotation and using appropriate graphical techniques for analyzing the data.

• To compare quantitatively the values of the moments of inertia determined by these two very different methods. Are they equal within the limits of expected uncertainties?

Introduction

In rotational motion the moment of inertia, $I$, plays a role similar to that of mass in translational (or linear) motion but with some important differences. If a point mass $m$ is located a perpendicular distance $r$ away from its axis of rotation, its moment of inertia is just $mr^2$. If more than one point mass is present, then the total moment of inertia is just the sum of the individual moments of inertia. If there are just two masses, labeled A and B, the total moment of inertia is:

$$I_{Total} = I_A + I_B \quad (10.1)$$

If there are a large number $N$ of point masses, labeled $i = 1, 2, ... N$, rotating about a common axis, and each mass $m_j$ is located a perpendicular distance $r_j$ from their common axis, the total moment of inertia is:

$$I_{Total} = \sum_{j=1}^{N} I_j = \sum_{j=1}^{N} m_j r_j^2 \quad (10.2)$$

For an extended object like a solid sphere, the moment of inertia can be computed by dividing the object into small (infinitesimal) masses and summing the moments of inertia of each mass. Since
the moment of inertia of an object depends on the distances \( r_j \), changing the axis of rotation usually changes the moment of inertia.

The apparatus consists of a Rotary Motion Sensor with a bar attached. Additional cylindrical masses can be attached to the bar. As the rod rotates, the masses move in a circle around the axis of the apparatus. The moment of inertia (for a single mass) corresponding to this motion is \( md^2 \), where \( d \) is the distance from the center of the mass to the axis of rotation. If this were a point mass, this would be its entire moment of inertia. However, the cylinder is extended and rotates about its own center-of-mass. This rotation increases the torque required to rotate the apparatus. The Parallel Axis Theorem states that the total moment of inertia of an object located a distance \( d \) from the axis of rotation and rotating rigidly around it is:

\[
I_{Total} = md^2 + I_{CoM}
\]  

where \( I_{CoM} \) is the moment of inertia of the object (here, a cylinder) about an axis through its center-of-mass (CoM) and parallel to the axis of rotation (here, the axis of the Rotational Dynamics Apparatus).

The torque required to accelerate the Rotational Dynamics Sensor is provided by a string with a small mass attached to the end and wrapped around a pulley. Initially the system is at rest with the mass, \( m_h \), suspended by the string some distance above the floor. When the mass is released, it speeds up as it falls to the floor, and the Rotary Motion Sensor spins correspondingly faster. The goal is to determine the moment of inertia of the two cylindrical masses that clamp on the bar attached to the sensor \( (I_{Cyl} = I_A + I_B) \). This will be done in two ways: (1) calculating the moment of inertia using a geometric expression for moment of inertia, and (2) by analyzing its motion due to the forces and torques acting on the system.

The nylon screws are easily broken if overtightened or if the masses are dropped. Please use appropriate care when you handle the masses.

Calculating the moment of inertia from the system geometry

It is convenient to calculate \( I_{CoM} \) first. From the CRC Handbook of Chemistry and Physics, find an expression for the moment of inertia of a cylindrical mass with a hole through the CoM parallel to the length of the cylinder. Choose the axis of rotation perpendicular to the length of the cylinder—called the transverse axis. Use this formula to calculate \( I_{CoM} \) for the brass masses. Ignore the error introduced by including the nylon screw as part of the mass. When you weigh the masses, be sure to include the mass of the screw as well. Then assume that each mass is simply a solid cylinder with a hole through the center along the axis of the cylinder. Use the calipers to measure the necessary dimensions of the cylinder. You can assume that the dimensions of both cylinders are the same, but the masses must be measured separately.

Choose the value of \( d \) (0.12 m ≤ \( d \) ≤ 0.18 m) that you will use in the experiment, then calculate the moment of inertia of the brass masses from the system geometry using the Parallel Axis Theorem. Use a ruler to measure the distance \( d \) and electronic balances to measure the masses of each cylinder. Remember that all of the distance/size measurements you make have some inherent
uncertainties. You will need to use these uncertainties later to find the overall uncertainty in your calculated value of $I_{Cyl}$. Determine the appropriate uncertainties for each measurement and record them for future reference.

### Relating the moment of inertia to the system dynamics

Find an expression for the moment of inertia, $I_{NoCyl}$, of the rotary motion sensor itself, the attached pulley, and the bar (without the cylindrical masses) by setting the initial potential energy of the hanging mass, $m_hg$, equal to the final kinetic energy of the system just at the instant before the hanging mass hits the floor. Remember that the linear speed of the string, $v$, is related to the angular speed of the pulley, $\omega$ (in radians per second), by the relation $v = \omega R$ where $R$ is the radius of the pulley. Show that your expression is equivalent to:

$$I_{NoCyl} = m_h R^2 \left( \frac{2gh}{v_{NoCyl}^2} - 1 \right) \quad (10.4)$$

where $v_{NoCyl}$ is the speed of the hanging mass just before it strikes the floor with no cylindrical masses on the rod, and $h$ is the distance traveled by the hanging mass from its point of release to the floor.

Derive a similar expression for the case when the two cylindrical masses are clamped on the rod. We will denote the moment of inertia of the two cylindrical masses by themselves as $I_{Cyl}$.

$$I_{NoCyl} + I_{Cyl} = m_h R^2 \left( \frac{2gh}{v_{Cyl}^2} - 1 \right) \quad (10.5)$$

where $v_{Cyl}$ is the speed of the hanging mass just before it strikes the floor with both cylindrical masses clamped to the rod.

Subtract Equation 10.4 from Equation 10.5 to find an expression for $I_{Cyl}$ alone. This expression can be further simplified by using the relationship between the linear speed of the hanging mass and the angular speed of the system ($v = \omega R$). Your expression should reduce to

$$I_{Cyl} = 2ghm_h R^2 \left( \frac{1}{v_{Cyl}^2} - \frac{1}{v_{NoCyl}^2} \right) = 2ghm_h \left( \frac{1}{\omega_{Cyl}^2} - \frac{1}{\omega_{NoCyl}^2} \right) \quad (10.6)$$

where $\omega_{Cyl}$ is the angular velocity of the Rotary Motion Sensor with the cylinders attached just before the hanging mass hits the ground. Likewise, $\omega_{NoCyl}$ is the angular velocity of the Rotary Motion Sensor without the cylinders just before the hanging mass hits the ground. The subtraction in Equation 10.6 is valid only when the value of the hanging mass, $m_h$, is the same in both expressions. Equation 10.6 allows us to calculate the moment of inertia on the basis of system dynamics and the concept of energy.
Measuring the final angular velocities

To obtain reliable values for the maximum angular speeds it is necessary that time between angular speed measurements be small. When you set up the Rotary Motion Sensor in the “Hardware Setup” tool, navigate to the “Properties” menu and increase the sensor resolution from “Low” to “High: 1440 counts per revolution.” Before acquiring data, increase the sample rate to 20 Hz in the tool bar along the bottom of the Display Area.

Use Capstone to measure the values of both $\omega_{Nocyl}$ and $\omega_{w/Cyl}$ for each hanging mass value from 5 to 30 g in 5 g increments. Since it is difficult to attach the cylinders precisely at distance $d$ from Sensor’s axis of rotation, the best results are obtained if you do this in two steps: work through all the hanging mass values with the cylinders attached, then remove the cylinders and work through the hanging mass values again.

Make sure that the hanging mass actually falls from rest at precisely the same height for each run. If you are careless here, the values for the angular speeds at the bottom will not be consistent. Check them for consistency by taking multiple measurements at each mass. Use the mean values for your calculations.

The Rotary Motion Sensor is sensitive enough that the paper clip attached to the end of the string makes a measurable difference in the torque. Therefore you should account for the mass of the paper clip itself in the value of $m_h$ you use in your calculations. For instance, the $m_h$ value with a 10 g mass on the string will actually be 10 g plus one paper clip mass. Extra paper clips identical to the one on your string are available near the electronic balances in the back of the lab. Average the mass of several paper clips to reduce the round-off error of the balances.

Calculate $I_{Cyl}$ for each hanging mass value, $m_h$ in an Excel spreadsheet. Are your computed values the same, assuming reasonable uncertainties, or do they show a trend as the mass value changes? Any trend points to some systematic error that may be fixable. Look carefully at your results. Finishing the calculations in this paragraph before proceeding to the next.

Every bearing has some friction associated with it, so that a sufficiently small mass on the end of the string will not produce any acceleration. If the friction due to the bearings is constant, the torque applied by friction will equal the torque applied by some small, constant value of hanging mass. For instance, if the frictional force is equivalent to 1 g on the end of the string, and if I put 10 g on the end of the string, only 9 g is “left over” to cause rotational acceleration. In the Newton’s Second Law laboratory, we repotted the acceleration versus force data to determine a “frictional mass equivalent” that reflected the magnitude of the frictional force. A similar trick is useful here.

Solve Equation 10.6 for $m_h$. Plot whatever is necessary to get $I_{Cyl}$ as a slope of a linear graph with $m_h$ on the y-axis and whatever else remains on the x-axis. The intercept should give the “frictional mass equivalent” and the slope should give an unadulterated (without the effects of friction) value for $I_{Cyl}$. Uncertainties in both the slope and the intercept should be determined and recorded. The “frictional mass equivalent” is the amount of hanging mass required just to overcome the friction of the pulley and Rotary Motion Sensor bearings. Since the mass of the paperclip alone is sufficient to overcome the force of friction on the Sensor, it is especially important to include the paperclip
mass in the values of $m_h$ used in making this graph.

**Summary**

Summarize your findings. Compare the moment of inertia value you calculated on the basis of system geometry with the value determined from your measurements of angular velocity. How big is the difference relative to the uncertainty of the difference? If the two values differ by more than three times the expected uncertainty, carefully examine your calculations and procedures for possible sources of error.

Before you leave the lab please:
- Straighten up your lab station.
- Report any problems or suggest improvements to your TA.
Lab 11. Spring-Mass Oscillations

Goals

• To determine experimentally whether the supplied spring obeys Hooke’s law, and if so, to calculate its spring constant.

• To find a solution to the differential equation for displacement that results from applying Newton’s laws to a simple spring-mass system, and to compare the functional form of this solution to the experimental oscillation you observe.

• To determine the spring constant by another method, namely, by observing how the oscillation frequency changes as the mass hanging on the end of the spring is varied.

• To add additional air resistance to the oscillating system and compare the resulting displacement as a function of time with the theoretical prediction given.

Introduction

If you hang a mass from the bottom end of a spring, then pull the mass down and release it, the mass will oscillate up and down. In this experiment we explore the nature of the force exerted by a “real” spring when it stretches. We determine if the resulting force oscillation is “simple harmonic” and examine the effect of energy loss on its motion. The term “simple harmonic” is applied to oscillatory motion that can be characterized by a sinusoidal function; that is, the displacement follows a simple sine or cosine function.

This lab makes extensive use of curve fitting routines, where the computer fits a model to your experimental data. Normally a model that omits important features of the experiment will fail to describe the data well. Unfortunately, making a model more complex can give it the ability to fit data for which the theory does not apply. (Some models can “fit an elephant”!) One defense against this is to examine any predictions of the model (or theory) for reasonableness.

Force exerted by a stretched spring

In this exercise, you are to plot the force exerted by a spring as a function of its “stretch” (not the overall length). Suspend the spring from a force sensor. Start by adding a 50-g mass to the mass hanger, which also has a mass of 50 g, to make a total of 100 g of mass hanging from the spring. Then increase the hanging mass in 100 g increments up to a total of 1200 g. Devise a
method to measure the stretch of the spring. Then take the data. Remember to zero the force sensor appropriately and to use SI units.

From your graph determine a mathematical equation relating the spring force to the stretch of the spring. Compare your result to the Hooke’s Law model described in your textbook. Can you characterize your real spring with a unique value of the spring constant (sometimes called the force constant) as in Hooke’s Law? Based on your data and analysis, how “ideal” is your spring?

**Spring-mass oscillations (neglect damping)**

Applying Newton’s Second Law to a mass hanging on a “massless” spring that can be modeled by Hooke’s Law, one finds that:

\[
M \frac{d^2 x}{dt^2} = -kx \quad \text{that is,} \quad ma = F \quad (11.1)
\]

where \( m \) is the mass hanging on the spring and \( x \) is the distance the spring has been stretched from its equilibrium position with the mass hanging at rest.

What does the function \( x(t) \) that satisfies Equation [11.1] look like? Guessing happens to be a “tried and true” technique for solving so-called differential equations. What common function do we know whose second derivative gives us the original function back again except with a minus sign? Hint: Try a function like \( x(t) = A \cos(Gt) \). Under what conditions does it satisfy the equality of Equation [11.1]?

From your knowledge of oscillatory motion you should recognize that \( G \) corresponds to the angular frequency of oscillation (often represented by the Greek letter \( \omega \)). And of course, the angular frequency in radians/sec is just \( 2\pi \) times the ordinary frequency in Hz.

**Experiment set-up**

Hopefully you are convinced you that the spring you are using is well described by Hooke’s Law. Since the spring force is directly proportional to the displacement from equilibrium, Equation [11.1] implies that both the displacement and the spring force should be sinusoidal for the oscillating mass.

Hang a 1-kg mass from the spring and set up the force sensor to measure the force oscillations. Increase the sampling rate of the force sensor to 20 Hz. Zero the force sensor when the mass is hanging at equilibrium. Displace the mass about 5 cm from its rest position and release it. Display the force on a graph. Select 10 or 15 seconds of data showing the oscillation using the “Highlight range of active points” tool in the toolbar across the top of the of the Display Area. Then select the “Sine: Asin(\( \omega t \) + \( \phi \)) + C” function from the “Apply selected curve fits from active data” menu in the tool bar across the top of the Display Area. This tells the software to attempt fitting a function of the form \( A \sin(\omega t - \phi) + C \) to your data, where \( A \) is the amplitude of the oscillation, \( \omega \) is the angular frequency of the oscillation, \( \phi \) adjusts the phase of the sine wave to accommodate positive
or negative values of the function at time $t = 0$, and $C$ accounts for small errors in taring the force sensor.

After this operation, Capstone should display a curved “best-fit” line follows the selected data and the values of the constants $A$, $\omega$, $\phi$, and $C$ (and their uncertainties) as determined by a least squares technique. Comment on how accurately the fitted sinusoidal function matches your actual data. (If the curve fit fails, it generally does a very poor job of describing your data. If your curve fit fails, discuss the situation with your TA.) If the uncertainty of $\omega$ is less than the least significant digit of the best fit value, increase the precision of your data in the Data Summary tool from the Tools Palette. The value of the period of the oscillation is equal to $2\pi/\omega$. Check to make sure that the period value calculated from $\omega$ is reasonable. For instance, you can estimate the period using a watch or the computer clock over several periods. When a curve fitting routine makes an error, it is usually a large one. You do not need a very precise period measurement to perform a useful check on the results of the curve fitting routine.

Perform a simple test of the effect of amplitude on the period of oscillation by displacing the mass about 20 cm from its equilibrium position and releasing it. Display the force on a graph and fit the Sine function to this data. Again comment on how accurately the fitted function matches the data and note the value of the period of oscillation. Based on this limited data, how does the oscillation period depend on the amplitude of the oscillation? Compare this behavior with the effect of amplitude on the period of the simple pendulum, which you measured in an earlier lab.

**Effect of mass on the oscillation period**

Vary the total hanging mass including the mass of the 50-g hanger from 200 g to 1200 g, determining the period of the oscillation for each mass value using the techniques from the previous section. It is a good idea to zero the force sensor with the mass at equilibrium prior to each run. For an ideal spring, the angular frequency, $\omega$, of an oscillating spring-mass system is related to the spring constant, $k$, and the hanging mass, $m$, by the relation:

$$\omega = \left( \frac{k}{m} \right)^{1/2}$$

(11.2)

We hope to determine $k$ by measuring the period $\omega$ as a function of the mass $m$ on the end of the spring. Because the slope of a line can be determined with low uncertainties, we want to modify Equation 11.2 to get the equation of a line with slope $k$. Solve Equation 11.2 for the mass, $m$. Show/explain why making a graph of $m$ (on the vertical axis) as a function of $1/\omega^2$ (on the horizontal axis) should be a straight line with a slope value equal to the spring constant, $k$, and a vertical axis intercept of zero. Make such a graph with your data. Compare the slope value of your graph with the spring constant determined using forces measurements. Do they agree within the uncertainty limits of each? Does your graph have a zero intercept as predicted by the analysis of an ideal spring? What is the intercept value and corresponding uncertainty? What are the units of the intercept value?

Hooke’s Law applies to “ideal”, that is, massless springs. For ideal springs, the oscillation period
goes to zero as the hanging mass is reduced to zero. Thus the oscillation frequency would approach infinity as the hanging mass approaches zero. Remove the hanging mass from your spring, stretch it a small amount (0.5–1.0 cm), and let it go. Is the oscillation frequency of your spring by itself infinite? How does it differ from an ideal spring?

When \( m \) (on the vertical axis) is plotted as a function of \( 1/\omega^2 \) (on the horizontal axis), the intercept value will be negative with a magnitude known as the “effective mass of the spring.” Using energy considerations and some simplifying assumptions, one can show that the effective mass should be about one-third of the total mass of the spring. How close is the magnitude of your intercept value to one-third of your spring mass?

**Spring-mass oscillations with damping**

The term “damping” is just a short way of saying that there are frictional forces which convert the mechanical energy (potential and kinetic) of a system into heat. In our case we will use a thin piece of cardboard moving back and forth through the air to provide damping. The cardboard piece has a small hole punched in the middle allowing it to slide over the top of the mass hanger. Then you can place any additional mass on top of the cardboard piece to hold it firmly in place. For this part of the experiment, put a total of 500 ± 5 g (including the mass of the cardboard and the 50-g hanger; record the actual value used) hanging on the spring.

With the hanging mass and cardboard in place stretch the spring downward 8–10 cm from equilibrium and release it. Use Capstone to plot the force as a function of time for 40–60 s. You will notice that the amplitude of the oscillation decreases significantly during the experiment due to the damping effect of air on the piece of cardboard.

As discussed in your textbook, when the damping force is relatively small and proportional to the velocity of the object, the oscillations can be described by a sinusoidal function with an amplitude that decreases exponentially (that is a negative power of \( e \)). We can check whether this is true for the present system by fitting such a function to our data and determining whether it fits appropriately.

After selecting about 50 s of your data with the “Highlight range of active points” tool (enough to show the decay clearly), choose the “Damped Sine: \( Ae^{-Bt}\sin(\omega t + \phi) + C \)” option from the “Apply selected curve fits from active data” menu. This tells the software to attempt fitting a function of the form \( Ae^{-Bt}\sin(\omega t - \phi) + C \) to your data. The value of “\( B \)” is the decay constant for the decay. The time \( \tau = 1/B \) is the time required for the amplitude of the oscillation to decrease from its value of \( A \) (at time \( t = 0 \)) to \( A/e \approx A/3 \) (at time \( t = \tau \)).

When the fit is done, the actual “best-fit” values of the four constants will be shown in the fit window to the left of your initial guesses and also in a small box within the graph window. Uncertainties (in the form of standard errors) will also be displayed for each value. In the graph window check to make sure that the fitted function actually does a good job of fitting the data. If the amplitude or phase of the fitted function differs significantly from the data (check this carefully!), then you will need to choose some other initial values of the constants and try again.

How closely does the fitted curve match your actual data? You may wish to expand the time scale
of a portion of your data so the fitted curve and the actual data can be seen more clearly and print it out to support your conclusions here. Estimate the time required for the oscillation amplitude to reach $1/e$ of its initial amplitude using a watch or the computer clock. Is the value of $B$ reported by the damped sine fit consistent with your estimate? Again, a precise estimate is not required.

Compare the period of the damped pendulum with the period without damping, but with the same total mass ($\pm 5$ g) on the end of the spring. Use your uncertainties in making this comparison. Explain any differences you see. You may want compare the oscillation frequencies with and without damping using the relationship for damped oscillations given in your textbook. To do this, you will have to covert the $B$ you measured to the equation for the damping constant used in the text.

**Summary**

Summarize your findings.

Curve fitting routines are powerful tools in science and engineering. They are simple examples of computer models or simulations. Normally one derives a model from theoretical considerations, then tests it against experiment. If the model is missing important features, it will generally fail to describe the data well. Unfortunately, making a model more complex can give it the ability to fit data for which the theory does not apply. The best defense against this is usually to examine predictions of the model for reasonableness, as you did above when you checked to see if the output parameters $\omega$ and $B$ were reasonable.

Before you leave the lab please:

- Quit Capstone.
- Straighten up your lab station.
- Report any problems or suggest improvements to your TA.
Lab 12. Vibrating Strings

Goals

• To experimentally determine the relationships between the fundamental resonant frequency of a vibrating string and its length, its mass per unit length, and the tension in the string.

• To introduce a useful graphical method for testing whether the quantities $x$ and $y$ are related by a “simple power function” of the form $y = ax^n$. If so, the constants $a$ and $n$ can be determined from the graph.

• To experimentally determine the relationship between resonant frequencies and higher order “mode” numbers.

• To develop one general relationship/equation that relates the resonant frequency of a string to the four parameters: length, mass per unit length, tension, and mode number.

Introduction

Vibrating strings are part of our common experience. Musical instruments from all around the world employ vibrating strings to make musical sounds. Anyone who plays such an instrument knows that changing the tension in the string changes the resonant frequency of vibration. Similarly, changing the thickness (and thus the mass) of the string also affects its frequency. String length must also have some effect, since a bass violin is much bigger than a normal violin. The interplay between these factors is explored in this laboratory experiment.

Water waves, sound waves, waves on strings, and even electromagnetic waves (light, radio, TV, microwaves, etc.) have similar behaviors when they encounter boundaries from one medium to another. In general all waves reflect part of the energy and transmit some into the new medium. In some cases the amount of energy transmitted is very small. For example a water wave set up in your bathtub moves down the length of the tub and hits the end. Very little energy is transmitted into the material of the tub itself and you can observe a wave of essentially the same size as the “incident” wave being reflected. The clamps at the ends of a string provide similar boundaries for string waves such that virtually all the energy of the wave is reflected back and the wave travels from one end to the other. The wave “bounces” back and forth. If waves are sent down a string of some length at a constant frequency, then there will be certain frequencies where the reflected waves and the waves being generated on the string interfere constructively. That is, the peaks of the incident waves and the peaks of the reflected waves coincide spatially and thus add together. When
this occurs, the composite wave no longer “travels” along the string but appears to stand still in
space and oscillate transversely. This is called a “standing wave” for obvious reasons. A marching
band that is marching “in place” but not moving is a fair analogy. You can easily demonstrate
this phenomenon with a stretched rubber band. These standing waves occur only at particular
frequencies, known as resonant frequencies, when all the necessary conditions are satisfied. These
necessary conditions depend on the factors mentioned above, such as whether the string is clamped
tightly at the ends or not (i.e., the boundary conditions), the length of the string, its mass per unit
length, and the tension applied to the string.

With this in mind, we will systematically explore how the resonant frequency depends on three of
the four factors listed above. In all cases our strings are clamped or held tightly at both ends; we
consistently use the same boundary conditions. Finally, we will search for a single equation that
describes the effect of length, tension, and mass per unit length on the resonant frequency.

**Equipment set up**

A schematic diagram of the set up is shown in Figure 12.1. Connect the speaker unit to the output
terminal (marked with a wave symbol) and the ground terminal (marked with the ground symbol)
of the Pasco Model 850 interface unit. The interface unit can be configured to produce a voltage
that varies sinusoidally at a known frequency. In the Experimental Setup window, click on the
image of the output terminal (marked with a wave symbol). In the window that appears, make sure
that the waveform pull down menu is set to Sine Wave. Use the frequency and voltage windows to
set the frequency and output voltages, respectively. Keep the output voltage below 4.5 V. Click the
“Auto” button (which toggles the Auto function off), then click the “On” button to start the voltage
generator.

This voltage drives an audio speaker mechanism that lacks the diaphragm that normally produces
the sound. You will nevertheless hear some sound from the speaker drive mechanism. This sound
can be irritating, so use the minimum voltage required to make a good measurement. This speaker
drive oscillates in synchrony with the drive voltage and is connected to the string via an “alligator”
clip.

**Caution:** Do not apply loads greater than 10 kg to the end of the string!

**Effect of string length on resonant frequency**

Start with the 1.3 g/m string (see the tag attached to the end of string) and hang a total mass of 5
kg, including the mass of the mass hanger, on the end of the string. Determine the fundamental
resonant frequency for five or six different string lengths. Plucking the string with your finger near
the middle point excites a vibration of the string primarily in its fundamental resonant mode (also
called the first harmonic). Pluck the string and note how the string vibrates. The vibration of the
string stops a short time after you pluck it because of energy losses due to air friction. The speaker
drive allows you to pump energy into the vibrating system at the same rate that it is lost, so that the
vibration can be maintained for as long you wish. The string will vibrate strongly only at certain
well-defined frequencies. By adjusting the frequency of the speaker drive slowly while watching
Figure 12.1. Typical apparatus for the vibrating string experiment. The Pasco Model 850 interface unit can be used to control the mechanical vibrator in place of the digital function generator.

the string you should be able to find the frequency that makes the string vibrate in its fundamental resonant mode. You can recognize the fundamental resonant frequency easily because the whole middle portion of the string oscillates up and down like a jump rope; the fundamental resonance can be thought of as the “jump rope mode.” For best results you must continue adjusting the speaker drive until you have found the “middle” of the resonance, where the amplitude of the vibration is maximized.

Note that the distance from the alligator clip to the top of the pulley where the string is held tightly determines the length of the vibrating string. The alligator clip does vibrate slightly but the string behaves very nearly as if the clip defines a clamped end. (The motion of the alligator clip cannot be ignored for very heavy strings. For these, you may have to visually locate the point near the alligator clip which appears to be clamped and doesn’t vibrate.)

Make sure that the string lengths that you test are approximately uniformly spaced between 0.4 m and approximately 1.7 m. (The maximum string length is limited by the length of the table.) By graphical means determine a mathematical function for the fundamental resonant frequency, \( f \), as a function of \( L \), where \( L \) is the length of the vibrating string as determined by the placement of the
alligator clip. Do you get a linear graph if you plot $f$ on the y-axis and $L$ on the x-axis? Instead, try plotting $f$ on the y-axis and $1/L$ on the x-axis. What important property of the wave on the string can be determined from this graphical analysis? The units of the slope of this graph (assuming it is linear) provide information on what this quantity might be. Explain your reasoning!

**Effect of string mass-per-unit-length on resonant frequency**

For this set of experiments, use the maximum string length employed in above and hang a total of 5 kg on the end of the string. Test the four the strings in the box, noting the mass per unit length ($\mu$) indicated on the attached cards. Find the fundamental oscillation frequency for each of the strings at your station. Remember that you already took one data point while observing the effect of string length. Determine graphically whether the relationship between fundamental frequency, $f$, and the mass per unit length, $\mu$, that is $f(\mu)$, is a simple power function. If so, find the equation for frequency as a function of mass/unit length. Refer to the Uncertainty-Graphical Analysis Supplement in this lab manual for details.

**Effect of string tension on resonant frequency**

For these experiments, use a string with a mass/length between 1.0 and 6.0 g/m and a length of at least 1.5 m. Determine the fundamental resonant frequency of the string as the total mass on the end of the string is increased from 1.0 to 10.0 kg. The weight of the hanging mass will equal to the tension in the string, $T$. Graphically determine whether the relationship between the fundamental resonant frequency, $f$, and the string tension, $T$, is a simple power function. Again refer to the Uncertainty-Graphical Analysis Supplement in the lab manual.

**Effect of harmonic mode number on resonant frequency**

Using the 1.3 g/m string and the 3 kg hanging mass, set the length of the string to at least 1.5 m. So far you have looked at the fundamental frequency or first harmonic of the string vibration. The second harmonic (mode number $n = 2$) will have a “jump rope” mode on each half of the string but they will oscillate in opposite directions. Increase the driver frequency until you find this resonance and record it. The third harmonic will have three “jump rope” modes on the string, etc. At the very least you should collect the data for $n = 1, 2, 3, \text{ and } 4$. If time allows, determine frequencies for even higher $n$ values.

Determine the relationship $f(n)$ between the resonant frequency, $f$, and the mode number, $n$, by graphical means.

**Summary**

Summarize your findings clearly and succinctly. Can you write a single mathematical function that encapsulates all the relationships that you have discovered? That is $f(T, \mu, L, n)$. Note that taking the sum of the four relations you determined above will not work. Compare your experimental results with those theoretically predicted in your textbook. (This is sometimes included in a section
on musical instruments.) Show that the textbook formula is dimensionally correct. Be quantitative in your comparisons.

Before you leave the lab please:
- Turn off the power to all the equipment.
- Leave only the 1 kg mass hanger on the end of the 1.3 g/m string.
- Straighten up your lab station.
- Report any problems or suggest improvements to your TA.
Uncertainty and Graphical Analysis

Introduction

Two measures of the quality of an experimental result are its accuracy and its precision. An accurate result is consistent with some ideal, "true" value, perhaps a commonly accepted value from the scientific literature. When a literature value is not available, we often perform an additional measurement by other methods. Different methods are usually prone to different errors. We can hope that, if two or three different methods yield consistent results, our errors are small. However, measurements made by different methods never agree exactly. If the discrepancy is small enough, we claim that the results are consistent and accurate. Most of our work with uncertainties will address the question, "How small is small enough?"

Precision refers to the reproducibility of a result made using a particular experimental method. When random variations are large, the precision is low, and vice versa. While we should work hard to reduce the size of random effects, they cannot be entirely eliminated. When we claim that two measurements are consistent, we are claiming that their difference (the discrepancy) is smaller than these random variations. Since many quantities of interest are calculated from measured values, we also need to know how random variations in measured quantities affect the results of these calculations.

Measurements in the presence of random deviations

Mean and standard deviation of the mean

In the presence of random variations, the best estimate of a physical quantity is generally given by the average, or mean. The average value of a set of $N$ measurements of $x$, $(x_1, x_2, x_3, \ldots, x_N)$, is given by

$$x_{\text{avg}} = \frac{x_1 + x_2 + x_3 + \ldots + x_N}{N} = \frac{1}{N} \sum_{i=1}^{N} x_i$$  \hspace{1cm} (13.1)

\footnote{A good reference for much of the information in this section is John R. Taylor, An Introduction to Error Analysis—The Study of Uncertainties in Physical Measurements, 2nd Edition (University Science Books, Herndon, Virginia, USA, 1987), especially Chapter 5.}
The individual measurements of $x$ will generally deviate from $x_{\text{avg}}$ due to random errors. The standard deviation of $x$, denoted $\sigma(x)$, indicates how far a typical measurement deviates from the mean. The value of $\sigma(x)$ reflects the size of random errors.

$$
\sigma(x) = \sqrt{\frac{(x_1 - x_{\text{avg}})^2 + (x_2 - x_{\text{avg}})^2 + (x_3 - x_{\text{avg}})^2 + (x_4 - x_{\text{avg}})^2 + \ldots + (x_N - x_{\text{avg}})^2}{N - 1}}
$$

$$
= \frac{1}{\sqrt{(N - 1)}} \left[ \sum_{i=1}^{N} (x_i - x_{\text{avg}})^2 \right]^{1/2} \quad \text{(13.2)}
$$

A small standard deviation indicates that the measurements ($x$-values) are clustered closely around the average value, while a large standard deviation indicates that the measurements scatter widely relative to the average value. Thus a small standard deviation indicates that this particular quantity is very reproducible—that is, the measurement is very precise. Note that the units of the standard deviation are the same as the units of the individual measurements, $x_i$.

The relation between the standard deviation to the deviation of the data from its average value is illustrated in Figure 13.1. Figure 13.1 is a histogram of 100 scores, chosen from a set of over 1000 random scores with an average was 85 and a standard deviation of 7.5. Because of their random distribution, the average of the 100 scores is not exactly 85, and their standard deviation is not exactly 7.5. Because we cannot take an infinite number of measurements, Equations 13.1 and 13.2 are only approximations to the true average and standard deviation. On average, the approximations improve as the number of measurements, $N$, increases.

![Histogram of 100 Student Scores](image)

Figure 13.1. Histogram of 100 scores with an average of 85 and a standard deviation of 7.5. The smooth curve is the Gaussian function corresponding to the same number of measurements, average, and standard deviation.

The Gaussian function, $G(x)$, corresponding to 100 scores with an average of exactly 85 and a standard deviation of exactly 7.5 is also shown in Figure 13.1. According to the Central Limit Theorem
of statistics, the Gaussian function represents the ideal distribution of scores for a given \( N, \ x_{\text{avg}}, \) and \( \sigma(x) = \sigma \) if the scores have a finite average and the measurements are statistically independent. These conditions apply to most of the measurements made in lab. (Important exceptions are found in the stock market, among other things.)

\[
G(x) = \frac{N}{2\pi\sigma} \exp \left[ -\frac{(x - x_{\text{avg}})^2}{2\sigma^2} \right] \quad (13.3)
\]

The value of the standard deviation in the context of uncertainties is that the probability of finding a score at some distance from the average falls in a predictable way as the distance increases. For an ideal Gaussian distribution, 68% of the measurements lie within one standard deviation of the mean \( (x_{\text{avg}}) \). In Figure 13.1, 63 scores (63% of 100) lie within 7.5 points of 85. Ideally, 95% of the scores lie within two standard deviations (here, \( \pm 15 \) points) of the average. Ideally, one would expect 99.7% of the points to lie within three standard deviations (here, \( \pm 22.5 \) points) of the average. No score in Figure 13.1, is more than three standard deviations from the average. (All of the scores lie between \( x_{\text{avg}} - 3\sigma = 62.5 \) and \( x_{\text{avg}} + 3\sigma = 107.5 \).) Unless the total number of scores is very high, the probability of finding a score more than 3\( \sigma \) from the average is quite low.

Since the standard deviation characterizes random errors, we can pretty much rule out random errors as the source of any difference greater than 3\( \sigma \). We will make this assumption in the physics labs, although the precise probabilities will usually differ from those given by the ideal Gaussian function. For instance, when the number of measurements is small, our estimates of \( x_{\text{avg}} \) and \( \sigma(x) \) may be poor. In more advanced work, it can be important to correct for this lower precision.\(^2\) When one is attempting to show that one measurement out of a large number differs significantly from the others, a higher threshold for significance (4\( \sigma \) or 5\( \sigma \)) may be necessary.

Since the result of an experiment is generally an average value, we need a measure of the precision of the average. This is called the “standard deviation of the mean,” \( \sigma(x_{\text{avg}}) \). Although one can repeat the entire set of \( N \) measurements several times to compute \( \sigma(x_{\text{avg}}) \), statistics allows us to estimate \( \sigma(x_{\text{avg}}) \) using the original \( N \) measurements alone:

\[
\sigma(x_{\text{avg}}) = \frac{1}{\sqrt{N(N-1)}} \left[ \sum_{i=1}^{N} (x_i - x_{\text{avg}})^2 \right]^{1/2} = \frac{\sigma(x)}{\sqrt{N}} \quad (13.4)
\]

The standard deviation function of most spreadsheet programs (Excel, OpenOffice), Capstone, and calculators gives \( \sigma(x) \), from Equation 13.2. To calculate the standard deviation of the mean from this number, you must divide by the square root of \( N \), the number of measurements.

On the other hand, spreadsheet Regression functions and Capstone’s curve fit function provide the standard deviation of the mean, \( \sigma(x_{\text{avg}}) \) from Equation 13.4.

\(^2\)Student’s \( t \)-test is used to make this adjustment in more advanced work. This is described at the end of Chapter 5 in John R. Taylor, *op. cit.*, and in many statistics books.
Other methods for estimating the effect of random errors

When several measured quantities are used in a calculation, a relatively crude measurement of one quantity may contribute little to the overall uncertainty. If so, there is little point in improving the measurement. To demonstrate that the uncertainty is small, we must provide an upper bound on the uncertainty and show that the effect of this uncertainty is indeed relatively small.

Smallest division

Most measuring devices have a smallest division that can be read. In this case, one can use the size of the smallest division as an upper bound on the uncertainty. In some cases, it is appropriate to use one-half of this smallest division. For instance, the smallest division displayed on a meter stick is usually 1 mm. The distance \( d \) is read to the nearest mark. Suppose, for example, you look at the meter stick a few times and read \( d = 85 \text{ mm} \) each time. Because you never measured 84 or 86 mm, you are confident that \( 84.5 \leq d \leq 85.5 \). That is, the magnitude of the uncertainty in \( d \) is less than 0.5 mm. This is a useful upper bound. You must use your judgement in cases where the measurement cannot be practically made with this precision. For instance, your precision can be much worse if you don’t have a clear view of the ruler.

Interpolation

If the uncertainty in such a measurement is not small relative to the other uncertainties in an experiment, a better estimate of the uncertainty is needed. In this case, taking the standard deviation of the mean of multiple measurements is necessary. For instance, you can estimate \( d \) to one-tenth of a mm using a meter stick. (Estimating values between the marks is called interpolation.) In this case, repeated estimates, made with care, will disagree, and you can calculate the standard deviation of their mean.

Manufacturer’s specification

The user manuals for many instruments (electronic ones in particular) often include the manufacturer’s specification as to the “guaranteed” reliability of the readings. For example, the last digit on the right of digital voltmeters and ammeters is notoriously inaccurate. In this case, it makes sense to use the manufacturer’s specification as a simple upper bound.

Terminology—Uncertainty and significant digits

Because the standard deviation is not the only measure of random variation, it helps to have another name and symbol for this quantity. We will call the the expected effect of random variation on \( x_{\text{avg}} \) its uncertainty, and represented it by the symbol \( u(x_{\text{avg}}) \). If the average and standard deviation of \( x \) are available, the best estimate of \( x \) is \( x_{\text{avg}} \), and the best estimate of the uncertainty of \( x_{\text{avg}} \) is the standard deviation of its mean, \( \sigma(x_{\text{avg}}) \). Then \( u(x_{\text{avg}}) = \sigma(x_{\text{avg}}) \). The uncertainty is often indicated by a ± sign after the average value. For instance, you might specify a length measurement as “1.05 ± 0.02 mm. Because there is more than one way to estimate the uncertainty, you must also specify how your estimate was made. For instance, the result of a length measurement may be reported as “1.05 ± 0.02 mm, where the uncertainty is the standard deviation of the mean of five length
readings;” or “24 ± 1 mm, where the uncertainty is the distance between marks on the meter stick.”

With or without a formal uncertainty estimate, you are expected to have a general idea of the uncertainties of the numbers you use. These uncertainties are communicated by the number of significant digits you provide with the number. For instance, a length written as 3.14 mm has an implied uncertainty of less than 0.1 mm; the inclusion of a digit in the second decimal place means that you have some knowledge of it. In your lab notebook and reports, you should not use more significant digits than are justified by your knowledge. Since rounding operations slightly increase the uncertainty in the last decimal place, it is appropriate to keep one extra significant digit in each step of a calculation. However, the final result must be rounded to an appropriate number of significant digits. Most physics texts include a discussion of significant figures.

Uncertainties in calculated quantities—the Derivative Method

Derivatives can be used to estimate the uncertainty associated with a function of the measured quantity, \( f(x) \), due to uncertainty in the measured variable, \( x \). We normally have an experimental value of \( x_{\text{avg}} \). To see how the uncertainty in \( x \) affects \( f(x_{\text{avg}}) \), we can plot \( f(x) \) as shown in Figure 13.2. The change due to small variation in \( x \) is given by \( \Delta f \approx f'(x)\Delta x \), where \( f'(x) \) is the slope (and the derivative) of \( f(x) \) at \( x_{\text{avg}} \).

![Plot of \( f(x,y,z) \), with \( y \) and \( z \) held constant](image)

Figure 13.2. Diagram relating the uncertainty in \( y = f(x) \) due to the uncertainty in \( x \).

For the simple function \( f(x) = 1/x \), with, \( x_{\text{avg}} = 2.0 \) and \( u(x_{\text{avg}}) = 0.1 \), the uncertainty in \( f(x) \), \( u[f(x)] \), is

\[
u[f(x)] = \sqrt{\left(\frac{df}{dx}u(x_{\text{avg}})\right)^2} = \sqrt{\left(-\frac{1}{x^2}u(x_{\text{avg}})\right)^2} = \sqrt{\frac{1}{4.0^2}}(0.1)^2 = 0.25 \quad (13.5)
\]

If \( f \) is a function of more than one variable, say \((x, y, z)\), where \( x, y, \) and \( z \) represent three measured quantities, the uncertainty in \( f(x, y, z) \) is found by computing uncertainties for each variable alone and adding them in quadrature, as explained below. The uncertainty due to \( x \) is computed by treating \( f(x, y, z) \) as a function of \( x \) only. Then from Equation 13.5

\[
 u[f(x)] = \sqrt{\left( \frac{\partial f}{\partial x} u(x_{\text{avg}}) \right)^2} \tag{13.6}
\]

where we introduce the \( \partial \) symbol to indicate that the \( y \) and \( z \) variables are being treated as constants when the derivative is taken. This is equivalent to assuming that the variables are independent; that is, none of the variables are completely determined by any subset of the others. Likewise:

\[
 u[f(y)] = \sqrt{\left( \frac{\partial f}{\partial y} u(y_{\text{avg}}) \right)^2} \quad u[f(z)] = \sqrt{\left( \frac{\partial f}{\partial z} u(z_{\text{avg}}) \right)^2} \tag{13.7}
\]

where \( u[f(x, y, z)] \) is the estimated uncertainty in \( f(x, y, z) \); \( u(x_{\text{avg}}), u(y_{\text{avg}}) \), and \( u(z_{\text{avg}}) \) are the uncertainties in the measured values of \( x_{\text{avg}}, y_{\text{avg}}, \) and \( z_{\text{avg}}, \) respectively, all evaluated at \( (x_{\text{avg}}, y_{\text{avg}}, z_{\text{avg}}) \). Again, the \( \partial \) symbols indicate that \( x \) and \( z \) are treated as constants when the derivative with respect to \( y \) is taken; likewise \( x \) and \( y \) are treated as constants when the derivative with respect to \( z \) is taken.

If you draw a two dimensional version of Figure 13.2, the Pythagorean theorem can be used to show that the uncertainties add like the edges of a right triangle, that is, in “quadrature.” (This is how individual deviations add when a standard deviation is calculated.) For a function of three variables, the uncertainties add in the same way:

\[
 u[f(x, y, z)] = \sqrt{\left( \frac{\partial f}{\partial x} u(x_{\text{avg}}) \right)^2 + \left( \frac{\partial f}{\partial y} u(y_{\text{avg}}) \right)^2 + \left( \frac{\partial f}{\partial z} u(z_{\text{avg}}) \right)^2} \tag{13.8}
\]

This technique can be generalized to account for as many measured parameters as necessary. When uncertainties from difference sources are added in this way, the result is called the “combined standard uncertainty,”\(^4\) or the “standard uncertainty.”\(^5\)

Consider the function \( f(x, y, z) = x^{1/2} y^2 \sin(z) \). To illustrate the difference between the derivatives used to calculate uncertainties, consider the regular (or total) derivative of \( f(x, y, z) \) with respect to \( x \), calculated using the product rule for derivatives.

\[
 \frac{df}{dx} = \frac{y^2 \sin(z)}{2x^{1/2}} + x^{1/2} y^2 \sin(z) \frac{dy}{dx} + x^{1/2} y^2 \cos(z) \frac{dz}{dx} \tag{13.9}
\]


However, in an experiment, \( x, y, \) and \( z \) are independent variables. Therefore we expect \( \frac{dy}{dx} = \frac{dz}{dx} = 0 \). For the purposes of calculating the contribution of \( u(x_{\text{avg}}) \) to the uncertainty of \( f, y \) and \( z \) might as well be constants. The three required derivatives of \( f(x, y, z) \) from Equation 13.8 are:

\[
\frac{\partial f}{\partial x} = \frac{y^2 \sin(z)}{2x^{1/2}} \quad \frac{\partial f}{\partial y} = 2x^{1/2}y \sin(z) \quad \frac{\partial f}{\partial z} = x^{1/2}y^2 \cos(z) \quad (13.10)
\]

In practice, taking derivatives can be a lot of work. However, many calculations involve products, which are simplified by starting with the natural logarithm of the calculated quantity. Since

\[
\frac{\partial}{\partial x} \ln(f) = \frac{1}{f} \frac{\partial f}{\partial x},
\]

we can calculate the derivatives we need from the derivatives of the logarithm. Since the logarithm function splits our function into terms with simple (partial) derivatives, they are easy to compute. In our example, \( \ln(f) = (1/2) \ln(x) + 2 \ln(y) + \ln(\sin(z)) \), so

\[
\frac{\partial}{\partial x} \ln(f) = \frac{1}{2x},
\]

\[
\frac{\partial}{\partial y} \ln(f) = \frac{2}{y},
\]

\[
\frac{\partial}{\partial z} \ln(f) = \frac{\cos(z)}{\sin(z)} = \cot(z) \quad (13.12)
\]

Substituting these partial derivatives into Equation 13.8 yields

\[
\frac{u[f(x, y, z)]}{f(x, y, z)} = \sqrt{\left[ \frac{u(x_{\text{avg}})}{2x_{\text{avg}}} \right]^2 + \left[ \frac{2u(y_{\text{avg}})}{y_{\text{avg}}} \right]^2 + \left[ \cot(z_{\text{avg}})u(z_{\text{avg}}) \right]^2} \quad (13.13)
\]

While this expression is not pretty, it is much simpler than the one obtained by substituting the derivatives of Equation 13.10 directly into Equation 13.8. For simplicity, the uncertainty is in Equation 13.13 is expressed as a fraction of the value of \( f(x, y, z) \). This is called the “relative uncertainty,” or more completely, the “relative combined standard uncertainty.”

**Using uncertainties to compare measurements or calculations**

Suppose you have measured a cart’s mass, \( m_{F/a} \), from force and acceleration measurements and Newton’s Second Law, \( F = ma \). To check for systematic errors, you have also measured the cart’s mass using an electronic balance, with the result \( m_{\text{bal}} \).

A straightforward way to determine whether these two measurements is to compare the discrepancy between the two measurements, say \( \Delta = |m_{F/a} - m_{\text{bal}}| \), with the expected uncertainty of \( \Delta \),
that is \( u(\Delta) \). As illustrated in Figure 13.1, the probability of \( \Delta \) being more than three standard deviations from the mean because of random errors alone is quite small. Therefore, if \( \Delta > 3u(\Delta) \) most of the discrepancy is almost certainly due to systematic problems. In this case, we say that the measurements of \( m_{F/a} \) and \( m_{bal} \) are not consistent.

The ratio between the discrepancy and its combined standard uncertainty is a useful measure of the seriousness of a discrepancy. Because this ratio is similar to the \( t \)-statistic of classical statistics, we call it the \( t' \)-score. In this example,

\[
t' = \frac{\Delta}{u(\Delta)} = \frac{\Delta}{\sqrt{u(m_{F/a})^2 + u(m_{bal})^2}}
\]

When you compare experimental results and find \( t' > 3 \), you should carefully review your calculations and measurement procedures for errors. If systematic errors appear to be significant, and you know what they might be, you should describe them in your lab notes. If time permits, repeating a portion of the experiment is in order. Whatever your conclusion, your lab notes must indicate how you estimated your uncertainties.

In the United States, the general authority on the reporting of uncertainties is the National Institute of Standards and Technology. These standards have been developed in consultation with international standards bodies. When the potential consequences of a decision are critical or when the data are unusual in some way, one should consult a statistician.

### Determining functional relationships from graphs

Linear relations are simple to identify visually after graphing and are easy to analyze because straight lines are described by simple mathematical functions. It is often instructive to plot quantities with unknown relationships on a graph to determine how they relate to one another. Since data points have not only measurement uncertainties but also plotting uncertainties (especially when drawn by hand), slopes and such should not be determined by using individual data points but by using a “best-fit line” that appears to fit the data most closely as determined visually. If graphing software is used, then the slope of the line can usually be determined by a computer using a “least squares” technique. We won’t go into detail about these methods here.

#### Linear functions \((y = mx + b)\)

If \( x \) and \( y \) are related by a simple linear function such as \( y = mx + b \) (where \( m \) and \( b \) are constants), then a graph of \( y \) (on the vertical axis) versus \( x \) (on the horizontal axis) will be a straight line whose slope (“rise” over “run”) is equal to \( m \) and whose \( y \)-axis intercept is \( b \). Both \( m \) and \( b \) can be

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7Ibid, Barry N. Taylor and Chris E. Kuyatt.

8W. Edwards Deming, Out of the Crisis (MIT Press, Cambridge, Massachusetts, 1982). Some authors attribute the ability of Japanese automakers to break into the U.S. market to their skillful application of the principles of statistical quality control popularized by W. Edwards Deming and Joseph Juran.
determined once the graph is made and the “best-fit” line through the data is drawn. if $x = 0$ does not appear on your graph, $b$ can be found by determining $m$ and finding a point $(x, y)$ lying on the “best-fit” line; then equation $y = mx + b$ can be solved for $b$.

**Simple power functions** ($y = ax^n$)

In nature we often find that quantities are related by simple power functions with $n = \pm 0.5, \pm 1, \pm 1.5, \pm 2$, etc., where $a$ is a constant. Except for $n = +1$, making a simple graph of $y$ (vertical axis) and $x$ (horizontal axis) for simple power functions will yield a curved line rather than a straight line. From the curve it is difficult to determine what the actual functional dependence is. Fortunately it is possible to plot simple power functions in such a way that they become linear.

Starting with the equation $y = ax^n$, we take the natural logarithm of each side to show

$$\ln(y) = \ln(ax^n) = \ln(a) + \ln(x^n) = \ln(a) + n\ln(x) \quad (13.15)$$

If $\ln(y)$ is plotted on the vertical axis of a graph with $\ln(x)$ plotted on the horizontal axis (This is often called a doubly logarithmic, or log-log graph.), then Equation [13.15] leads us to expect that the result is a straight line with a slope equal to $n$ and a vertical axis intercept equal to $\ln(a)$. If the relationship between $y$ and $x$ is a simple power law function, then a graph of $\ln(y)$ as a function of $\ln(x)$ will be linear, where the slope is $n$, the power of $x$, and the intercept is the natural logarithm of the coefficient $a$. This is quite useful, because it is easy to determine whether a graph is linear. If we suspect a simple power function relationship between two quantities, we can make a log-log graph. If the graph turns out to be linear, then we are correct in thinking that it should be a simple power function and can characterize the relationship by finding values for $n$ and $a$.

**Exponential functions** ($y = ae^{bx}$)

Radioactive decay, the temperature of a hot object as it cools, and chemical reaction rates are often exponential in character. However, plotting a simple graph of $y$ (on the vertical axis) and $x$ (on the horizontal axis) does not generate a straight line and therefore will not be readily recognizable. A simple graphical method remedies this problem. Starting with an equation for the exponential function, ($y = ae^{bx}$). We can take the natural logarithm of each side to show

$$\ln(y) = \ln(ae^{bx}) = \ln(a) + \ln(e^{bx}) = \ln(a) + bx \quad (13.16)$$

If $\ln(y)$ is plotted on the vertical axis and $x$ is plotted on the horizontal axis (This is called a semi-log graph.), Equation [13.16] takes the form of a straight line with a slope equal to $b$ and a vertical axis intercept equal to $\ln(a)$. Thus any relationship between two variables of this simple exponential form will appear as a straight line on a semi-log graph. We can test functions to check whether they are exponential by making a semi-log graph and seeing whether it is a straight line when plotted this way. If so, the values of $a$ and $b$ that characterize the relationship can be found.
Using error bars to indicate uncertainties on a graph

When plotting points \((x, y)\) with known uncertainties on a graph, we plot the average, or mean, value of each point and indicate its uncertainty by means of “error bars.” If for example the uncertainty is primarily in the \(y\) quantity, we indicate the upper limit of expected values by drawing a bar at a position \(y_{\text{max}}\) above \(y_{\text{avg}}\), that is, at position \(y_{\text{max}} = y_{\text{avg}} + u(y_{\text{avg}})\). Similarly, we indicate the lower limit of expected values by drawing a bar at position \(y_{\text{min}} = y_{\text{avg}} - u(y_{\text{avg}})\). Figure 13.3 shows how the upper error bar at \(y_{\text{max}}\) and the lower error bar at \(y_{\text{min}}\) are plotted. If the quantity \(x\) also has significant uncertainty, one adds horizontal error bars (a vertical error bar rotated \(90^\circ\)) with the rightmost error bar at position \(x_{\text{max}}\) and the leftmost error bar at position \(x_{\text{min}}\).

Occasionally one encounters systems where the upper and lower error bars have different lengths. In this case, the upper uncertainty, \(u(y_{\text{avg}})\) does not equal the lower uncertainty, \(u_-(y_{\text{avg}})\).

Figure 13.3. Diagram of error bars showing uncertainties in the value of the \(x\)- and \(y\)-coordinates for point \((x_{\text{avg}}, y_{\text{avg}})\). When you print a graph in lab, the labels are omitted.
Computer Tools for Data Acquisition

Introduction to Capstone

You will be using a computer to assist in taking and analyzing data throughout this course. The software, called Capstone, is made specifically to work with the interface unit connected to your computer. This may be either the black PASCO Scientific Science Workshop 750 Interface, or the blue and gray PASCO 850 Universal Interface. These interface units can accept up to four digital inputs (the four receptacles on the front left, numbered 1–4), and at least three analog inputs (the three receptacles on the front right (labeled A, B, and C).

A digital input essentially detects either a “1” or “0”. In other words it can detect whether something is “on” or “off.” For compatibility with the integrated circuits inside the box, an electrical voltage of zero volts represents the “off” state and a voltage of 5 volts represents the “on” state. For example, a photogate consists of an infrared light source in one arm and an infrared light detector in the other arm, and sends a zero volt signal to the computer when something blocks the beam and sends a 5 volt signal when the beam is unblocked. This allows the computer to time objects as they pass through the gate. For the study of motion, timing is an important tool, so most of the sensors that we plug into the interface will be of this digital nature.

The analog inputs detect electrical voltages between +10 volts and –10 volts. Thus electrical circuits can be monitored directly since the signals are already electrical in nature. Other sensors can be constructed to convert forces, pressures, temperatures, etc., into electrical voltages. These kinds of sensors also use the analog inputs. If the computer software knows the relationship between the quantity of interest, say pressure, and the electrical voltage produced by the sensor, then the computer can display the pressure directly rather than simply displaying the voltage.

The Capstone software assumes that you are working in SI units (meters, kilograms, seconds, and coulombs). Any numbers that you enter are assumed to be in these units, so convert any values to SI units before entering them into the program.

Setting up a new experiment

Make sure that the interface unit connected to your computer is turned on. The on-off switch is located on the right rear of the ScienceWorkshop 750 (SW 750) units and on the left front of the 850 Universal Interface (850 UI) units. When the power is on, a small green light should be glowing on the far left side of the front panel. (Note: SW 750 interface units connected to the computer with USB adaptors are not recognized by the computer if the interface is turned on after
the computer has booted up. Restarting the computer will solve this problem. Interface units that
do not need special USB adaptors don’t have this idiosyncrasy.)

The icon for the Capstone software should be present on the left side of the desktop (the default
screen when the computer is first turned on) when the log-in process is complete. Start the Cap-
stone software by finding and clicking on the Capstone icon on the desktop of the computer. When
Capstone loads, the display screen appears. Use the Tools Palette on the left hand side of the Dis-
play Area to set up the sensors for your experiment and the Display Palette on the right hand side
of the Display Area to set up your data display. Both palettes can hidden or rendered visible using
the Workbook menu at the top of the screen.

Click on the uppermost icon in the Tools Palette is a picture of a 850 UI unit, labeled “Hardware
Setup”. If for some reason the software did not find the interface, a yellow warning triangle will
appear along with a message to that effect. If you get this message, check the USB connection and
make sure the interface unit is powered up. Then have the software scan again for the interface.
Interface units with USB adaptors will also require you to restart the computer. The Hardware
Setup screen should appear with a picture of your interface unit when the interface is recognized
and all is well.

Choosing a sensor or sensors

Now you must choose the appropriate sensor(s) to use for your particular experiment. Usually the
required sensors are provided along with the apparatus for the day’s exercises. Sensors come in
two varieties, digital and analog. By looking at the connector on a sensor and comparing it to the
digital and analog input receptacles on the front of the interface, one can easily determine whether
it is an analog or digital sensor. Most sensors have only one connecting cable to the interface, but
the rotary motion sensor and the ultrasonic motion sensor (described below) have two cables to
be connected to adjacent digital inputs on the interface. Multiple sensors are also used in some
experiments. Plug the connecting cable(s) from the sensor(s) into the interface. If you are using
multiple sensors, this must be done in a thoughtful way so that you know what each sensor will be
measuring.

Setting up Capstone for your sensor

After connecting the hardware, you must tell Capstone software which sensor(s) you have con-
nected. On the computer monitor make sure that you see the “Hardware Setup” screen with a
picture of your interface unit. Yellow circles will mark the position of each input and output jack.
If, for example, you have connected a digital sensor to digital Channel 1, move the cursor within
the yellow circle surrounding the Channel 1 input and click it. This reveals an alphabetical list of
all currently compatible digital sensors. Some of them have special names. The more complex
sensors are labeled with their names. Move the cursor to the name of the sensor of choice, high-
light it, and click OK. The software should now display the chosen sensor connected to Channel
1 along with a setup window specifically for that sensor. You may have to edit settings that are
specific to your sensor using the Properties window, which can be opened by clicking on the word
“Properties” to the right and below the picture of your interface unit. For instance, the resolution
of some sensors can be changed in the Properties window.
Analog sensors are set up in a similar fashion. You will often need to adjust the sampling rate—how often to take a reading. The sampling rate can be adjusted using the Controls Palette below the Display Area.

To exit the Hardware Setup screen, click again on the 850 UI icon. You can return to the Hardware Setup screen at any time to make changes.

**Displaying data**

The two most commonly used displays are Graph and Table. For instance, if you wish to graph you data, drag the Graph icon from the Display Palette into the Display Area. A graph will appear that fills the entire Display Area.

Below the horizontal axis is a <Select Measurement> button. Clicking on this will bring up a list of quantities that can be plotted on the horizontal axis. A similar button on the vertical axis allows you to choose the quantity to be plotted on the vertical axis. Only the quantities available for the sensors you have set up are shown. (Some of the options are calculated from the data reported by your sensor. For instance, velocity and acceleration values can be calculated from position measurements from a motion sensor.) Capstone will provide the appropriate axis labels, showing the quantity followed by the SI abbreviation for its units in parentheses. These labels are required for all graphs, whether they are drawn from Capstone or not. If you use other software or draw a graph by hand, you will have to manually provide these labels.

To plot more than one graph in the same Display Area, click on the “Add new plot area to the Graph display” icon along the top of the graph. These graphs will have the same horizontal axes, but different vertical axes. To plot a second quantity with the same units using the same horizontal axis, but with the y-axis values listed on the right hand side of the graph, click on the “Add new y-axis to the active plot area” icon along the top of the graph. Finally, to plot a second quantity using the same horizontal and vertical axis, click on the vertical axis label button and choose “Add similar measurement” from the menu that appears. A list of the available similar measurements (for instance, potential energy in a graph of kinetic energy versus time) will appear. Choose the quantity to be plotted from the list. Each of these formats will prove useful.

All graphs should have titles. In Capstone, the title can be typed into the lower left hand corner of the Display Area, to replace the text “[Graph title here]”. If you forget to type the title in, you can print the title across the top of your graph by hand. Similarly, horizontal and vertical axes labels may be printed by hand if they are not provided by the software.

**Preparing graphs for printing**

Graphs can be extremely useful device for displaying the results of an experiment. However, much of their value is lost if certain mistakes are made. Without titles or axes labels, the reader will not know what is plotted. Graphs that are too small, or are dominated by data that is irrelevant to the goal of the experiment, are almost useless. For lab notes, certain information must be recorded directly on the graph. For instance, the results of curve fitting procedures should be noted on the graph along with their uncertainties; you should also indicate on the graph which data were included in the curve fitting process. Observations about the plotted data—especially comments
about relationships between plotted quantities, are much more clear if they are written on the graph itself. For instance, a vertical line can show that the minimum of one quantity coincides with the maximum of another quantity. To make room for these notes, the important parts of your data must be plotted in as large a format as possible.

Before printing a graph, make sure that the horizontal and vertical scales are adjusted to show the data of interest in as large a format as possible. Capstone allows you to adjust both scales arbitrarily. When the cursor is moved over one of the numbers along the horizontal scale, it morphs into “a spring with an arrow on each end.” Click and drag the cursor and the scale expands or contracts. The vertical axis can be adjusted in a similar fashion. You can move the whole $x$ and $y$ axes horizontally or vertically by moving the cursor over one of the axis lines until it morphs into a small hand. Clicking and dragging now allows you to adjust the position of the axes horizontally and vertically to give the best presentation of the desired data.

The data in most graphs occupies a larger portion of the page if printed in the “landscape” format, as opposed to the “portrait” format, since the long edge of the graph is printed along the long dimension of the paper. This fills the sheet more efficiently and makes the graph bigger. Unfortunately “portrait” is the default setting. The “Print” command is in the drop-down menu under “File” on the very top left hand corner of the main Capstone window. “Print Page Setup” on the same drop-down menu can be used to specify the “landscape” format. It will be there somewhere, but the exact location is printer dependent. If it is not readily apparent, choose the printer “properties” tab and you should be able to find it under the options available in that window.

Full credit will not be awarded for graphs without a title or where labels are missing. Graphs that are smaller than necessary or include a lot of unnecessary data will also not receive full credit. In lab notes, graphs with curve fits will not receive full credit unless they include the results of the curve fit(s), their uncertainty, and a clear indication of which data were included in the curve fit.

**Uncertainty analysis with Capstone**

The mean, $x_{\text{avg}}$, and standard deviation, $\sigma(x)$, of a set of data are easily computed from tables and graphs. (On graphs, you can highlight the data you want to average with the cursor.) Click on the down arrow just to the right of the $\Sigma$ on the toolbars at the tops of the graph and table windows to calculate means and standard deviations. In Capstone and Excel, it is important to distinguish between the standard deviation of your data, $\sigma(x)$, and the standard deviation of the mean, $\sigma(x_{\text{avg}})$, which represents the uncertainty in $x_{\text{avg}}$. These are defined in the Uncertainty/Graphical Analysis Supplement to the lab manual. The standard deviation function in Capstone and Excel returns $\sigma(x)$, the standard deviation of the selected data. To calculate the uncertainty in $x_{\text{avg}}$, you must divide $\sigma(x)$ by the $\sqrt{N}$, where $N$ is the number of selected data points.

“Least squares fits” to graphical data are easily done. If you wish to fit only part of the data, first select the data you want to fit using the “Highlight range of points in active data” tool (icon with yellow pencil and blue dots) above the Display Area. For a linear fit, select “Linear: $y = mx+b$” from the “Apply selected curve fits to active data” tool (icon with red line and blue points) above the Display Area. The software displays a box showing the slope and intercept of the linear equation...
along with standard errors of the slope and the intercept values. The standard error corresponds to \( \sigma(x_{avg}) \).

**Changing data precision in Capstone**

Although Caption acquires and stores data at the maximum precision provided by the sensor, the precision of values in tables and graphs is often lower. When the least significant figure of the displayed data (or the least significant figure of a value determined from a curve fit) is larger than the indicated uncertainty, you need more precise data in the display. To increase the precision, select the graph or table with the data and click on the “Data Summary” icon in the Tools Palette on the left side of the screen. The Data Summary window displays a list of your data. Select the data you wish to modify, then click on the gear icon to its right. From the Properties window that opens, click on the “Numerical Format” tab and adjust the number of decimal places in the text entry box.

**More details for specific sensors**

**Motion sensor (ultrasonic, digital)**

Plug the leads from the motion sensor into the digital Channels 1 and 2 of the interface unit. Any two adjacent digital channels will work for the motion sensor, but the yellow plug must be to the left of the black plug. In the Hardware Setup window, assign the motion sensor to the input channel with the yellow-banded plug. The motion sensor sends out a short high-frequency pulse of sound waves at about the limit of human hearing and measures the time for the echo to return. Thus position, velocity, and acceleration can be computed with that information. The sampling rate for the motion sensor is limited to 50 Hz or to 50 readings per second. You may need to adjust this rate from the default setting of 20 Hz to optimize the data collected for your particular experiment. The sampling rate is set in the Control Palette along the bottom of the Display Area. Don’t hesitate to experiment a little to determine the best setting. Position, velocity, and acceleration are the default data quantities. Graphs are by far the most common display for this sensor. Objects less than 0.4 m (0.25 m for the newer model) or more than 4 m from the motion sensor are not reliably detected. At small distances the echo returns too quickly to be measured reliably while at large distances the echo is too weak. Relatively smooth, flat surfaces make better reflectors of sound. Beware of stationary objects close to your experiment that may reflect the sound waves and give you spurious results.

**Rotary motion sensor (digital)**

Plug the leads from the rotary motion sensor (RMS) into Channels 1 and 2 (digital) of the interface unit. Any two adjacent digital channels will work for the RMS, but the yellow plug must be to the left of the black plug. Internally this sensor consists of two photogates. You can get a brief explanation of how it works at http://www.sxlist.com/techref/io/sensor/pos/enc/quadrature.htm. You need to tell Capstone that the rotary motion sensor is connected to the interface unit, what quantity is to be measured with the sensor, and how you want the resulting data displayed.
On the image of the interface unit in the Hardware Setup window, click on the digital input channel with the yellow plug. A list of digital sensors will be displayed. Find the RMS in the sensor list. Highlight it by clicking on it with the cursor and hit OK. The RMS should now be displayed in the Hardware Setup window as connected to the interface. Click on the Properties button along the bottom half of the window. To measure position, velocity, and acceleration the software assumes that you are passing a string over a pulley on the shaft of the rotary motion sensor, and that the position, velocity, and acceleration will be the speed of the string. For a particular rotation rate, the string will move much faster if it passes over a large-radius pulley than over a small-radius pulley. Thus the circumference of the pulley must be indicated. If the standard black PASCO pulley is used, then you can choose “Large Pulley (groove),” “Med Pulley (groove),” or “Small Pulley (groove),” and the correct circumference will automatically be inserted in the “Linear Conversion Value” box. For some experiments, the lab manual will also direct you to change the “Resolution” setting.

When setting up a graph or a table, you will select the measurement you wish to display using the <Select Measurement> button. Note that all the angle measurements are in radians. The kinematic equations for angular motion conventionally use radians and not degrees. This makes it simple to convert angular displacements, velocities, and accelerations to their corresponding linear displacements, velocities, and accelerations. If you display a linear quantity, it is important to have the correct pulley selected in the Hardware Setup Properties window of the RMS.

Click on the “Record” button and spin the shaft of the rotary motion sensor. After a few seconds click the “Stop” button. Since the computer arbitrarily set the scales of the graphs, the displayed data may be too small or too large. Once the data is taken, Capstone can change the scale of the graphs to have the data fill the available space. The “Scale to Fit” feature must be applied to each graph separately. Click anywhere on the graph to highlight it with a line box around it. Then click on the “Scale to fit” icon on the far left in the toolbar at the top of the graph window or adjust the horizontal axis and vertical axis scales separately as discussed above. When the scaling is complete, you can print out the graphs. Be sure that the graph window is active so that’s what gets printed. If you reverse the direction of rotation of the sensor shaft, the signs (+, −) of the angular position and angular velocity change. Interchanging the input leads that are plugged into Channels 1 and 2 also changes the signs of these measured quantities. This feature allows you to choose the positive vertical axis for any experiment that you do.

**Photogate (infrared so you won’t see the light!)—digital**

Plug the ‘photogate” into one of the four digital channel receptacles, say Channel 1. Make sure that the plug is inserted all the way into the receptacle. The photogate consists of an infrared light source in one arm and an infrared light detector in the other arm. It outputs zero volts to the computer when something blocks the beam and 5 volts when the beam is unblocked. Now that the hardware is connected, you need to inform the software what is connected to the interface. If the Hardware Setup window is already displayed, click on Channel 1 on the image of the interface unit to display a list of digital sensors. (The Hardware Setup window can be opened by clicking on the interface unit icon in the Tools Pallete.) After clicking on Channel 1, select “Photogate” from the pull-down menu that appears. If done successfully, you should see a picture of the photogate connected to the chosen digital channel. Note that there are other sensor choices for specialized
applications of the photogate.

The software now understands that a photogate is connected to Channel 1, but does not know what you want to measure. To continue, click on the “Photogate Timer” icon just below the Hardware Setup icon in the Tools Palette. Capstone will guide you through the steps to set up your time. Photogates are often used to measure the speed of small objects that pass through the infrared beam. For this application, select the default “Choose a Pre-Configured Timer” option and click on the <Next> button to complete the first step of the setup. In Step 2, make sure the box next to your photogate “Photogate, Ch 1”, is checked and click on the <Next> button. To measure speed, choose “One Photogate (Single Flag)” option from the list in Step 3. In Step 4, make sure that the “Speed” option is checked. You may also want to check the “Time in Gate” option. Capstone will calculate the speed of your object by dividing the width of the object (the Flag Length) by the time the beam is blocked as the object passes through. Enter the width of the object (in meters) whose speed you wish to measure into the Flag Length block in Step 5. Finally, you can give your timer a name in Step 6. Exit the Photogate Timer setup window by clicking once on the Photogate Timer icon in the Tools Palette.

To display your speed measurement(s) in a table, drag the Table icon from the Display Palette on the right into the Display area. A table with two columns will appear. Select the measurements to display from the menus that appears when you click on the <Select Measurement> buttons. Putting the time in seconds in one column and the speed in m/s in the other will work for now. If you plan to print your Table for inclusion in your lab notes, describe of your data briefly on the [Table Title Here] line.

To take some test data find a pen or pencil, click on the “Record” button along the bottom left of the Display Area, and pass the pen or pencil back and forth through the photogate a few times; then click the “Stop” button that appears where the “Record” button had been. The table now displays some times along with the measured speeds. When you activate the “Record” button, an internal clock is started. When the light beam is blocked or unblocked, the time when it is blocked or unblocked is recorded relative to the arbitrary “zero time” when you hit the “Record” button. If necessary, repeat the data collection process to clarify the details. Notice that the second set of data is simply named “Run #2.”

You can connect a photogate to any of the four digital channels of the interface unit and they will work just the same.

**Photogate with pulley (digital)**

A photogate can be used to measure displacement and velocity of objects connected to a string that runs over a pulley. The photogate must be plugged into a digital Channel on the interface unit. Choose the “Photogate with Pulley” option when you set up the interface unit. As the pulley turns, the spokes of the pulley successively block and unblock the light beam from the photogate. With the pulley, the photogate works much like the Rotary Motion Sensor, but with one serious limitation. The photogate has no way of determining which direction the pulley is rotating, so distances are always considered to be positive. If the object that we are studying moves in only one direction, this limitation poses no problems. The software knows the circumference of the pulley and can compute position, velocity, and acceleration as a string passes over the pulley and turns it.
Angular quantities can also be measured. These quantities can be graphed, displayed in a table, or whatever.

**Force sensor (analog)**

The Force Sensor must be plugged into one of the analog Channels, A, B, or C. In the “Hardware Setup” window click on the analog channel with your sensor and choose the “Force Sensor” option [not “Force Sensor (student)"]. The default measurement of the sensor is force in newtons. The data from the sensor is usually most useful when shown in a table. You can change the “sampling rate,” that is, how often the computer reads the output of the force sensor. The default sampling rate is 10 Hz, or 10 times a second. For some applications, this can be reduced to 1 Hz, or one reading each second. The sampling rate is set in the Control Palette along the bottom of the Display Area. It is necessary to “zero” the sensor by pushing the small tare button located on the side of the sensor. This is exactly analogous to zeroing an electronic balance before weighing something. It is good practice to make sure that the force sensor really reads zero after pushing the tare button by clicking the “Record” button and checking the resulting data table. You will notice that the force value is not exactly zero, but varies a bit. This is normal. These variations will also be present if a mass is suspended from the sensor. So, what is the “real” value of the force? Answering this question will introduce you to some of the data analysis options available with the Capstone software.

If we measure something multiple times but get slightly different answers each time, then the average value or “mean” value is often the “best value.” When using a mean value, it is important that each time the measurement is made nothing significantly changes. In other words the conditions under which the measurements are made must remain essentially unchanged. As long as the hanging mass is not swinging or something, the conditions are unchanged. One question remains: how large are the variations from the mean value? Statistically speaking, the standard deviation is a measure of this variation. You can read more about how these quantities are defined and used in the “Uncertainty Analysis Using Capstone” section earlier in this document.

The standard deviation of the mean is a useful estimate of the uncertainty of the average value of \(x\). Throughout this course the standard deviation of the mean will play this role, and you will have occasion to use it again and again. On the toolbar along the top of the Table window, look for the Greek symbol \(\Sigma\). This Capstone’s “statistics” icon. Clicking on the down arrow just to the right of the \(\Sigma\) opens a drop-down menu that includes the mean and standard deviation. Make sure a check mark appears by each function you want to display. Then clicking on the \(\Sigma\) causes these numbers to appear at the bottom of your table. If you want increase the number of significant figures in the display (say, to minimize round-off error), you can increase the number of displayed digits by clicking on the “0.0 \(\rightarrow\) 0.00” icon along the top of your table. (To decrease the precision, click on the “0.00 \(\rightarrow\) 0.0” icon.) To calculate the standard deviation of the mean from the standard deviation, divide the standard deviation by the square root of \(N\), the number of measurements. Capstone will display \(N\) if you also check the “Count” function in the \(\Sigma\) menu.
**Voltage sensor (analog)**

The Voltage Sensor is simply two wires with a special plug that connects from the circuit of interest to one of the analog Channels A, B, or C of the interface unit. The interface then serves as a voltmeter. The voltage can then be observed using various displays. If the voltage remains constant with time, using the “Meter,” “Digits,” or “Table” display may serve you well. For voltages that vary with time the “Graph” or “Scope” display may be more useful. The “Graph” display is often best for a one-time event where the voltage varies with time. For repetitive voltage signals, that is, those that repeat in time, the “Scope” display is extremely useful.

**Geiger counter (digital)**

The Geiger counter is used to detect certain types of radiation, including beta particles and gamma rays. Our older Geiger counters have an AC power cord must be connected to a regular electrical outlet. The signal cable is inserted into one of the digital input channels. Our newer Geiger counters are powered by the cable that connects to the interface unit.

The device can be controlled to count for a specified length of time, then record the number of counts, then repeat the process for as many times as you like, before stopping. The sampling rate is the length of time for each individual counting period. The default is 1 Hz (counts per second), but it can be changed in the Control Palette along the bottom of the Display Area. To set the duration of data collection, click on the “Recording Conditions” icon in the Control Palette along the bottom of the Display Area; then set the “Stop Conditions” option for “Condition Type” to “Time Based” and the “Record Time” to 100 s.

When the resulting data is displayed in a table, you can use the statistics options to compute the average count rate, its standard deviation, and the standard deviation of the mean. The procedure is described above in the Force Sensor section. Geiger counter data is often plotted in a histogram, with the count rate (counts per second) along the horizontal axis, and the number of samples with each specified count rate along the vertical axis. To display a histogram, drag the “Histogram” icon from the Display Palette on the right into the Display Area. The table and histogram views fit nicely side by side.
Excel Spreadsheets and Graphs

Spreadsheets are useful for making tables and graphs and for doing repeated calculations on a set of data. A blank spreadsheet consists of a number of cells (just blank spaces surrounded by lines to make a little “box”). The cell rows are labeled with numbers while the columns are labeled with letters of the alphabet. Thus Cell A6 is the “box” in Row 6 of Column A, which is the first column. Text, numbers, and formulas of various kinds can be entered in each cell.

Tables

Making a table of, say, the force exerted by a spring as its length is changed requires entering the force values in the cells of one column and the length values in the corresponding rows of an adjacent column. Adding some explanatory text in the cells above each column can complete the table. It is sometimes useful or necessary to adjust row heights and/or column widths to accommodate more or less “stuff” in the cells. Clicking on “Help” in the main toolbar at the top of the screen opens a small window where you can type in your question. In this case type in the words “column width” (without the quotation marks) and click on “Search.” Several options will be displayed, including “Changing column width and row height.” Click on it and get detailed instructions how to make the desired changes. Don’t be afraid to use the help screens in Excel. Most of the time you can find answers to your questions fairly quickly.

Graphs

To make a graph in Excel, first select the data to be graphed by clicking on the upper-left cell of the $x$-data and dragging the cursor down to the lower-right cell of the $y$-data. A box should appear around your data and the selected cells will change color. Then select the Insert tab on the main toolbar, click on the Scatter icon, and select the “Scatter with Only Markers” icon from the pull down menu that appears. This icon appears first in the list and shows dots for data points, with no lines joining them. This choice is almost always the best choice for the graphs we make in lab. A graph of the data should appear on the worksheet. In addition, the “Chart Tools” ribbon should appear in the main toolbar. (If your $x$-values are not adjacent to your $y$-values, you will need to use the “Select Data” option to add data points to your blank graph. This option appears in the “Chart Tools” ribbon after clicking on the graph.)

If you do something unwanted, immediately stop the operation and click on “Undo” icon near the top-left corner of the Excel window. This icon is a blue arrow that curves to the left. Usually you can escape your predicament and try again.
Now you can add a descriptive title (“Graph 1” or “Exercise 1” is not sufficiently descriptive) to the graph and label the quantities (with their units!) plotted on the horizontal and vertical axes. Clicking on the “Layout” tab in the Chart Tools ribbon at the top of the Excel window will bring up icons labeled “Chart Title” and “Axis Labels”, among others. For the chart title, select the “Above Chart” option. A text box for the title will appear. Move your cursor to the text box and type your title. To label the horizontal axis, move your cursor to the “Axis Labels” icon and choose the “Title Below Axis” option for the “Primary Horizontal Axis Title”. To label the vertical axis, choose the “Rotated Title” option for the “Primary Vertical Axis Title.” In each case, a text box will appear in which you can type the axis label with units. For instance, if a cart velocity is plotted along the y-axis, you would want a label like “Velocity (m/s)” The velocity units should be indicated parentheses after the main label.

You may wish to add other features to your graph, such as legends, gridlines, best-fit curves to match the plotted data, different axis labels, etc. Even the size and aspect ratio of the graph can be changed. Some of these options appear when you right-click on an axis. Others can be accessed from icons under the Design, Layout, and Format tabs in the Chart Tools ribbon. Your best approach is to do some exploring. Only a few of the options will likely be useful to you on a regular basis, but you need to find where they are.

When you print a graph, don’t print the whole spreadsheet. Move the cursor over the graph and click it to highlight the graph. Then using the “Print” command in the drop-down menu under the “File” tab on the main toolbar will print just the graph. Selecting “Landscape Orientation” under “Settings” will make the graph as large as possible while still fitting on one page. Graphs printed for you lab notes should be printed in the landscape orientation. Excel will display a preview that shows exactly how the graph will appear on the paper when it is printed. Make any necessary adjustments, then print the graph by clicking on the printer icon in the top-left hand corner of the print window.

Making calculations on a set of data

For example let us say that you have data values in Cells A1 and B1 and you wish to take the product of these two numbers and put the result in Cell C1. In Cell C1 type =A1*B1 (the * symbol indicates multiplication). The “equal” sign tells Excel that a formula is to follow. When you hit “enter,” the calculation will be performed and the product displayed in Cell C1. The formula for calculating the number in the cell is still present but hidden behind the number in a sense. If you now change the number in Cell A1, as soon as you enter it, the number in Cell C1 will also change as it re-computes the product with the new number. Suppose that we have one set of numbers in Column A, Rows 1–10, another set of numbers in Column B, Rows 1–10, and that we want to calculate the following products, A1*B1, A2*B2, ..., A10*B10. After typing the product formula into Cell C1, we can click on Cell C1, making a dark outline appear around it. Move the cursor to the bottom right corner of Cell C1 until the cursor morphs into a little + sign. Click and drag down to Cell C10 copying the product formula to successive cells along the way. When you release the click button, the desired products should be displayed in Column C, Rows 1–10.

The symbols used for various mathematical functions are:
* = multiplication / = divide
+ = addition
- = subtraction
^ = powers (need not be integer values)

Use parentheses to make it perfectly clear to Excel what you want to do. The formula =A1+B1/C1 is computed as =A1+(B1/C1). If you wish to sum A1 and B1, then divide by C1, you need to write it as =(A1+B1)/C1. The operations of multiplying, dividing, and taking powers are done first before adding and subtracting.

Some other useful functions in Excel are:

AVERAGE(A1:A9) = calculates the average (mean) of the numbers in Cells A1–A9.
STDEV(A1:A9) = calculates the standard deviation, \( \sigma(x) \), of the numbers in Cells A1–A9.
SIN(A3) = assumes that A3 is in radians and calculates the sine of the angle.
COS(A3) = assumes that A3 is in radians and calculates the cosine of the angle.
TAN(A3) = assumes that A3 is in radians and calculates the tangent of the angle.
ASIN(A6) = calculates the angle in radians whose sine is the number in Cell A6.
ACOS(A6) = calculates the angle in radians whose cosine is the number in Cell A6.
ATAN(A6) = calculates the angle in radians whose tangent is the number in Cell A6.
SQRT(A11) = square root of the number in A11.
LN(A7) = natural logarithm of the number in A7.

Note: These functions must be preceded by the “equal” sign in order to be treated as a formula and do a calculation. For example, =SQRT(B9) typed into Cell C12 will calculate the square root of the number in cell B9 and record it in Cell C12. If the functions are part of a more complicated formula, then only the leading “equal” sign is required. For example, =A2+SIN(A4) typed in Cell B8 will add the number in Cell A2 and the sine of the number in Cell A4 and record it in Cell B8.

**Fitting data with straight lines—only if the data are linear!**

Often in physics the dependence of one variable on another is characterized by a linear relationship, meaning that the variables are related to one another through the equation of a straight line of the form \( y = mx + b \), with \( m \) being the slope and \( b \) the \( y \)-intercept of the graph. The slope and intercept often can be quantities of interest. When several data points, \((x, y)\), are related linearly, how can we calculate the best values of the slope and intercept of the relationship? “Least squares” methods minimize the sum of the squares of the deviations of the fitted line from each of the data points and thus give the “best” values for the slope and intercept of the line.
Excel spreadsheets and graphs

Excel is capable of doing these kinds of fits quite easily. If you have a graph that appears to be quite linear and thus suitable for fitting with a straight line, you can add a “Trendline” to the graph by moving the cursor over the symbol for one data point on the graph and right clicking on it. A drop-down menu should appear with “Add Trendline” as one of the options. Click on it and choose “Linear”. In the same small window click on the “Options” tab near the top and mark the little box for “Display equation on chart.” Clicking on “OK” will display the “best-fit” line on the graph and give the equation of the line as well on the graph. You can move the equation with your cursor by clicking and dragging if it obscures some of the data points. You can also add or subtract digits of precision to the numbers given for the slope and intercept by right clicking on the equation after highlighting it with the cursor. In spite of its applications in other disciplines, the $R^2$ value is seldom useful in the physical sciences and should not be displayed on the graph.

Finding the “standard error” (basically the standard deviation of the mean) for the slope and intercept values, respectively, is also important, because it gives information regarding how precisely we know the slope and intercept values. Excel can do this using the more advanced Regression feature of least-squares fitting. (In OpenOffice and LibreOffice, the LINEST function performs the same regression.) In Excel, the following steps are required:

1. Click on the “Data” tab in the Chart Tools ribbon and click on the “Data Analysis” icon in the “Analysis” group on the right.

2. In the pull-down menu that appears, scroll down to the “Regression” option and click on it to highlight it. After choosing OK, the Regression window should appear.

3. To input the $y$-values in the first blank text box, move the cursor to the box and click in it. Now move the cursor to the top of the $y$-data column in your spreadsheet and click and drag down to select the whole set of $y$-values. The corresponding cell numbers should appear in the $y$-value box in the Regression window. Now move the cursor to the box for inputting the $x$-values in the Regression window. Click and drag over the column of $x$-values in your spreadsheet and these cell numbers should appear in the $x$-value box in the Regression window.

4. In the Regression window under “Output options” mark the circle for “Output range.” Move the cursor into the blank space just to the right of “Output range” and click it.

5. Now move the cursor to an empty cell in the leftmost column of your spreadsheet near the bottom and click it. The corresponding cell number will appear in the box. This tells Excel where to put the results of the regression analysis.

6. Now you are ready to click OK in the Regression window. Excel will do the appropriate calculations and display them below and to the right of the cell that you chose for the Output range. The values of interest are displayed in the lower-left corner of the stuff displayed, just to the right of labels, “Intercept” and “$X$ Variable.” The first column to the right of the word “Intercept” shows the value of the $y$-intercept. This value should equal the value in the trendline equation on the graph—a nice check! The next column to the right shows the “standard error”, or uncertainty of the intercept value. In other words, the intercept will have a plus/minus uncertainty given by this standard error. Similarly the first column to the right of “$X$ Variable” shows the value of the slope (which should equal the slope in the trendline
equation) and the next column shows the plus/minus uncertainty of the slope value. How does Excel get from X Variable to slope? If you look carefully at the regression output, Excel is calling “slope” the coefficient of the X Variable, which is true in the equation of a straight line. A little awkward, but it works.

It is important to avoid fitting a straight line to data that is definitely curved. In this case, your eye is telling you that your model does not fit the data. Such fits are misleading at best. It is often acceptable to select part of your data that does appear to lie on a straight line and fit those points to a straight line.
Formal Lab Report Instructions

The following eight pages of instructions are formatted like your formal lab report. The format is deliberately plain to the point of being ugly. Reports generally undergo substantial revision after submission and before approval no matter where you work. The double spaced text allows room for edits. A uniform, plain look encourages the editor to focus on the presented information and logic. It is also designed to fit smoothly into the institution’s publishing workflow.

If you intend to include your report in your Junior Writing Portfolio, please follow the instructions with care. Avoid copying fragments of text from the lab manual or other sources—especially material from the goals and introduction sections. Those who evaluate the physics submissions often have experience as physics teaching assistants, and are likely to identify the material as plagiarized.
Formal Lab Report Instructions—Title of Lab Here

Authors names here. You will be the first author, with your lab partner’s name following.
Author address(es) here. Write your Course and Lab Section Numbers here in lieu of an address.

Put your abstract on the first page. Do not label it “Abstract.” It will be obvious that it is an abstract. The abstract is a brief summary of your report, including your results and conclusions. Normally, the length of your abstract should be about 5% the length of your report. Your abstract should make it implicitly clear what your report includes and what it omits. By implicit, I mean that you don’t say, “This report includes x and omits y.” You don’t even say that it is a report. A good summary of your results and conclusions will do the job implicitly. Readers use these summaries to decide whether to read your report. If they notice discrepancies between your abstract and what actually appears in your report, they feel cheated.
1. Introduction

**What to include in the introduction.** Like the abstract, your introduction should describe the subject of your paper. An introduction is longer than an abstract, so the subject is described in more detail. One includes the purpose of the experiment and describes its scope. For instance, you will often specify which parameters were explored and how much they were varied. Any background information that the reader needs to understand the rest of your introduction is also included. For the purposes of this exercise, assume that your reader is an introductory physics student like yourself who has not performed this particular experiment.

The introduction is usually written after the main body of your report is complete. Paradoxically, it is seldom clear exactly what you are going to write until you actually write it.

**Characteristics of good technical writing.** Good technical English is unified, coherent, clear and concise. Unity is achieved by enforcing a theme to the paper as a whole. Subjects that are not encompassed by the theme should be left out. The *choice* of theme is critical to the success of the writing operation. The theme is not stated explicitly. (Don’t write, “The theme of this paper is…”.) The abstract of the work should make it clear what belongs in the report and what does not. A good abstract helps the author maintain unity. Ideally, your theme should include everything you intend to write and exclude everything else. After writing the abstract, you may decide to add or delete material as appropriate to make the report a unified whole.

Coherence is achieved by providing logical transitions between the parts of your paper. The order of topics in your paper has a major effect on coherence. If you find yourself repeating ideas in different parts of the paper, you may have failed to order your topics appropriately. Cause and effect is a major part of technical writing. Be sure you state cause and effect relationships clearly.

Clarity is achieved by removing potential sources of ambiguity. Avoid text that can be interpreted inappropriately. In general, your statements should be as specific as possible. The
goal is to communicate as much information as possible. Do not hide information that should be available to the reader.

Conciseness is generally achieved by good editing. All other things being equal, you should use as few words as necessary to communicate what you have to say. Sentences that start with “There are” or “It is” can often be shortened by making an appropriate noun the subject of the sentence. This often resolves unintended ambiguities as to what “it” refers to. Verbs and adjectives with more specific meanings can shorten sentences and improve readability. Active verbs are better than passive verbs unless they shift your focus inappropriately. Your report is not about you. Similarly, do not write about your report in your report. Focus on your subject.

Formatting technical reports is mostly a matter of achieving conformity. Creative formats are not rewarded. (The nail that sticks up gets beaten down.) Your reader must focus on the content of your work, not the details of presentation. Any deviation from standard formatting must be well justified as an improvement (more clear, more concise, etc.). Although these standards are to a large degree arbitrary, many are related to the need for good-looking copy when reproduced. For the purposes of this assignment, the formatting requirements will follow those of the American Institute of Physics.²

With the exception of figures and equations, this assignment must be printed from a word processor. Use a 12 point serif font, preferably Times. (A point is about 1/72 of an inch. In this context, the font size refers to the intended spacing between single-spaced lines, not the size of the letters themselves.) Set the line spacing to exactly 24 points (not double-spaced), with no extra space before or after paragraphs. (Exceptions include lines with equations or figures, which often need more room. These usually need to be single-spaced, that is, with a line space of “at least 12 points”. Lines with equations and figures should be the only lines in your paper that are single-spaced.) Use one-inch margins on all four sides of the paper. For regular paragraphs, justify the text along both left and right margins. The first line of every paragraph should be indented 0.5 inches. Disable automatic formatting options like “format the next paragraph like
the one before it.” They often cause formatting faults and complicate inserting equations and figures.

Use a spell checker, but keep a dictionary at hand for unusual words.

2. Experiment

Title this section Experiment, not Experimental. Titles normally function as nouns, and “Experimental” is not appropriate in this role. Put an extra line break (24 point) before and after numbered headings (as well as figures and equations).

What to include in the experiment section. The experiment section should not include all the procedures that appear in your lab notes. Assume that your reader is familiar with the equipment. Omit most of the information that would normally be found in equipment manuals. Do include the manufacturer’s name and model numbers of any equipment with special features that might not be easily duplicated. Also include any details that might be necessary to the replication of the experiment but would not be clear from reading the manuals. For instance, it is often important to include sample rates for data collection, but not important to specify the units employed when acquiring data.

The experiment section is not the best place to describe some experimental details. Details that apply to only one section of the results can (and usually should) be included in the appropriate part of the results section. This reduces the strain on the reader’s memory and eliminates the temptation to repeat these details unnecessarily. Details that apply to more than one section of the results are generally included in an experiment section. The goal is to avoid repetition, not to collect all the experimental details in one place.

Equations and math. Any equations in your paper should be numbered in sequence on the right hand margin. The number should be in parentheses. Position the equation itself near the middle of the page (left and right). In a word processor, this is achieved by right-justifying the line to position the equation number [(1), (2), etc.] on the right hand margin, then inserting tabs
to center the equation. Equations are normally type-set using an equation editor. If necessary, hand-write your equations. Computer type-set equations must generally be inserted into lines that are single-spaced. For instance, the magnitude of the gravitational force of the earth on the moon, $|F_{E\text{on}M}|$, can be calculated using the equation:\(^\text{1}\)

$$|F_{E\text{on}M}| = \frac{G m_e m_M}{R_{EM}^2}, \quad (1)$$

where $G$ is the Universal Gravitational Constant ($6.67 \times 10^{-11}$ N-m$^2$/kg$^2$), $m_e$ is the mass of the earth ($5.97 \times 10^{24}$ kg), $m_M$ is the mass of the moon ($7.35 \times 10^{22}$ kg), and $R_{EM}$ is the distance between the earth and the moon (average $3.84 \times 10^8$ m).\(^\text{4}\) In text, symbols for variables and constants are italicized. If experimental uncertainties are available, specify them as well (for example, $1.01 \pm 0.01$ g).

All mathematical variables must be defined in the text immediately before or after the first time they are used, except for numbers like $\pi$ and $e$. (Similarly, acronyms must be defined the first time they are used.) If you define a variable in Equation (1), and the same variable is used in Equation (2), use the same symbol in both equations and define this variable only once, with Equation (1). In the text that follows an equation, refer to it as Equation (1) or Equation (2), etc. Equation can be abbreviated “Eq.”, except at the beginning of a sentence. Do not abbreviate the first word of a sentence. The first word of each sentence should be completely spelled out.

3. Results and Discussion

**Descriptive titles.** Papers of modest length do not need numbered subsections. Subheadings are useful. Mark a new subsection by placing a bold title at the beginning of the first paragraph of that section. Do not include the exercise number. Readers quickly lose count.
Short descriptive titles are a great help. When sections become longer than a few double-spaced pages, numbered subsections are appropriate.

For emphasis, use italic, not bold or underlined characters.

**What to include in the Results and Discussion section.** The general principle is to include all the data needed to support your conclusions, with enough discussion to convince the reader of the truth of your conclusions. If some of your data can be interpreted in more than one way, for instance, you will want to present data and/or explanations that support your interpretation. Although we must structure the labs so that they make maximum use of your data to teach physics, your report should be more focused. Everything needed to support your conclusions must be included, and everything that is not related to those conclusions must be excluded. A great deal depends upon what you choose to conclude. Conclusions that are overly broad or too narrow can ruin your entire report. Consider your conclusions carefully.

Unless each data point is of special interest to the reader, tables of data are generally inappropriate. (Data tables are important in your lab notes.) If you need to display your data, use a format that communicates not only the data, but any important relationships. In most cases, figures are the best way to display data. If a table is necessary, they should be numbered with Roman numerals (Table I, Table II, etc.) and provided with descriptive titles. Double lines run across the top and bottom, and a single line separates the column headings from the data. No other lines should appear. A properly formatted example appears below as Table I.

<table>
<thead>
<tr>
<th>Table I. Power loss versus frequency.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency (Hz)</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>100</td>
</tr>
<tr>
<td>1000</td>
</tr>
</tbody>
</table>
**Figure formats.** In your lab notes, your figures generally should be as large as possible. You may, for instance, want to add handwritten notes or slope calculations. In a formal report, you want them to fit comfortably with the text. Figures are normally less than 3.2 inches across, including all labels. If a figure has many parts that can be arranged in two columns, you can double the width. Large figures must fit on a single page with their captions while maintaining the normal one-inch margins. In reports, titles are optional, but captions are mandatory. Figure labels should normally use the same font as the text or a sans serif font like Arial. The figure and caption must contain sufficient information so that the reader does not need to refer to the text to understand what is being presented. Figure captions start with a phrase that serves as a title. This introductory phrase is not a complete sentence. The text that follows consists of complete sentences. Captions should not normally include a discussion of the data. The implications of your data should appear in the text.

All figures must be described in the text. Figures must appear as soon as possible after they are mentioned in the text. The word Figure may be abbreviated (Fig.) in the middle of a sentence, but never at the beginning.

If possible, embed your figures as high resolution bitmap files—at least 300 dots per inch (dpi). TIF files (Tagged Image Files) are compatible with many word processes. To ensure that your graphics files are readable, all lines should be at least 1 point (about 0.014 inch) thick. The smallest letter (including superscripts and subscripts) should be at least 1 mm high. This rules out most superscript and subscript fonts unless you can manually control the size. Do not use open symbols (○) for data points; always use closed symbols (●). Remove all grids and backgrounds. (The background should be transparent.) Center your figure left and right on the page on a single-spaced line. Not long ago, figures were traced by hand for publication. You may trace your figures and label them neatly by hand, if the size requirements are met.
Fig. 1. Hanging mass required to move a pine block across a clean aluminum surface at a constant velocity of 0.2–0.3 m/s as a function system mass (the sum of the mass of the block and any added masses). The slope of the graph corresponds to the kinetic coefficient of friction.$^5$

**Discussion Section.** Relatively short papers do not need a separate discussion section. (Section 3 is then a “Results and Discussion” section, as above.) Otherwise, the third section is titled “3. Results” and the fourth section is titled “4. Discussion.” In your report, a separate discussion section is probably not warranted. Discuss your results as they are presented in the results section.

Generally, it is inappropriate to answer questions for further discussion in a formal lab report. Although these questions are designed to help you learn, your report must be more focused. You should include everything necessary to support your conclusions and nothing more. If the answers form a logical part of your report, and you have data to back them up, they are probably appropriate. If this is true, it would be superfluous to have a subsection entitled, “Questions for further discussion.” You would need to provide other, more descriptive, titles.

**Traceability.** As a rule, formal reports do not contain the details needed to fully verify whether your conclusions are valid. The reader will assume a reasonable level of competence on
the part of the authors. If questions arise, the reader will need access to your lab notes. Do not put anything in your report that is not supported in your lab notes. Your lab notes must in general be recorded at the same time the work was performed. That is, notes about experiment details must be made during the course of the experiment. Notes about data analysis must be made when you analyze the data. Notes about conclusions should be made when you are prepared to conclude. Notes made after the fact are not reliable records. Turn in your lab notes along with your lab report. You teaching assistant should be able to support your conclusions from your report’s Results and Discussion section, which in turn must be supported by the data in your lab notes. This is called traceability.

4. Conclusion

Never conclude anything you don’t discuss. Do not discuss anything that does not relate to your results. Raising new issues in the conclusion is a bad idea. Conclusions must be supported by your work, not merely be related to it. Medical misinformation, for instance, is often presented in conclusions that are not supported in the rest of the report. As noted above, your conclusions determine what you choose to put in the rest of your report.

Acknowledgments

Acknowledgments are optional. If someone or some organization has supported your work financially or provided significant assistance, say so here. Example:

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References


Some publications permit paragraph-length notes in the references section. Few publications require article titles in their citation lists, but they help the reader. Please include them. If you do not know the official abbreviation of a journal title, write out the entire journal title. Article and section titles are enclosed in quotes. Book titles are italic. Journal volume numbers are bold. Journal issue numbers, if provided, should be in parentheses. Page numbers follow, with the year of publication in parentheses. Normally one does not cite URL’s (Uniform Resource Locators for web-based material) unless the links are permanent. To address this problem, many publishers provide each article with a Digital Object Identifier (DOI). One can often locate an article on the web by searching on its DOI at [http://www.doi.org](http://www.doi.org). If you know the DOI, provide it. Do not cite unpublished work unless absolutely necessary.