Lab 12. Vibrating Strings

Goals

- To experimentally determine the relationships between the fundamental resonant frequency of a vibrating string and its length, its mass per unit length, and the tension in the string.
- To introduce a useful graphical method for testing whether the quantities x and y are related by a "simple power function" of the form $y = ax^n$. If so, the constants a and n can be determined from the graph.
- To experimentally determine the relationship between resonant frequencies and higher order "mode" numbers.
- To develop one general relationship/equation that relates the resonant frequency of a string to the four parameters: length, mass per unit length, tension, and mode number.

Introduction

Vibrating strings are part of our common experience. Musical instruments from all around the world employ vibrating strings to make musical sounds. Anyone who plays such an instrument knows that changing the tension in the string changes the resonant frequency of vibration. Similarly, changing the thickness (and thus the mass) of the string also affects its frequency. String length must also have some effect, since a bass violin is much bigger than a normal violin. The interplay between these factors is explored in this laboratory experiment.

Water waves, sound waves, waves on strings, and even electromagnetic waves (light, radio, TV, microwaves, etc.) have similar behaviors when they encounter boundaries from one medium to another. In general all waves reflect part of the energy and transmit some into the new medium. In some cases the amount of energy transmitted is very small. For example a water wave set up in your bathtub moves down the length of the tub and hits the end. Very little energy is transmitted into the material of the tub itself and you can observe a wave of essentially the same size as the "incident" wave being reflected. The clamps at the ends of a string provide similar boundaries for string waves such that virtually all the energy of the wave is reflected back and the wave travels from one end to the other. The wave "bounces" back and forth. If waves are sent down a string of some length at a constant frequency, then there will be certain frequencies where the reflected waves and the waves being generated on the string interfere constructively. That is, the peaks of the incident waves and the peaks of the reflected waves coincide spatially and thus add together. When this occurs, the composite wave no longer "travels" along the string but appears to stand still in space and oscillate transversely. This is called a "standing wave" for obvious reasons. A marching band that is marching "in place" but not moving is a fair analogy. You can easily demonstrate this phenomenon with a stretched rubber band. These standing waves occur only at particular frequencies, known as resonant frequencies, when all the necessary conditions are satisfied. These necessary conditions depend on the factors mentioned above, such as whether the string is clamped tightly at the ends or not (i.e., the boundary conditions), the length of the string, its mass per unit length, and the tension applied to the string.

With this in mind, we will systematically explore how the resonant frequency depends on three of the four factors listed above. In all cases our strings are clamped or held tightly at both ends; we consistently use the same boundary conditions. Finally, we will search for a single equation that describes the effect of length, tension, and mass per unit length on the resonant frequency.

Equipment set up

A schematic diagram of the set up is shown in Figure 12.1. Connect the speaker unit to the output terminal (marked with a wave symbol) and the ground terminal (marked with the ground symbol) of the Pasco Model 850 interface unit. The interface unit can be configured to produce a voltage that varies sinusoidally at a known frequency. In the Experimental Setup window, click on the image of the output terminal (marked with a wave symbol). In the window that appears, make sure that the waveform pull down menu is set to Sine Wave. Use the frequency and voltage windows to set the frequency and output voltages, respectively. Keep the output voltage below 4.5 V. Click the "Auto" button (which toggles the Auto function off), then click the "On" button to start the voltage generator.

This voltage drives an audio speaker mechanism that lacks the diaphragm that normally produces the sound. You will nevertheless hear some sound from the speaker drive mechanism. This sound can be irritating, so use the minimum voltage required to make a good measurement. This speaker drive oscillates in synchrony with the drive voltage and is connected to the string via an "alligator" clip.

Caution: Do not apply loads greater than 10 kg to the end of the string!

Effect of string length on resonant frequency

Start with the 1.3 g/m string (see the tag attached to the end of string) and hang a total mass of 5 kg, including the mass of the mass hanger, on the end of the string. Determine the fundamental resonant frequency for five or six different string lengths. Plucking the string with your finger near the middle point excites a vibration of the string primarily in its fundamental resonant mode (also called the first harmonic). Pluck the string and note how the string vibrates. The vibration of the string stops a short time after you pluck it because of energy losses due to air friction. The speaker drive allows you to pump energy into the vibrating system at the same rate that it is lost, so that the vibration can be maintained for as long you wish. The string will vibrate strongly only at certain well-defined frequencies. By adjusting the frequency of the speaker drive slowly while watching

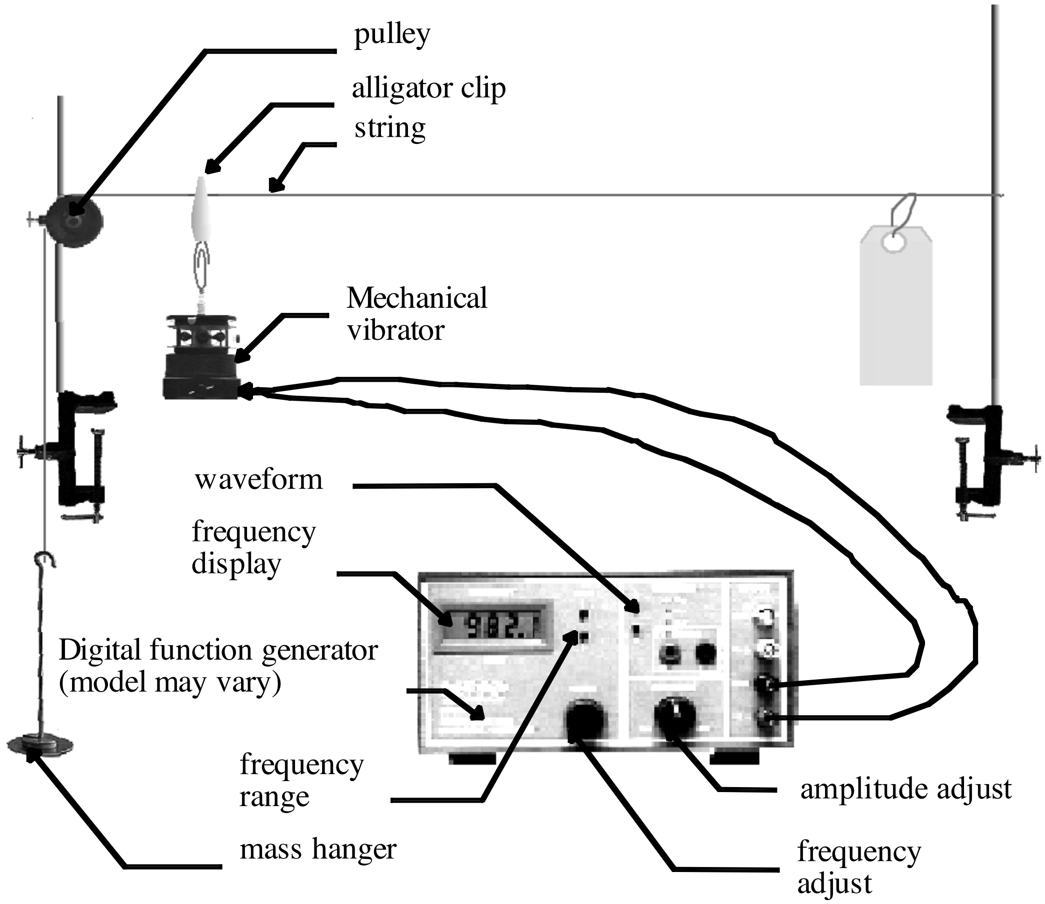

Figure 12.1. Typical apparatus for the vibrating string experiment. The Pasco Model 850 interface unit can be used to control the mechanical vibrator in place of the digital function generator.

the string you should be able to find the frequency that makes the string vibrate in its fundamental resonant mode. You can recognize the fundamental resonant frequency easily because the whole middle portion of the string oscillates up and down like a jump rope; the fundamental resonance can be thought of as the "jump rope mode." For best results you must continue adjusting the speaker drive until you have found the "middle" of the resonance, where the amplitude of the vibration is maximized.

Note that the distance from the alligator clip to the top of the pulley where the string is held tightly determines the length of the vibrating string. The alligator clip does vibrate slightly but the string behaves very nearly as if the clip defines a clamped end. (The motion of the alligator clip cannot be ignored for very heavy strings. For these, you may have to visually locate the point near the alligator clip which appears to be clamped and doesn't vibrate.)

Make sure that the string lengths that you test are approximately uniformly spaced between 0.4 m and approximately 1.7 m. (The maximum string length is limited by the length of the table.) By graphical means determine a mathematical function for the fundamental resonant frequency, f, as a function of L, where L is the length of the vibrating string as determined by the placement of the alligator clip. Do you get a linear graph if you plot f on the y-axis and L on the x-axis? Instead, try plotting f on the y-axis and $1/L$ on the x-axis. What important property of the wave on the string can be determined from this graphical analysis? The units of the slope of this graph (assuming it is linear) provide information on what this quantity might be. Explain your reasoning!

Effect of string mass-per-unit-length on resonant frequency

For this set of experiments, use the maximum string length employed in above and hang a total of 5 kg on the end of the string. Test the four the strings in the box, noting the mass per unit length (μ) indicated on the attached cards. Find the fundamental oscillation frequency for each of the strings at your station. Remember that you already took one data point while observing the effect of string length. Determine graphically whether the relationship between fundamental frequency, f, and the mass per unit length, μ, that is $f(\mu)$, is a simple power function. If so, find the equation for frequency as a function of mass/unit length. Refer to the Uncertainty-Graphical Analysis Supplement in this lab manual for details.

Effect of string tension on resonant frequency

For these experiments, use a string with a mass/length between 1.0 and 6.0 g/m and a length of at least 1.5 m. Determine the fundamental resonant frequency of the string as the total mass on the end of the string is increased from 1.0 to 10.0 kg. The weight of the hanging mass will equal to the tension in the string, T. Graphically determine whether the relationship between the fundamental resonant frequency, f, and the string tension, T, is a simple power function. Again refer to the Uncertainty-Graphical Analysis Supplement in the lab manual.

Effect of harmonic mode number on resonant frequency

Using the 1.3 g/m string and the 3 kg hanging mass, set the length of the string to at least 1.5 m. So far you have looked at the fundamental frequency or first harmonic of the string vibration. The second harmonic (mode number $n = 2$) will have a "jump rope" mode on each half of the string but they will oscillate in opposite directions. Increase the driver frequency until you find this resonance and record it. The third harmonic will have three "jump rope" modes on the string, etc. At the very least you should collect the data for n = 1, 2, 3, and 4. If time allows, determine frequencies for even higher n values.

Determine the relationship $f(n)$ between the resonant frequency, f, and the mode number, n, by graphical means.

Summary

Summarize your findings clearly and succinctly. Can you write a single mathematical function that encapsulates all the relationships that you have discovered? That is $f(T,\mu,L,n)$. Note that taking the sum of the four relations you determined above will not work. Compare your experimental results with those theoretically predicted in your textbook. (This is sometimes included in a section on musical instruments.) Show that the textbook formula is dimensionally correct. Be quantitative in your comparisons.

Before you leave the lab please:
- Turn off the power to all the equipment.
- Leave only the 1 kg mass hanger on the end of the 1.3 g/m string.
- Straighten up your lab station.
- Report any problems or suggest improvements to your TA.