

Lab 6. Work and Energy

Goals

- To apply the concept of work to each of the forces acting on an object pulled up an incline at constant speed.
- To compare the total work on an object to the change in its kinetic energy as a first step in the application of the so-called Work-Energy Theorem.
- To relate the work done by conservative forces to the concept of potential energy.
- To apply the concept of conservation of mechanical energy, where mechanical energy is defined as the sum of kinetic and potential energy, to a system where the work done by nonconservative forces is zero or cancelled out, as in this experiment.

Introduction

The notion of “work” has a special meaning in physics. When the applied force is constant in magnitude and direction, and the motion is along a straight line, the formula for work reduces to:

$$W = Fd \cos \theta = (F \cos \theta)d = F(d \cos \theta) \quad (6.1)$$

where F is the magnitude of the force, d is the magnitude of the displacement, and θ is the angle between the force vector and the displacement vector. Since magnitudes are always positive, F and d are always positive, and the sign of the work is determined by the factor of $\cos \theta$.

If the force is not constant, then one must sum the work done over each of a series of very small displacements, where the force is approximately constant over each small displacement. In calculus, this process is described in terms of integration.

The concept of work is most useful for point particles in the presence of conservative forces (no friction). Because work is a scalar and forces are vectors, problems that can be solved using the work concept are usually easier to solve by using work than by using Newton’s Second Law.

Work done on a cart moving at constant velocity

Carefully place the wooden block on edge under the end of the track opposite the pulley so that the track is inclined at an angle of $4\text{--}5^\circ$ to the horizontal. “Hooking” the small rubber feet on the bottom of the track over the edge of the block will keep the track from slipping and changing the angle if the track is bumped. Determine the angle of the ramp to within about one-tenth of a degree. A protractor can’t be read accurately enough; use trigonometry! Measure the mass of the cart and the mass of one of the aluminum bars. Two aluminum bars will be placed in the cart for this experiment.

Work done by you on the cart with spring scale parallel to track

Using the small spring scale held parallel to the ramp, pull the cart with the aluminum bars on board at a slow constant velocity up the ramp a distance of 0.5 m. From the definition of work in your textbook or the Introduction above, calculate the work done by you on the cart as you pull the cart up the ramp. Be careful of units! The gram readings of the spring scale must be converted to newtons.

Repeat the measurement as you carefully lower the cart down the ramp at constant velocity.

Work done by you on the cart with spring scale inclined 60° to track

Pull the cart up the track through the 0.5 m distance at constant velocity while holding the spring scale at an angle of 60° with respect to the ramp. Again calculate the work done by you on the cart as you pull the cart up the ramp. Compare these values and comment on the results.

Repeat the measurement as you carefully lower the cart down the ramp at constant velocity.

Work done by gravity on the cart as it moves up and down the ramp

Draw a free-body force diagram of the cart being pulled up the ramp. (Ignore friction.) You have already computed the work you did as the cart was pulled up the ramp. Now calculate the work done by each of the other forces. Show the steps of your analysis carefully and be careful of signs.

Repeat the free-body diagram and work calculations for the cart as it moves down the ramp. Use a table to show the values of the work done by each force acting on the cart for the 0° and 60° orientations of the spring balance. Sum the values of the work to find the total work done by all the forces acting on the cart for each of the two cases.

When a net force begins to act on an object at rest, the object begins to move. One can argue mathematically (see your textbook for the details) that the work done on the object (neglecting friction) is equal to the change in its kinetic energy if we define the kinetic energy to be $mv^2/2$, where m is the mass of the object and v is its speed. Remember that the net force on the cart is zero when it moves with constant velocity.

Based on the Work-Energy Theorem, what total work would you expect for each case? Did your calculated total work values behave in accordance with these expectations? Make quantitative comparisons between your expected and experimental results. Explain.

Applying the Work-Energy theorem to an accelerating cart

Before beginning this investigation, level the track and take the aluminum bars off the cart. Then add or subtract paper clips on the end of the string as necessary to cancel the frictional forces acting on the pulley and cart as the cart moves toward the pulley end of the track. When you have achieved the correct balance between the weight of the paper clips and the friction, the net force on the cart will be very close to zero. Then the acceleration of the cart should also be very close to zero. (What will the velocity-time graph look like if the acceleration is zero?) Give the cart a gentle push toward the pulley and use Capstone with the “Photogate with Pulley” sensor to measure the acceleration. Adjust the number of paper clips as necessary to “fine-tune” the apparatus, so that the average acceleration is as close to zero as possible.

Place a 20-g mass on the end of the string in addition to the paper clips. The net force on the system (cart plus hanging mass) is now the weight of the 20-g mass. As the cart moves and the 20-g hanging mass descends, work is done by the gravitational force on the cart-hanging mass system. Is work done by any other forces acting on the system? Remember that we have “cancelled out” the frictional force with the paper clips, so friction need not be considered here.

Move the cart to the end of the track opposite the pulley and release it from rest. Click the “Start” button in Capstone a couple seconds before releasing the cart. This ensures that you get some data before the cart is released. With the “Photogate with Pulley” sensor, Capstone defines the position of the cart at the beginning of data collection to be zero. This will be helpful below.

Take appropriate data to address the question of whether the total work done on the system is equal to the change in kinetic energy. Note that the computer can calculate the total work done since the system was released from rest and the instantaneous value of the kinetic energy in real time as the cart moves. Use the Capstone “Calculator” tool from the Tools Palette to define expressions for the work done on the system and for the kinetic energy. Your TA can assist if necessary. **Be sure to show the reasoning used to get these expressions in your lab notes.** These defined quantities can then be displayed on a graph just like other measured quantities. Displaying the work and the kinetic energy on the same graph provides the simplest method for comparing the two as a function of time.

Print out the results and discuss your findings. Be sure to address the issue of “change in kinetic energy” versus just “kinetic energy.” Are they ever the same?

Kinetic and potential energy

A slightly more complicated arrangement is produced by raising the end of the track opposite the pulley to make an angle of about 2° with the horizontal. (Laying the block of wood flat on the table under the feet of the track should be adequate.) With the 20-g mass still hanging from the end of the string, the net force on the system of the cart and hanging mass now involves more than one

force. (Note: The assumption that the friction force is not changed much by raising the ramp to a small angle should be very good.) Again address the question, does the total work done by the forces equal the change in the kinetic energy of the system? Collect appropriate data to determine the validity of this hypothesis.

Potential energy and the conservation of energy

Where does the idea of “potential energy” come from? In many ways potential energy is an intuitive concept from everyday experience. For example, if you are hit by a falling apple, you know instinctively (or by experience?) that the damage it does depends on the height from which it falls. We might even be tempted to think about the notion of “conservation of energy.” While the apple is falling and losing energy of position (potential energy), is it possible that the energy of motion (kinetic energy) increases so that the sum of the two energies remains constant? One approach to answering this question is to assume that the sum of the kinetic and potential energies remains the same, and then try to discover how the potential energy would have to be defined to obey such a conservation law.

In the previous exercises you have hopefully shown that the work (W) done on an object by a net force is equal to the change in kinetic energy (KE) of that object. Mathematically we say:

$$W = \Delta KE = KE_{final} - KE_{initial} \quad (6.2)$$

If our energy conservation idea is to be correct, then something like potential energy (denoted PE) must exist so that the sum of KE and PE is constant:

$$KE_{final} + PE_{final} = KE_{initial} + PE_{initial} \quad , \text{ or} \quad (6.3)$$

$$KE_{final} - KE_{initial} = PE_{initial} - PE_{final} \quad \Rightarrow \quad \Delta KE = -\Delta PE \quad (6.4)$$

The energy conservation idea from Equation 6.4 can be reconciled with the work-energy relationship from Equation 6.2 only if the change in PE is equal to the negative of the work done. That is,

$$\Delta PE = -W \quad (6.5)$$

This relationship cannot be used in the presence of forces like friction since the position of an object in space does not uniquely specify the work done by friction in the process of moving the object to that position; thus the potential energy cannot be uniquely defined either. Fortunately for us, the work done by the gravitational force and the electric force, two of the most common forces in nature, are both defined uniquely as objects move from one place to another. (Refer to your textbook for a discussion of conservative forces, such as gravity and electric forces, and non-conservative forces, such as friction.) Let’s apply Equation 6.5 to a simple case of an object of mass, m , near the earth’s surface falling vertically from a position, $y + h$, to a lower position, y .

Since the gravitational force is downward, the work done by gravity is positive and equal to mgh . Therefore

$$\Delta PE = PE_y - PE_{y+h} = -mgh \quad \text{or} \quad PE_{y+h} - PE_y = mgh \quad (6.6)$$

Interestingly, Equation 6.6 only specifies that the difference in the potential energy from the initial to the final state must be mgh . This can be satisfied by setting PE_{y+h} to mgh and setting PE_y to zero. An equally valid solution would be to choose $PE_{y+h} = mg(y+h)$ and $PE_y = mgy$. In this case the zero of potential energy will occur when $y = 0$. There is a certain amount of latitude in choosing the “zero” of potential energy. This is always the case!

If an object moves horizontally near the earth’s surface, the gravity force has no component along the direction of the displacement. Thus no work is done, and the potential energy of the object does not change. In cases where both horizontal and vertical displacements occur, only the vertical displacement leads to a change in potential energy of the body.

Plotting the cart’s total mechanical energy as a function of time

Use Capstone’s Calculator Tool to calculate and plot the sum of the kinetic and potential energies of the cart/mass system as the cart moves down the track. Is the total mechanical energy of the system is conserved?

Conclusion

Summarize all your findings carefully and succinctly. Where possible, discuss your results in terms of the measurement uncertainties.

Before you leave the lab please:

Quit all computer applications you may have open.

Place equipment back in the plastic tray as you found it.

Report any problems or suggest improvements to your TA.