Physics 202

Lab Manual

Summer 2017

Manual Owner ____________________

Lab Section Number _______________

TA Name _________________________

TA e-mail ________________________

Lab Group Rotation Number ______

Lab Schedule

Date	Lab Title	Knight Chapters
June 6	Electrostatics	Ch. 22
June 8	Electric Fields	Ch. 23
June 13	**No Labs**	
June 15	Gauss's Law Tutorial	Chs. 24 and 25
June 20	Ohm's Law	Ch. 27
June 22	Series and Parallel Resistors	Ch. 28
June 27	RC Circuits	Ch. 28
June 29	Magnetic Fields	Ch. 29
July 4	**No Labs (Fourth of July)**	
July 6	Current Balance	Ch. 29
July 11	Electromagnetic Induction	Ch. 30
July 13	AC Circuits	Ch. 32
July 18	Interference of Light	Ch. 33
July 20	Images with Thin Lenses	Ch. 34
July 25	**Lab Exam** during your regularly scheduled lab period	
July 27	**No Labs (Finals Week)**	

Lab rooms change frequently. Consult the bulletin boards across the halls from the elevators on the second, third, or fourth floors of Webster Physical Sciences Building for the latest lab room information.

Contents

Lab Syllabus

Lab Director	Dr. Stephen Langford
Office	Webster Hall Room 348
Phone	335-3398
Email	physlabs@wsu.edu

To apply what you learn in the lecture, you will need some skills and concepts that are best learned in the laboratory. These skills include model building, data collection and analysis, laboratory record keeping, and formal reporting of results. You will also need enough statistics to perform elementary hypothesis testing. These skills apply to quantitative work in many fields, including the health- and life-sciences, math, and engineering. Although these activities should improve your understanding of the lecture material, our primary goal is to turn theory into practice.

Most students in introductory physics courses have had lab experience in chemistry and other disciplines. We build on that experience. Your teaching assistants will not be as specific about their requirements as your chemistry teaching assistants were. You will often be expected to figure things out on your own in consultation with your lab partner; you will be graded on the quality of those decisions. Since you will be working more independently, you will be required to document your work more carefully, with less input from your teaching assistant.

To accomplish these goals, you will be expected to:

- Build simple physical models that incorporate lecture material.
- Design and perform simple experiments to test or improve these models.
- Employ representative software packages for data collection and analysis.
- Document your experimental methods, results, and data analysis in a lab notebook.
- Evaluate and compare results using uncertainties.
- Communicate your work in writing (short and long formal assignments).

Student responsibilities

- Read the syllabus. The regulations/guidelines in this syllabus take precedence over any oral commitments that may be made. The lab director is responsible for the final interpretation of these policies.

- Arrive at your lab on time. Note that the lab rooms change from week to week. The room schedules are posted on the bulletin boards across from the elevators on the second, third and fourth floors of Webster Hall.
- Complete all labs and the lab exam. If you miss or expect to miss a lab due to sickness or another valid reason, arrange for a make up laboratory as described in the Requests for Make-Up Laboratories section of this syllabus.
- Make sure that all submitted work is your own. Academic dishonesty is not tolerated and is grounds for failing the course.
- Before each lab, read the lab manual and related course material, particularly if the material has not already been covered in lecture.
- Bring your lab manual, calculator, pen and pencil, a lab notebook with carbonless copies, and scratch paper to lab each week.
- Come prepared to perform mathematical calculations based on the level of math appropriate for the course. This includes algebra, geometry, and trigonometry. Physics 201 also requires one semester of calculus. Physics 202 requires two semesters of calculus.
- Do not bring food, tobacco, or beverages into a lab room.

Final lab grades

Your final lab grade is awarded on the basis of:

Lab Assignments	80%
Lab Exam	20%

Consult your lecture instructor for the weight given to the lab grade in the total course grade.

Lab assignments include (1) lab notes recording during lab, (2) complete or partial formal reports of laboratory work; and (3) tutorials and quizzes performed during lab. Although each lab partner in a group will report the same data, your data analysis, discussion of results, and conclusions must be your own. For more information regarding lab notes and reports, refer to the "Lab Notes and Reports" section immediately following the syllabus.

All laboratory exercises are important. No scores are dropped at the end of the semester. A penalty equal to one-twelfth of the lab assignment points (80/12) will be assessed for each missing exercise.

The lab exams are administered during Closed Week, in your regularly scheduled laboratory. The exam may include any experimental techniques, methods of data analysis, and/or concepts covered during the semester. You may refer to your graded lab work for the current semester and the lab manual during the exam. You may not refer to the textbooks or other references. Work on the exam is individual (no lab partners). Bring your calculator.

Questions regarding grades on lab assignments need to be discussed with your teaching assistant within two weeks of receiving the graded material (earlier at the end of the semester). Final lab grades will be posted on the bulletin board on the 3rd floor of Webster Hall during Finals Week. To affect the lab grade submitted to your instructor, changes must be made by Friday morning of Final Exam week. Errors that affect your physics course grade will be corrected after final grades are submitted to the Registrar, if necessary.

By Physics and Astronomy Department policy, students earning lab grades below 50 will receive an F grade for the course, irrespective of their performance in the lecture portion of the course. On the other hand, students who fail the course but achieve a laboratory grade of at least 80 may choose to "carry over" this score when the course is retaken. To take advantage of this option, you must notify the lab director no later than the first week of the semester that you are repeating the course. 100-level labs cannot substitute for 200-level labs.

Computation of final lab grade

At the end of the semester, the average student scores will inevitably differ from lab section to lab section. The lab director will adjust the assignment and exam scores to make the grading more equitable. Students who were graded more rigorously will be raised in grade, while those who were graded more leniently will be lowered in grade. The lab assignment and lab exam scores will be adjusted (normalized) using the following formula:

$$\text{Normalized Score} = \left[\frac{\text{(Student's Average – Section Average)}}{\text{(Section Standard Deviation)}} \times 7.5\right] + 85 \tag{1}$$

After normalization, the average assignment score (and the average exam score) for each section will be 85 and the section standard deviation will be 7.5. This mean and standard deviation correspond to the standard 90-80-70-60 grading scale, where 90–100 is an A, 80–89.99 is a B, 70–79.99 is a C, 60–69.99 is a D, and below 60 is an F (for the lab).

The final lab score is formed by adding 80% of the normalized assignment score to 20% of the normalized exam score. From this total, 6.667 points will be subtracted for each week of missing lab work. The resulting final lab score will be sent to your lecture instructor, who will incorporate it into your final course grade. If your lecture instructor uses a different grade scale, your lecture instructor will make an additional adjustment.

Assignment submission policies

Records of work completed during lab, including tutorials, must be turned in to your teaching assistant before you leave the room. Records of work completed after lab are normally due at the beginning of the next lab session. Work completed after lab (but before the next lab session) can be turned in using the homework cabinet in the hall on the third floor Webster Hall. All work must be submitted to your laboratory teaching assistant by Monday afternoon, the day before the lab exam, to be counted toward your final lab grade.

Turn in all lab assignments promptly. A penalty of ten points per working day is assessed for late work. It is important that you submit work on time even if it is not complete. You are free to complete your work in your lab notebook after the deadline, provided that the late entries are dated with the day the work is actually performed. You will not receive credit for this late work, but it may help you on the lab final. If you have reason to believe that an assignment that you submitted has been lost, report it immediately to your teaching assistant.

Requests for make up laboratories

Do not attend lab if you are ill with something contagious. When you are well enough to attend, contact your teaching assistant to arrange for a make up laboratory. Unless your illness lasts more than a few days, laboratories must be made up within one week of the missed lab.

If you expect to miss your regularly scheduled lab to attend a university-approved activity, you are also expected to make up the missed laboratory. University-approved activities include music and athletic events in which you perform. Make up labs are not available for study sessions, extra credit activities, or exams in other courses. Make up laboratories for scheduled absences must be requested the week before the scheduled absence. Make up space is limited, and may not be available if you request a make up lab later.

To schedule a make up laboratory, contact your teaching assistant. Your request should include your name, your course catalog number (Physics 101, 102, 201, 202, etc.), your regular lab section number, the name of the missing lab (e.g., Buoyancy), the reason why the lab is being missed, and your e-mail address. You must also indicate when you might be available to make up the lab. Make up labs are usually offered during other lab sections of the same course, but sometimes in another course.

Except for the last lab of the semester, make-up work must be submitted to the laboratory teaching assistants by Monday, July 24, to be considered for credit. The physics labs will close Friday, July 21, to prepare for lab exams. Make up labs for the last lab of the semester and the lab exam can sometimes be scheduled after your lab exam.

Student conduct

"Washington State University, a community dedicated to the advancement of knowledge, expects all students to behave in a manner consistent with its high standards of scholarship and conduct. Students are expected to uphold these standards both on and off campus and acknowledge the University's authority to take disciplinary action. The purpose of these standards and processes is to educate students and protect the welfare of the community."—Quoted from the Preamble to the Washington State University Standards of Conduct for Students (http://apps.leg.wa.gov/WAC/default.aspx?cite=504-26).

A partial list of prohibited conduct appears in Washington Administrative Code (WAC) Section 504-26 (http://apps.leg.wa.gov/wac/default.aspx?cite=504-26). Of special importance to the laboratories is the false reporting of data, experiment results, information, or procedures. The data and results in your lab notebook and reports must result from your own work in the current semester. Reporting data acquired by others (including your lab partner if you did not contribute) or in previous semesters is academically dishonest. Fabrication of results, information, or procedures, and sabotaging other students' work is also prohibited. Likewise, sharing information about the end-of-semester lab exam with students yet to take the exam is prohibited. Violations of this policy will affect your lab grade and may be reported to the Student Conduct Committee as instances of academic dishonesty.

Students are expected to avoid behavior that unnecessarily interferes with the learning of other students. We expect students to be on time to labs and lab exams and to mute their cell phones for the duration. Some physics concepts are subtle, and even the most intelligent students make mistakes. It is important that students be willing to ask questions if they don't understand what their lab partners say or do. To this end, we require that students and teaching assistants alike avoid behavior that discourages communication. This includes threats and insults. Students who repeatedly disrupt lab may be directed to leave the room and may receive a zero grade for that week's lab.

Disability accommodations

Reasonable accommodations are available for students with documented disabilities. If you have a disability and need accommodations to fully participate in the lecture, contact the Access Center (Phone: 335-3417, e-mail: access.center@wsu.edu, URL: http://accesscenter.wsu.edu). All accommodations must be approved through the Access Center. You must notify the lab director during the first week of laboratories concerning any approved accommodations. Late notification may cause the requested accommodations to be unavailable.

Safety resources

General information on campus safety is posted at http://safetyplan.wsu.edu—the Campus Safety Plan. Information on how to prepare for potential emergencies is posted on the Office of Emergency Management web site (http://oem.wsu.edu/). Safety alerts and weather warnings are posted promptly at the WSU Alerts site (http://alert.wsu.edu/). Urgent warnings that apply to the entire University community will also be broadcast using the Campus Outdoor Warning System (speakers mounted on Holland Library and other buildings) and the Crisis Communication System (e-mail, phone, cell phone). For this purpose, it is important to keep your emergency contact information up to date in MyWSU. To enter or update this information, click the "Update Now!" link in the "Pullman Emergency Information" box on your MyWSU home page (https://my.wsu.edu/).

Safety information that applies to the laboratories appears in the Lab Manual. Your teaching assistant will also present any safety information that applies to the current laboratory at the beginning of the laboratory. Students are expected to conduct themselves responsibly and take no unnecessary risks. Students who disobey the safety instructions will be directed to report to the lab director. All accidents and injuries must be reported promptly to your teaching assistant.

An Emergency Guide is posted by one door of each lab room. If faced with an emergency, follow the "Alert, Assess, Act," protocol: Remain ALERT (through direct observation or emergency notification), ASSESS your specific situation, and ACT to ensure your own safety and the safety of those around you. In case the fire alarm sounds, leave the building promptly in an orderly fashion. If you are not on a ground floor, use the stairs. Do not use the elevators. After exiting the building, gather across from the basketball court behind Waller Hall (down the hill, south of Webster Hall, see Fig. 1) with the other members of your lab. A representative of the Department of Physics and Astronomy will tell you when it is safe to re-enter the building. If this does not happen before the end of the lab period, you are free to leave for your next class. If the emergency involves an active shooter, your options are to RUN, HIDE, or FIGHT (https://www.youtube.com/watch?v=5VcSwejU2D0). Each lab room door can be locked from the inside in case of a lock down.

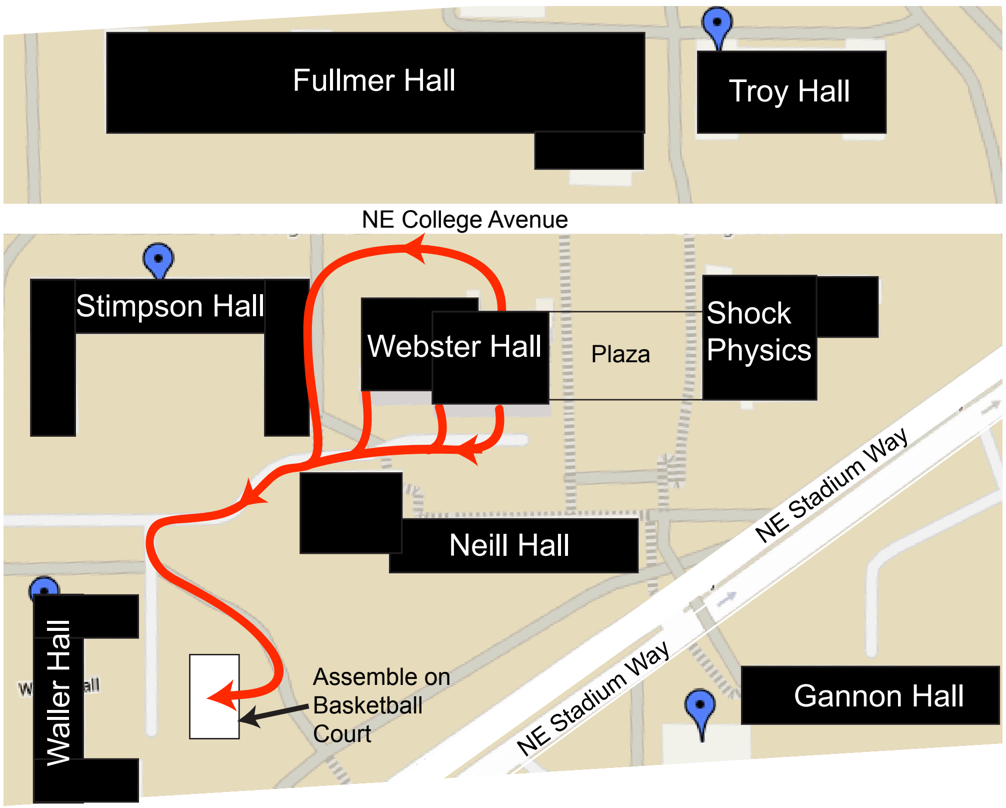


Figure 1. Physics and Astronomy assembly point. In case of a fire alarm, exit the building and gather at the basketball court behind Waller Hall. Use the stairs. Do not use the elevators in case of fire. A department representative will tell us when it is safe to re-enter the building.

Possible changes

The lab director reserves the right to correct errors in the syllabus and to modify lab schedules and room assignments.

The lab director has delegated some authority to modify assignments and due dates to your teaching assistant. This helps ensure that your are graded according the criteria stated during your lab meeting.

Lab Notes and Reports

Written communication of laboratory work

Records of laboratory work take at least two forms. For legal and reference purposes, the primary record of lab work is the lab notebook. The notebook includes notes you make before, during, and after performing an experiment. For grading purposes, we require that you use a commercial notebook with index pages at the front, and numbered, carbonless copy pages for notes. Many introductory chemistry laboratories use suitable notebooks. If your chemistry notebook is otherwise suitable and has blank pages left, you are free to use it for this course. At the end of each laboratory, you will submit the copy pages from your notebook to your teaching assistant. You will submit the copies for any work you do outside of class with the rest of the lab assignment. You will retain the original copies for your record and study. When you fill up one notebook, you are expected to obtain another.

For communication within a broader technical community, lab work is summarized in technical reports. These reports communicate your main results and omit many details recorded in your lab notebook. Because the preparation of proper lab reports require considerable time and effort, we will not require a complete report for each laboratory. However, to satisfy UCORE requirements, some formal writing is necessary. For many labs, we will ask that you submit a well written, partial report, where you focus on particular communication tasks.

These two forms of communication employ different standards that can be only partially implemented in an instructional lab. What we require is described below.

Lab notes—official record of attendance and work performed[1]

Although neatness is important, the content of your lab notes is the main criterion for grading. Lab notes must be sufficiently legible to make it easy for you and others to read and understand exactly what you did. Your notes must include all your raw data, and explain how it was analyzed (for instance, using sample equations). You will often type numerical data into Excel spreadsheets for analysis, but the original numbers must appear in your lab notebook as well. *Your notebook is the official record*—and a backup in case your computer crashes. At the end of the semester, you will take a lab exam in which parts of a few selected experiments are to be reproduced—usually with small changes. You are expected to refer to your lab notebook during the lab exam. (The lab exam

[1] A detailed introduction to the lab notebook is found in: Howard M. Kanare, *Writing the Lab Notebook*, (American Chemical Society, Washington, DC, 1985).

is a practical test of your lab notes.) The exam can be relatively easy if your notes are complete. Parts of the exam will be impossible if you omit important details.

With the exception of computer-generated graphs and tables printed during lab, lab notes must be handwritten in pen. Although lab notes are not formal documents, they are legal records. Any attempt to remove information from the record after the fact destroys this value and is considered scientific misconduct. *If you decide that any original data or notes are in error, put a single "X" through it, make short note in the margin explaining why it is in error, then record the new information in a new entry.* Both sets of data must be legible in your lab notes. Your grade will not lowered by including these marked errors. This practice conforms to standard scientific and engineering practice. You are free to work through any derivations that should appear in your lab notes on scratch paper before entering them in your lab notebook.

In case of a dispute over lab attendance or what you did in lab, pages torn from your lab notebook will not be accepted as evidence. Likewise, notes on regular notebook paper will not be accepted as evidence. A computer printout is evidence only if it is permanently attached (taped, not stapled) to an original page in your notebook or shows the signature of the supervising teaching assistant. Missing original pages are evidence for suspicious activity and carry a "presumption of guilt": we will assume you are guilty of something—the only question is what.

If you rewrite or type your notes, understand that your original notes are the official record, not the rewritten notes. Notes made after the fact are not valid records and will not be treated as such. The copy pages with your notes must submitted in order to receive a grade for laboratory work.

Each entry in your lab notebook should start with the current date and time in the left margin. If you work on your lab notes at home after lab, the entries made at home must also begin with the current date and time (the time of writing, not the time of the lab). Each entry must be recorded at the same time the work is performed. Entries must be sequential. Leaving one or more blank pages or part of a page in your notebook for later work is not acceptable. When you move on to a new page, draw a diagonal line through any large blank areas of the previous page. To work on an earlier lab after you have started work on a later lab, start your addition on first blank page in sequence. Mark the top of the new page, "Continued from page ..." and another note at the bottom of the old page, "Continued on page ...". Many lab notebooks provide spaces for these notes. You lab notebook should also have an index for this information.

Unlike lab reports, lab notes do not have formal sections. It is appropriate to write out questions you have about the lab and one or two sentences of introductory material in your notebook before coming to lab; these entries must be dated at the time or writing. Each step of your procedure must be recorded as you actually perform it. Do not copy procedures from the manual into your lab notes before coming to lab. (When pre-recorded procedures are absolutely necessary, draw a vertical line down the center of the notebook page, with your intended procedure on the left and your record of what you actually did on the right.) Likewise you should record your data as you take the data. There is no data section. To help you avoid missing important points, the lab manual includes some questions about each lab; these questions should be answered in your lab notes where the questions arise in the lab. If you print a graph or data table in lab, attach it to your other notes as close as possible to the handwritten notes that describe the data and how it was collected. Do not collect your computer printouts at the end. Submit your notes in chronological order.

Your lab notes must be sufficiently detailed that you or another student with your background can reproduce your work. The reader must be able to "trace" your work from the original data, through your analysis, to your conclusions. Your notes should leave no doubt about how the data were collected, what sensors and sensor settings were used (if any), and which equations were used to calculate the quantities you report. Define any symbols used in your equations and include appropriate units for numerical data. Sample calculations are often necessary.

Each graph printed during lab should fill a full sheet of paper to allow room for notes. To provide this room, computer-generated graphs should normally be printed in the "landscape" (rather than the "portrait") mode. Landscape mode will print the x-axis along the longer dimension of the paper and thus makes most graphs about 50% larger. In some cases it is useful to display computer-generated graphs, for example, showing position, velocity, and acceleration as functions of time, on the same page to facilitate comparison. These graphs should be printed in the mode that most completely fills the page. All graphs must have a *descriptive* title that indicates what is being graphed. ("Graph 1" or "Exercise 1" is not sufficient.) Labels and units are required for both the x- and y-axes. If you are asked to draw a "curve" through your data points, this should always be a best-fit curve (for example, a straight line if appropriate) that best represents your data. Best-fit lines can be drawn by eyeball and a ruler, or with the help of the computer. If you are asked to calculate the slope (or perform other analysis) of the graph by hand, show the results of this analysis directly on the graph, clearly identifying which points are being used to calculate the desired quantities. When a computer-generated best fit curve is displayed on a graph, the resulting equation (with parameters and uncertainties) should also be displayed on the graph. This allows the reader to evaluate the curve fit results without referring back to the text. Refer to the "Uncertainty/Graphical Analysis Supplement" near the back of your lab manual for more information about using graphs to find mathematical relationships between graphed quantities.

Keeping good records during lab takes time, and it is virtually impossible using formal English, with complete sentences and paragraphs. Record your actions and data in the most clear, efficient way possible. Use phrases instead of sentences. Annotated diagrams—simple sketches with the parts labeled and notes—can save time and be more clear. Descriptive titles for graphs and table columns also help. If an equation is used to describe the data in a graph, write the equation on the graph. Putting it elsewhere usually requires additional text.

Lab reports—formal communication with peers

Although lab notebooks are the primary records of lab work, they are poor communication devices. Experimental results are usually communicated in technical reports. Unlike lab notes, these reports omit most "historical" aspects of the work: false starts are omitted. While one often reports the manufacturer and model number for important pieces of equipment, operational details are usually omitted. (The operational details must be recorded in your lab notes.) While lab notes often include derivations, technical reports normally include only the result. As communication devices, we expect lab reports to conform to the standards of formal written English, with appropriate word choice, grammar, and structure.

Because writing formal lab reports is time consuming, an entire report will not be required for each lab. Rather, most labs will require short writing assignments that focus on one element of an entire report—perhaps an introduction or an experiment section. If the teaching assistant believes a submission is inadequate, the teaching assistant may require that it be rewritten and resubmitted for partial credit. As time permits, we will require complete, formal reports for one or two labs. The deadline for the submission of complete reports will be at least a week after the lab is performed. Your teaching assistant will inform you of the report requirements on a week by week basis.

Lab reports (partial or complete) must be typewritten or printed from a text editor, using the format specified in the "Formal Lab Report Instructions" supplement near the back of the lab manual. You will have the original copies of your lab notes to use in preparing your report. Carbon copies of all relevant lab notes must be submitted to your teaching assistant for credit. The statements and conclusions in your formal report must be supported by the data and analysis in your lab notes. Omissions and gaps in logic, when observed, will lower your grade.

Special requirements for lab assignments

Cover Page

A cover page is required for every submission. It must include:

- The title of the experiment
- Your name and student ID number
- The name of your lab partner
- The date that the lab was performed
- The name of your teaching assistant
- The course and lab section numbers (for example, Physics 101, Lab Section 5)

Nothing else should appear on this page. Lab reports that are submitted in the wrong slot or are otherwise misplaced take much longer to reach your teaching assistant if the information on the cover page is incorrect or incomplete. Work submitted during lab might not require a cover page.

Uncertainty analysis

Many experiments involve a quantitative comparison between values of the same quantity determined by two or more distinct methods. When you compare two values, you must address the question of whether or not they agree within the limits of the expected or measured uncertainties. The Uncertainty/ Graphical Analysis Supplement near the back of your lab manual defines important quantities, such as the standard deviation, and supplies details about determining uncertainties. As the semester progresses, you will need to make decisions by yourself on appropriate methods for calculating the uncertainties in your various measured and calculated quantities. Physics 102 and 202 students are expected to be aware of the uncertainty methods learned in Physics 101 and 201, respectively, and to use them appropriately.

Lab 1. Electrostatics

Goals

- To understand and verify the behavior of the two kinds of charge, denoted “positive” and “negative”, respectively.
- To understand the response of the electroscope when a charged rod is brought near, so that the electrical charges on the rod interact with charges already present in the electroscope.
- To visualize charge transfer between charged rods, the electroscope, and other objects, and to understand how the electroscope is used to compare the net charges on two objects.

Introduction

Electroscopes are used to detect the presence or absence of electric charge. They come in various forms, but a picture of a typical electroscope is shown in Figure 1.1. Inside the electroscope a metal needle pivots on a wire support shaped something like a paper clip. This structure inside the electroscope is connected to the outside by a metal rod passing through a plastic insulator. The metal disk on top simply allows charge to be detected more efficiently; otherwise its geometry is not too important. The term “electrostatics” refers to charges that are basically stationary, rather than continuously moving as in a wire carrying an electric current. An analogy may be made to water in a bathtub as opposed to a flowing stream of water.

Some important things to remember are:

- Electric charges come in two varieties that are designated positive and negative.
- Charges of the same variety repel one another while charges of the opposite variety attract one another.
- Charges exert greater forces on one another when closer together (Coulomb’s law).
- All materials are composed of positive and negative charges.
- In metal objects, a small fraction of the negative charge is relatively free to move from one place to another within the object. (This is why metals are called conductors.)
- Electric charges in insulators such as rubber and glass are essentially fixed in place.
- The positive charges in solid materials are in the atomic nuclei and are not free to move.

- Electric charges in static equilibrium have no net force acting on them.
- When rubbed with silk, a glass rod acquires a net positive charge on its surface by giving up electrons to the silk, which has a stronger affinity for electrons.
- The plastic (polyvinyl-chloride, or PVC) acquires a net negative surface charge when rubbed with wool by "stealing" electrons from the wool.

Caution: The glass rod is brittle. Return it to the tray when not in use. If placed on the table, the rod can roll off and break. Avoid handling the glass rod, the plastic tube, and the wool and silk fabrics any more than necessary. Their electrostatic properties are degraded by moisture and oil from your hands.

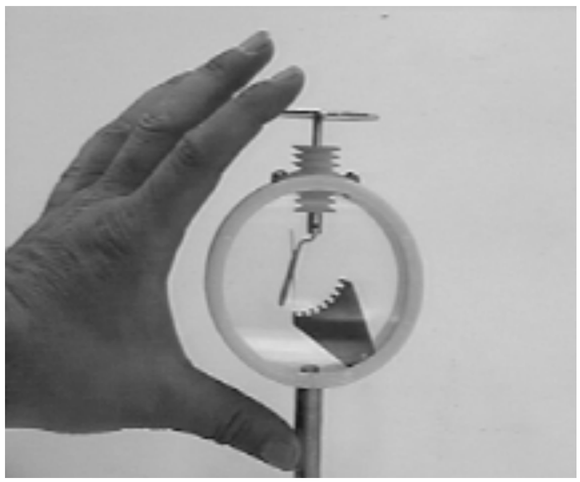


Figure 1.1. "Grounding" the electroscope.

Holding a charged rod close to the electroscope plate

Ground the electroscope as illustrated in Figure 1.1. This works because your body can absorb or give up small amounts of charge without suffering any ill effects. You could use a wire connected to the earth (or ground, hence the term "ground"), but your body is handier in this case. To charge the glass rod, hold the silk cloth by the edge so that it hangs below your hand and stroke the hanging silk with the glass rod. This procedure keeps moisture from your hand from damping the silk. Then position the part of the glass rod that touched the silk just above the circular disk on top of the electroscope without touching the disk with the rod.[1] What do you observe? As the rod is moved away from the disk, what happens? Hypothesize what is happening to the charges. If at any time you suspect that the needle is stuck, gently tap the case of the electroscope with your finger.

[1]The effectiveness of the charging procedure depends strongly on the ambient humidity and the cleanliness of the glass rod. On a humid day, it may take some time to properly charge the rod. Cleaning the glass with a glass cleaner helps considerably. On a dry day, the charging procedure can produce much more charge. As you move the rod toward the electroscope, stop when you see the needle move. Sparks between the rod and the electroscope will invalidate this part of the experiment. If the needle moves more than half the distance up the scale, you have probably produced a spark. Sparks transfer charge to the electroscope. The effect of transferred charge will be studied below.

The case is not connected to the top plate. Tapping the case will not affect the charge on the plate or needle.

Repeat the same sequence with the plastic tube after rubbing it with wool. The wool cloth is thicker than the silk, and is less susceptible to moisture. The best procedure is to put to wool in the palm of your hand and rub it against the rod. Take care to avoid sparks, as described in the footnote above. Again record your observations.

Now explain your hypothesis with the aid of some simple "cartoons"—a series of pictures with words of explanation; your TA will have some helpful suggestions for making simple drawings. Show what the electric charges on the electroscope are doing as the charged rods are brought close and then moved away. You will need a sequence of several cartoon pictures to show the locations of the charges on the electroscope for different positions of each rod. If you can't support your hypothesis by your observations and pictures, you may need to make another hypothesis.

Charging the electroscope by direct contact

Ground the electroscope again. This time touch the charged glass rod to the disk, and then move the rod away. What happens to the needle of the electroscope? Make a hypothesis about what happened when you touched the disk with the rod using some "cartoons" as visual aids. Without grounding the electroscope, test your hypothesis by bringing the charged glass rod near the disk at the top of the electroscope but without touching it. What happens to the electroscope needle? Explain whether this observation supports your hypothesis or not. If the observation doesn't support your hypothesis, redo the whole procedure and make sure that the observed behavior is repeatable—an important aspect of the scientific process. Record all your hypotheses, whether they turn out to be correct or incorrect. By using the scientific method we hope to reach the correct explanation in the end. If the behavior is repeatable, then make another hypothesis to explain your observations and test it again. To double check your understanding, bring the plastic tube close to (but not touching) the electroscope that was touched at the outset with the charged glass rod and observe what happens. Is your hypothesis consistent with these additional observations? Explain with the aid of another cartoon sequence.

Repeat the entire process outlined in the previous paragraph, but start this time by touching the initially uncharged electroscope with the charged plastic tube.

Can a net electric charge be left on the electroscope by touching it with the rods? How does the sign of the charge on the electroscope compare to the charge on the rod that touches it? Summarize your findings for this exercise.

Charging the electroscope by induction

Ground the electroscope again, as in Figure 1.1. With your finger still touching the edge of the disk and your thumb still touching the body of the electroscope, bring the charged glass rod up close to the the other side of the disk (away from your finger) without touching the disk. Now remove your finger from the disk first and then move the glass rod away. What do you observe on the electroscope? Make sure that it is repeatable. Make a hypothesis about what happened to the charge in the electroscope and record it. Then test your hypothesis using what you learned so far. Modify your hypothesis as necessary. Explain your reasoning with another cartoon sequence.

Now beginning with the charged plastic tube repeat the process described in the previous paragraph. Summarize your results for this section.

Effect of lit match on electroscope charge

Using your knowledge of the behavior of the electroscope and the charged rods, determine what variety of charge is released when a match is burning. Hold the burning match about 2 cm above the disk. (Hold the match at least 1 cm from the plate.) Try it with the electroscope initially uncharged, positively charged, and negatively charged. Explain in detail your procedure, results, reasoning, and conclusions. More cartoons are needed here.

Summary

Summarize your findings concisely. Provide a brief explanation of your most important observation in each experiment.

Before you leave the lab please:
- Straighten up your lab station.
- Report any problems or suggest improvements to your TA.

Lab 2. Electric Fields

Goals

- To understand how contour lines of equal voltage, which are easily measured, relate to the electric field produced by electrically charged objects.
- To learn how to identify regions of strong and weak electric fields from maps of electric field lines.
- To quantitatively estimate the magnitude and direction of an electric field using experimental voltage measurements.

Introduction

The concept of the electric field is useful in determining the force on a charged object due to the presence of other charges. The purpose of this laboratory is to quantitatively map, in two dimensions, a set of equipotential lines for two different charge distributions using a voltmeter. An equipotential line connects the set of points for which the potential difference or voltage has a constant value. The two-dimensional charge distributions will be established by applying a potential difference between a pair of conducting electrodes. The electrodes are attached to a board covered with conducting paper. From these equipotential lines the electric field can be determined. Electric field lines always cross equipotential lines at right angles as a consequence of the definition of electric potential. By convention, electric field lines start on positive charges and end on negative charges.

You will use a voltmeter to locate different points on the black conducting paper for which the voltage differences between the points in question and a reference point (say, at zero potential) are the same. These points are recorded on a white sheet of paper with the same grid pattern as the conducting paper. Then connect these points of equal voltage to form an equipotential line. From a set of equipotential lines you can create a map of the vector electric field following the rules stated in the previous paragraph. Since electric field lines start from and end on electrical charges, higher densities of field lines near the electrodes indicate regions of higher charge concentration. From a complete electric field map, the charge densities on the electrodes themselves can be deduced.

Electric field of a long plate parallel to a long rod

Equipment set-up

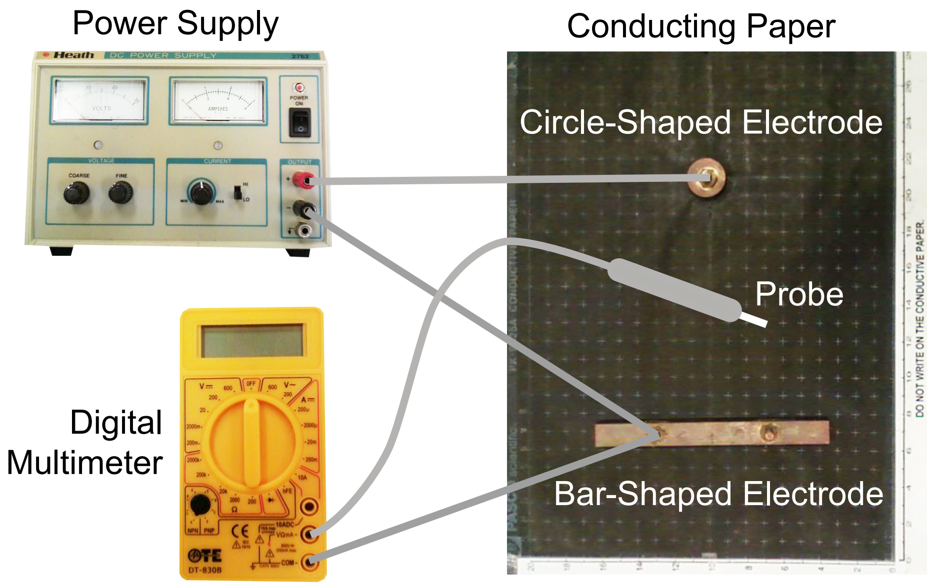


Figure 2.1. Electrical connections.

1. On the white paper grids provided, carefully draw the outlines of the brass electrodes at the same positions as they appear on the black conductive paper.

2. Connect the positive terminal (red jack) of the power supply to the circle-shaped electrode on the conductive paper, as shown in Figure 2.1. This will produce a net positive charge on the circular electrode. Connect the negative terminal (black jack) of the power supply to the bar-shaped electrode on the conductive paper. This will produce a net negative charge on the bar electrode. Use "alligator clips" to connect the wires from the power supply to the electrodes. This configuration simulates the electric field between a positively charged rod and a negatively charged plate.

3. Adjust the current knob on the power supply to the straight-up or 12 o'clock position. Then turn the power supply on and adjust the COARSE voltage control knob to set the voltage to about 5 V as read on the voltmeter on the front of the power supply.

4. Connect the common (COM) terminal of the digital multimeter (DMM) to the bar-shaped electrode. Connect the wire lead with the probe to the V-Ω (volt-ohm) terminal of the DMM, and set the range knob to 20 DCV.

5. Now fine tune the adjustment of the power supply by: (1) touching the probe to the circle-shaped electrode, and (2) turning the FINE voltage control knob on the power supply until the voltage reading on the DMM lies between 4.90 and 5.10 V, making sure that the reading is stable. Once set, this voltage should remain constant for the mapping of all the equipotential lines for a given electrode configuration. Check it from time to time as you make your map,

and adjust the voltage as necessary to maintain this voltage reading. Be sure to record the actual measured voltage.

6. Verify that you have a good electrical connection between the bar-shaped electrode and the power supply by touching the probe to the bar-shaped electrode. The voltage reading should be zero. If this is not the case, ask your TA for assistance.

7. Touch the probe to the conductive paper at a few random points. The voltage readings on the DMM should lie between zero and the value you measured on the circle-shaped electrode. If this is not the case, ask your TA for assistance.

Caution: Do not mark the conductive paper with pencils or pens, or poke holes in it with the pointed probe.

Data collection

Choose some convenient voltages between 0 and 5 V, say 0.50, 1.00, 1.50, etc.

1. Using the probe find a point on the conducting paper that gives a voltage of 0.50 ± 0.01 V. Mark this point on the white grid paper using a symbol of your choice (such as a small x). Now move the probe 1–2 cm away from the point you just located and search for another point on the conducting paper that gives a reading of 0.50 ± 0.01 V. Mark this point on the white grid paper using the same symbol. Continue this process until you reach the edge of the conducting paper or you run into points already located. Now connect these points with a smooth line (Don't just connect the dots with straight line segments!) and label this line "0.50 V". This is the first equipotential line for this electrode configuration.

2. Repeat the process outlined in (a) above for points with a voltage of 1.00 ± 0.01 V, using another symbol to mark these points on (such as a small o). Alternating the plot symbols will clearly distinguish the various lines of equal potential. Repeat this process for the other voltage values.

3. If you have any large blank regions on your map, choose an intermediate value of potential (one that falls between the voltages of previously drawn equipotential lines) and fill in the "blanks."

4. Each electrode is also an equipotential. Try it by touching the probe to the electrode at various points; you may have to rub the probe on the brass gently to make good electrical contact because of the layer of tarnish that forms on brass. Record the voltage of each electrode on your white grid paper.

Data analysis

First sketch in the electric field lines associated with the equipotential lines measured previously by following the "rules" for field lines as outlined in the Introduction. Since each conducting electrode is an equipotential surface, electric field lines that start or end on a conducting surface must be perpendicular to the surface where they touch it. A suggestion is to start at a point on the positive electrode and draw a smooth continuous line which crosses all equipotential lines at right angles. Extend each line until you either reach the edge of the paper or the negative electrode. Pick other points on the positive electrode and repeat this process.

From the definition of electric potential, the magnitude of the electric field, $|\mathbf{E}|$, is related approximately to the electric potential (or voltage), V, in the following way:

$$|\mathbf{E}| = \frac{\Delta V}{\Delta s} \tag{2.1}$$

where ΔV is the difference in voltage between two equipotential lines and Δs is the distance between the two equipotential lines measured along an electric field line. This approximation becomes exact in the limit as the distance between the two equipotential lines approaches zero. In our case we must be content with approximate values for the electric field. The electric field is perpendicular to nearby equipotentials, and points from high potential to low. Be sure to indicate the direction of each field line with arrows. Don't leave any large regions of your map devoid of field lines.

Pick 8–10 points on your electric field map and calculate the approximate values of the electric field using the above equation. Be sure to use adjacent equipotential lines in order to make the approximation better. When you do this, you are finding the average electric field between the two equipotentials, which will closely approximate the actual value of the electric field midway between the two equipotentials. Use a special plot symbol (a different color pen or pencil would be good) to indicate on your map the locations of the points at which you calculate the magnitudes of the electric field. Label the points P_1, P_2, etc. Show the calculations for each point in your report. Try to locate the places on your map where the electric field is largest and where it is smallest by this process.

Another electrode configuration

Replace your conducting board with another board with a different configuration of electrodes.

Choose the polarity of each electrode and connect the power supply appropriately. Some electrodes may be left neutral or unconnected. Your TA will have special instructions for some electrode configurations.

Repeat the process above to create and analyze another map.

Summary

Based on your electric field maps and calculations of the magnitude of the electric field, make some general observations about where the electric field tends to be largest and smallest. Is it possible to predict from the electric field lines alone where the field will be large or small? Explain your reasoning.

Before you leave the lab please:

Straighten up your lab station.

Report any problems or suggest improvements to your TA.

Lab 3. Gauss's Law Tutorial

Goals

- To understand and explain in words the physical meaning of Gauss's law.
- To employ symmetry arguments to determine the direction of the electric field for simple charge distributions.
- To use Gauss's law to calculate the the electric field produced by an appropriate charge distribution.

Introduction

In mathematical form, Gauss's Law can be written as:

$$\oint_S \mathbf{E} \cdot \mathbf{dA} = \frac{q_{Enclosed}}{\varepsilon_0} \tag{3.1}$$

Problem 1

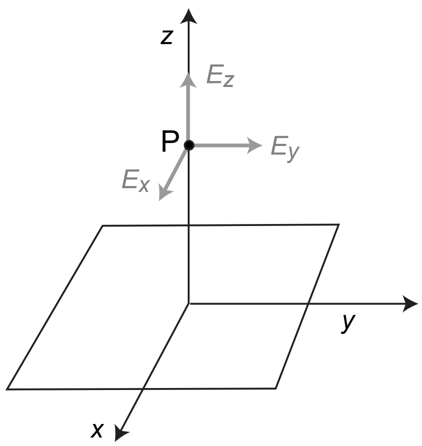
Describe a procedure for applying Gauss's Law of electromagnetism in your own words, without using equations. Define the following terms carefully:

Surface element:	$\mathbf{dA}$
Closed surface:	S
Enclosed charge:	$q_{Enclosed}$

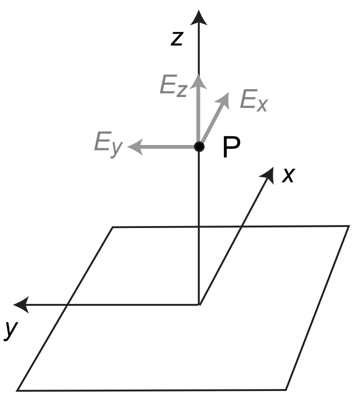
Describe the dot product in Equation 3.1 in terms of vector components. Describe the process of integration in terms of a sum over many small surface elements. It is a good idea to sketch out your solutions on scratch paper before entering them in your lab notebook.

Using symmetry to determine the direction of the electric field

Gauss's Law can be used to determine the magnitude of the electric field in several important geometries. Gauss's Law is always true, but it doesn't help you determine the magnitude of the electric field unless you know the direction of the field. In the special cases where Gauss's Law can be used to calculate the electric field, the direction of the field can be determined from the symmetry of the charge distribution. Figure 3.1 shows a uniformly charged, square plate in the x-y plane with its center at the origin. Rotating the charge distribution by 180° around the z-axis does not change the distribution of charge. Every charge element that is moved by rotation is replaced by another, identical charge element after rotation. Therefore, we say that the charge distribution is symmetric with respect to a 180° rotation about the z-axis.



(a) Original charge distribution.



(b) Charge distribution after 180° rotation.

Figure 3.1. A thin, uniformly charged square plate in the x-y plane, with its center at the origin of an x-y-z coordinate system before and after rotation by 180°. The gray vectors at Point P show the three components of **E** at Point P. The charge distributions before and after rotation are identical. Therefore we expect identical electric fields. Although the z-component of **E** is the same before and after rotation, the x- and y-components change sign. The only way these observations can be reconciled is if $E_x = E_y = 0$.

Symmetry arguments rely on two principles. First, changing the orientation of the source of an electric field (the charge) changes the orientation of its electric field in the same way. Second, identical charge distributions produce identical fields. Figure 3.1 shows that E_x and E_y at Point P change sign after a 180° rotation about the z-axis. They rotate along with the plate. But this rotation does not change the charge distribution, so it cannot change the electric field. The only way these two facts can both be true is if the x- and y-components of the electric field at Point P are zero. In contrast, E_z is not changed by this rotation. Therefore the z-component of the electric field does not have to be zero. The electric field at Point P must point in the $+z$ or $-z$ directions.

This procedure yields the same result for all points on the z-axis. The procedure fails for points with nonzero x- and y-components, unless the square is infinite in extent. We cannot use this procedure to find the direction of the electric field for points that are not on the z-axis, except for the special case of a plate that extends to infinity in the x- and y-directions.

Three useful groups of symmetry operations are described below.

Translations (straight-line displacements)

Charge distributions with translational symmetry in the x-direction are not changed when the object is moved in the x-direction. A infinite, charged plate perpendicular to the z-axis has translational symmetry in the x- and y-directions. Movements in the x- and y-directions do not change the charge distribution, because every patch of moved charged is replaced by an identical patch of charge from elsewhere on the plate. Since the charge distribution is not changed if you move the plane in the x- or y-directions, the electric field must not be changed by this motion. That is, the electric field vector at any Point P above a charged, infinite plane cannot depend on the x- or y-components of that point.

Rotations

Charge distributions with rotational symmetry are not changed by a rotation about some axis. The axis and the angle of rotation must be specified, although some shapes are symmetric about a special axis for all angles. The square in Figure 3.1 is symmetric under rotations of 90°, 180°, and 270° about the z-axis.

Reflections

The difference between a vector and its reflection is similar to the difference between a vector and its reflection in a mirror. Reflection through the x-y plane changes the sign of vector z-components without changing their magnitude. The x- and y-components are not changed. You will not need to use reflections in this exercise, but they can be useful.

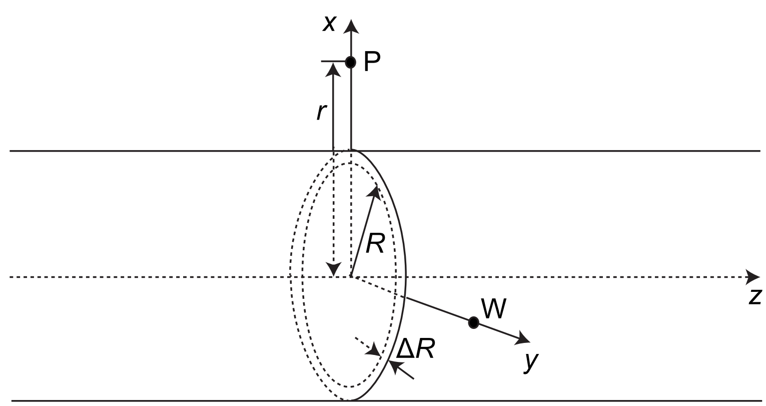


Figure 3.2. Sketch representing an infinitely long cylindrical shell (like a pipe) of radius R and thickness ΔR, centered on the z-axis. The capital R indicates the position of a point in the charge distribution. The lower case r indicates the position of a point at which the electric field is to be determined.

Problem 2

Figure 3.2 represents an infinitely long cylindrical shell of radius R and thickness ΔR, centered on the z-axis. Assume that it is made of a material with a uniform, positive volume charge density ρ.

Sketch "before" and "after" pictures of this charge distribution and the electric field components at the point of interest for each symmetry operation, as in the example above.

(a) Use symmetry arguments to show that the electric field outside the cylinder at Point P on the x-axis is directed in the $+x$-direction. (Rotate the cylinder 180° about the x-axis.)

(b) Use symmetry arguments to show that the electric field outside the cylinder at Point W on the y-axis is directed in the $+y$-direction.

(c) Use symmetry arguments to show that the electric field is directed along a cylinder radius—that is, pointing directly toward or away from the z-axis. Further, show that the magnitude of the electric field is constant on circles of radius $r = \sqrt{x^2 + y^2}$, the distance from the z-axis to Point P.

(d) Use additional symmetry arguments to show that the electric field outside the cylinder does not depend on the z-component of position.

Constructing a Gaussian surface

To perform the integral in Gauss's Law, one must be able to compute the dot product inside the integral:

$$\oint_S \mathbf{E} \cdot \mathbf{dA} = \frac{q_{Enclosed}}{\varepsilon_0} \quad \text{where} \quad \mathbf{E} \cdot \mathbf{dA} = E dA \cos\theta \tag{3.2}$$

and θ is the angle between $\mathbf{E}$ and $\mathbf{dA}$. The circle on the integral sign indicates that the surface must be closed. Like cubes or spheres, closed surfaces are composed of pieces whose orientations vary. Since the direction of $\mathbf{dA}$ is along the surface normal direction, it changes depending where you are on the closed surface. From Problem 2, we know the direction of $\mathbf{E}$ due to an infinitely long cylindrical shell. We get to choose the surface, and so have some control over the direction of $\mathbf{dA}$. The best we can hope for is to make $\mathbf{E} \cdot \mathbf{dA}$ constant for each part of the surface. This requires that θ be constant for each part of the surface. For closed surfaces, the only workable angles are 0°, 90°, 180° and 270°. Then $\cos\theta = 0$ or ± 1.

Problem 3

Sketch a Gaussian surface for the infinitely long cylindrical shell in Figure 3.2, making sure that the angle between $\mathbf{E}$ and $\mathbf{dA}$ is one of the approved angles for each side of the closed surface. Since the surface must be closed, your Gaussian surface cannot be infinitely long. Show the dimensions of the Gaussian surface on your sketch. Write the Gauss's Law integral as the sum of the integrals over each part of your surface, using the fact that $\cos\theta = 0$ or ± 1.

Calculating the flux through the Gaussian surface

Even though we have determined θ for each of the required flux integral(s), we still must be careful of the functional dependence of the remaining scalars E and dA. You can't calculate an integral unless you know the function. This is not a problem where $\cos\theta = 0$, as the integral for those parts must equal zero. But for surfaces where $\cos\theta = \pm 1$, the integral can be a problem. Practically, the only hope for a solution is if the magnitude E is constant. Fortunately, you showed in Problem 2 that for a fixed r, the magnitude of the electric field E is indeed constant. If (and only if) r is constant for the parts of your Gaussian surface where $\cos\theta = \pm 1$, you can perform the integral. After factoring out the constant factor of E, the remaining integral should have the form:

$$\int dA = A \tag{3.3}$$

where A is the area of a part of your Gaussian surface where $\cos\theta = +1$ or -1.

Problem 4

Calculate the total flux through your Gaussian surface S.

Calculating the charge enclosed by the Gaussian surface

Since S is a closed surface, with a definite inside and outside, it encloses a well defined volume. If all the charges in the system are simple point charges, one can simply identify which point charges are inside the volume and sum their values. Another simple case is when the charge density in the volume is uniform, or constant. Then the enclosed charge is given by the product of the volume V inside S and the charge density ρ; that is, $q_{Enclosed} = \rho V$. Care must be taken to include only the charge inside S. If part of a charge distribution is not inside S (that is, some parts poke through the surface), only the part inside S contributes to $q_{Enclosed}$.

If the charge density is a function of R only, it can still have rotational symmetry. (In this case, the shape is not changed by any rotation about the axis of symmetry.) Then the enclosed charge may be found by integration. To minimize confusion, we will use the variable R to refer to the radial coordinate of a position in the charge distribution (the cylinder). We will use the variable r to refer to the radius of our Gaussian surface.

In your calculus class, you used the method of cylindrical shells to determine the volume of shapes with rotational symmetry. You can use the same method to determine the total charge in such an object by introducing a factor of ρ, the volume charge density. In the shell method, the volume of a thin cylindrical shell is given by[1]

$$\Delta V = 2\pi hRdr \tag{3.4}$$

where h is the (constant) length of the shell. To see that this must be true, consider a solid cylinder with radius R_{Cyl} whose charge density might be a function of R. In this case one can approximate

[1] See, for example, the current calculus text: William Briggs, Lyle Cochran, and Bernard Gillet, *Calculus—Early Transcendentals* (Pearson, Boston, 2015), Section 6.4, on the shell method. Note that Briggs treats *much* more complicated shapes than we do. We have a simple cylinder with a fixed height. We will eventually treat the case of variable charge density. The charge density, however, will be constant inside every thin shell of radius R. That is all we need.

the volume of the cylinder as the sum of the volumes of a series of N thin cylindrical shells of radii $R_1, R_2, R_3 ... R_N$. If we take the thickness of each shell to be $\Delta R = R_{Cyl}/N$, we can construct a series of shells with radii $R_J = J\Delta R$, where $(J = 1, 2, 3...N)$. As N goes to infinity the sum of the shell volumes V_J becomes an integral, and the integral yields the exact value of V_{Cyl}. (This is the definition of an integral according to Riemann.) The progression from thin shells to integrals can be written:

$$V_{Cyl} = \lim_{x \to \infty} \sum_{J=0}^{N} \Delta V_J = \lim_{x \to \infty} \sum_{J=0}^{N} 2\pi h R_J \Delta R = \int_0^{R_{Cyl}} 2\pi h R dR \tag{3.5}$$

To find the charge enclosed in the *entire* cylinder, q_{Cyl}, one need only add a factor of ρ to the integral.

$$q_{Cyl} = \int_0^{R_{Cyl}} 2\pi \rho h R dR \tag{3.6}$$

You can find q_{Cyl} for almost any charge distribution $\rho(R)$ that depends only on R. If the radius of your Gaussian surface is greater than the radius of the cylinder, $q_{Enclosed} = q_{Cyl}$; the upper limit of integration is then R_{Cyl}, as in Equation 3.6. If the radius of your Gaussian surface is less than the radius of the cylinder, you must include only the charge inside the Gaussian surface. To get $q_{Enclosed}$, you reduce the upper limit of the integral from R_{Cyl} to r, the radius of your Gaussian surface.

Problem 5

Compute the total charge inside in a cylinder of length h and radius R_{Cyl} when $\rho(R) = \alpha R$. Use the result to compute the electric field produced by the cylinder at points outside the cylinder $(r > R_{Cyl})$. Note that since $r > R_{Cyl}$, the Gaussian surface (with radius r) encloses all the charge in the cylinder. State the direction of the electric field inside and outside the cylinder when $\alpha > 0$, that is, when the cylinder carries positive charge.

Problem 6

Find the charge enclosed by a Gaussian surface as a function of its radius, r, when $\rho(R) = \alpha R$, for the case of $r < R_{Cyl}$. Since $r < R_{Cyl}$, a Gaussian surface with radius r encloses only part of the cylinder's charge. Use the result with the rest of Gauss's Law to compute the magnitude of the electric field inside the cylinder as a function r for $r < R_{Cyl}$.

Before you leave the lab please:

Turn your work in to your teaching assistant.

Work done at home will receive no more than 50% credit.

Lab 4. Ohm's Law

Goals

- To understand Ohm's law, used to describe the behavior of electrical conduction in many materials and circuits.
- To calculate the electrical power dissipated as heat.
- To understand and use a rheostat, or variable resistor, in an electrical circuit.
- To learn how to connect electrical components so that the current can flow around the circuit, and to learn how to use, connect, and read ammeters (current reading instruments) and voltmeters (voltage reading instruments).
- To measure and observe the behavior of the voltage across and the corresponding current through a simple resistor (electronic component) and a tungsten-filament light bulb.

Introduction

One of the most basic electrical circuits is a resistor connected to a voltage source, such as a battery or power supply. A quantity called the resistance, R, of a component is defined as the ratio of the potential difference, ΔV, across the component to the current, I, flowing through the component, or

$$R = \frac{\Delta V}{I} \tag{4.1}$$

When ΔV is expressed in volts and I is expressed in amperes (amps), then R is in the SI units of ohms (Ω). The power, P (in the SI unit of watts), dissipated by that component in the form of heat is given by

$$P = I(\Delta V) = I^2 R = \frac{(\Delta V)^2}{R} \tag{4.2}$$

The resistance of some materials is constant over a wide range of voltages and currents. When a material behaves in this way, it is called "ohmic." Electrical components made from ohmic materials are called resistors.

By measuring the current flowing through a component as a function of the voltage across the component, one can determine whether the ratio $\Delta V/I$ is a constant or not. If it is constant, then the component is ohmic and the constant resistance in ohms can be determined. If the voltage to current ratio is not constant, the device is not ohmic and does not obey Ohm's law. A **voltmeter** is used to measure voltage and an **ammeter** is used to measure current. Ideal voltmeters and ammeters will not affect the currents or voltages in the circuit as the measurements are being made. Real meters only approximate this ideal.

An ammeter measures the electrical current that flows through it. To measure the current flowing through a particular device in a circuit, the ammeter must be connected in such a way that the same current flows through the ammeter as through the device. The ammeter is simply a flow meter for the electrical current, so the wire at one end of the device must be disconnected and the ammeter inserted. The disconnected wire end is now connected to one terminal of the ammeter and a new wire is connected between the second terminal of the ammeter and the device to restore the flow of current through the circuit. This type of connection is called a "series" connection. The ammeter in Figure 4.1 is represented by a box marked with the letter "A".

Current versus voltage for a 100 Ω (nominal) resistor

In this exercise the voltage across and the current through a known resistor are measured as the current through the circuit is varied. The power supply voltage is kept constant, but the current flowing in the circuit is controlled with a variable resistor, also called a rheostat. (See Figure 4.1.)

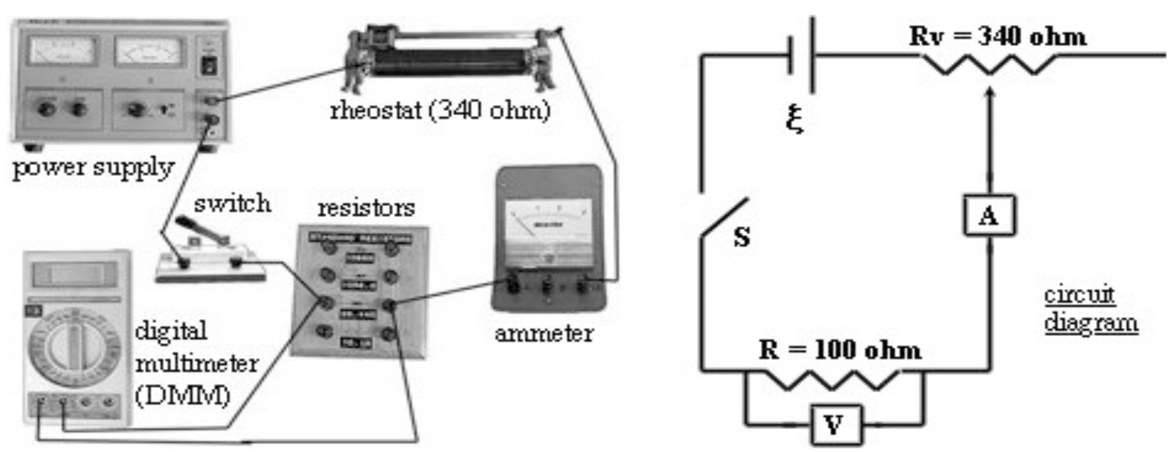


Figure 4.1. Circuit connections.

The rheostat has three terminals. Two terminals are on the ends of the device and are fixed, and the third is connected to a sliding contact that can be moved from one end of the device to the other. The resistance between the end terminals has a fixed value, but the resistance between the one of the end terminals and the sliding contact can be varied from zero to the fixed value of whole device.

Preliminary calculations

Assuming that the power supply voltage is fixed at 5.0 V, calculate the following quantities to two significant digits:

1. The current through the nominally 100 Ω resistor when the resistance of the rheostat is a maximum (340 or 360 Ω—your rheostat is marked with the value to use here), and when the resistance of the rheostat is zero.

2. The maximum power dissipated as heat by the 100 Ω resistor. The rated maximum power for this resistor is 0.50 W. If the power you calculate exceeds 0.50 W, please ask your TA for help before proceeding!

Equipment set-up

Caution: If current flows backwards through the ammeter, the ammeter tries to respond by registering a negative current. Since the meter needle can show only positive values, this can damage the meter. The ammeter can also be damaged if the magnitude of the current is much larger than the current rating of the chosen scale. To check that the ammeter is connected with the correct polarity and to a safe current scale, quickly tap the knife switch (See Figure 4.1) without closing it completely. If the meter tries to deflect in the negative direction, exchange the connections of the two wires connected to the ammeter. If the meter tries to deflect off-scale in the correct direction, use a current scale with a higher current rating. If the meter passes these two tests, close the knife switch completely and proceed to make measurements.

1. Turn the current knob on the power supply to the straight-up or 12 o'clock position, and set the power supply voltage to 5.0 V.

2. Build the circuit shown in Figure 4.1, leaving the switch open, that is, not making electrical contact. Be sure to use an ammeter scale with a current rating large enough to measure the maximum current you calculated above. By convention ammeters read positive when electrical current flows into the positive terminal (red) of the meter and then flows out of the negative terminal (black) of the meter.

3. Set the rheostat for maximum resistance by moving the slide so that the current must travel through the entire coil.

4. Tap the knife switch to make sure that the ammeter connections are correct. If all is well, then close the switch. Both the ammeter and voltmeter should read non-zero values. If the measured current is below the current rating of a more sensitive scale, open the knife switch, move the connection to the more sensitive scale, and tap the knife switch closed to test the new scale. Use the most sensitive current scale that can handle the current safely (reading stays on-scale).

Data collection

1. Make at least ten different measurements of the voltage and corresponding current by adjusting the rheostat between its minimum and maximum resistance. To obtain data points at low currents, you can lower the voltage supplied to the circuit by the power supply to some value less than 5 V. Ask your TA for help as necessary.

2. How does the current measured by the ammeter change if the ammeter is connected between the power supply and the rheostat instead of between the rheostat and the resistor? What if it is connected between the power supply and the switch? Verify your answers experimentally.

Data analysis

1. Draw a graph of the voltage across the nominal 100 Ω resistor as a function of the corresponding current flowing through it.

2. Is the graph linear? Draw a best fit smooth line through your data points, and from your graph find an equation for ΔV as a function of I in SI units.

3. Does the resistor exhibit ohmic behavior? Explain your reasoning. If so, what is the "real" value of the resistance? How does your value compare to the nominal 100 Ω value indicated by the "color code" painted on it?

Current versus voltage for an incandescent light bulb

Equipment set up

Caution: Be sure to leave the switch open while you construct the new circuit. Before closing the switch, have your TA check your circuit.

1. Build a circuit analogous to the one in Figure 1, but use the 22 Ω rheostat instead of the 340 or 360 Ω one used above and replace the 100 Ω resistor with the small light bulb.

2. Use the highest current scale on the ammeter to begin with. You can always change to a more sensitive scale if the measured current is low enough.

3. Make sure that the power supply is still set to 5 volts.

Data collection

1. Make at least ten different measurements of the voltage and corresponding current by adjusting the rheostat between its minimum and maximum resistance.

2. Does current flow through the light bulb even when the bulb is not glowing? Be sure to take data over the full range of possible values, whether the bulb glows or not.

Data analysis

1. Make a graph of the voltage difference between the light bulb terminals as a function of current. What is the current flowing through the light bulb if the voltage across it is zero? Be sure to plot this point on your graph!
2. Is the light bulb ohmic? Explain your reasoning. If so, what is its resistance? If not, what are the minimum and maximum values of its resistance?
3. What is the maximum power dissipated by the light bulb? (This power is dissipated primarily in the form of heat, but some also appears in the form of visible light.) What is the power dissipated by the bulb when it first begins to glow?

Summary

Compare and contrast the electrical behavior of the resistor and the light bulb. Consult a textbook and try learn why the light bulb exhibits a more complicated behavior than the resistor. Explain this in your notes.

Before you leave the lab please:

Turn off the power to all the equipment.
Disassemble the circuit and place the small components in the plastic tray.
Straighten up your lab station.
Report any problems or suggest improvements to your TA.

Lab 5. Series and Parallel Resistors

Goals

- To understand the fundamental difference between resistors connected in series and in parallel.
- To calculate the voltages and currents in simple circuits involving only resistors using the rules for "adding" series and parallel resistors.
- To learn to connect components correctly according to a circuit diagram and then to make valid current and voltage measurements with ammeters and voltmeters.
- To compare the predicted and measured currents and voltages for three circuits.

Introduction

Circuits are often composed of multiple resistors connected in various ways. Two general configurations that recur again and again are the so-called "series" and "parallel" combinations. Many resistor networks can be broken down into these simple units. For the sake of the following discussion, assume that the terminals of each resistor are labeled Terminal 1 at one end and Terminal 2 at the other end.

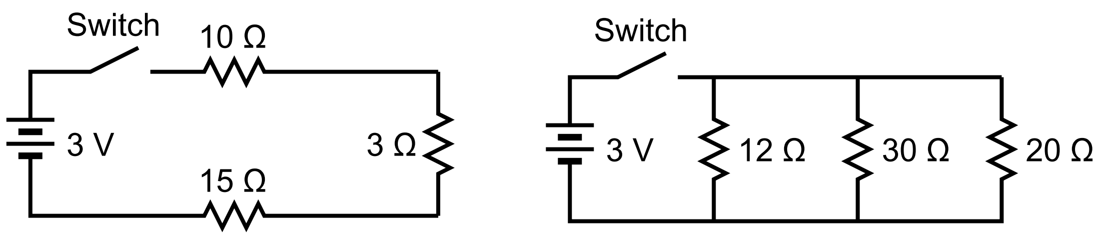
A "series" connection is when Terminal 2 of one resistor is connected to Terminal 1 of the next resistor and so on. This is like adding lengths of garden hose to reach the far corner of the yard. A battery or power supply is connected between Terminal 1 of the first resistor in the chain and Terminal 2 of the last resistor in the chain. Just like the water hose, where water flows into one end of the hose at the same rate as water flows out of the other end, the same electrical current (charge flow) flows through each of the resistors connected in series. It is important to note that in series connections, no other electrical connections can be made anywhere along the chain to add more current or take some away. If extra connections are present, even though the resistors may appear to be in a chain, our assumptions are invalid and the circuit is no longer a simple series combination. It is straightforward to show that resistances connected in series can be summed together to get the total resistance of the whole chain. In other words

$$R_{total} = R_1 + R_2 + R_3 + R_4 + \ldots \tag{5.1}$$

A "parallel" connection is when all of the Terminal 1's of several resistors are connected together. Likewise, all of the Terminal 2's are connected together. A battery or power supply is then connected between the combined Terminal 1 and the combined Terminal 2. In this case the applied voltage ("pressure" if you will) across each resistor is the same. Using this observation it again is straightforward to show that the total resistance of such a parallel combination is

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} + \ldots \tag{5.2}$$

Simple series and simple parallel resistor configurations



(a) Circuit 1. (b) Circuit 2.

Figure 5.1. Diagrams of (a) series and (b) parallel circuits for study.

Analyze Circuits 1 and 2

Answer the following questions for both Circuits 1 and 2. Be sure to explain your reasoning and show your calculations in your notes! You can summarize your numerical results in the provided table.

1. Which circuit contains the series combination and which the parallel combination?
2. What is the value of current through each resistor?
3. What is the voltage across each resistor?
4. What is the total current flowing through the power supply into the entire circuit?
5. What is the power dissipated (as heat) in each resistor? If any value exceeds 2 W, talk with your TA before proceeding to the next step.

Construct and study Circuits 1 and 2

Caution: Set the power supply to 3 V *before* connecting it to your circuit!

1. Measure the current through each resistor, showing on a circuit diagram exactly how and where the ammeter is connected in the circuit for each of the measurements.
2. Measure the voltage across each resistor, showing on a circuit diagram exactly how and where the voltmeter is connected in the circuit for each of the measurements.

3. Measure the total current flowing through the circuit, showing on a circuit diagram exactly how and where the ammeter is connected in the circuit.
4. Measure the total voltage across the whole circuit, showing on a circuit diagram exactly how and where the voltmeter is connected in the circuit.

Compare measured and predicted potential differences and currents

Compare your calculated and measured values using table at the end of the lab. (Remove this table from the manual and turn it in with your lab notes.) Percent differences are a good way to compare. Note whether the measured values are larger or smaller than the calculated ones. This is a good way to determine whether the differences are due to a systematic error or to some random process. If all the calculated values are larger than the measured ones, this suggests a systematic error, perhaps due to an non-ideal measuring device. If some values are a little high and others are a little low, the cause of variation is more likely to be random, such as variations in reading the meters.

Use these results to address the following questions. Explain your reasoning and justify your conclusions based on your data.

1. How are the currents through each resistor related to the total current flowing through the power supply in a series circuit? Look for a general rule that will apply to all series circuits.
2. How are the voltages across each resistor related to the total voltage across the power supply in a series circuit? Look for a general rule that will apply to all series circuits.
3. How are the currents through each resistor related to the total current flowing through the power supply in a parallel circuit? Again, look for a general rule that will apply to all parallel circuits.

Combined series and parallel configuration of resistors

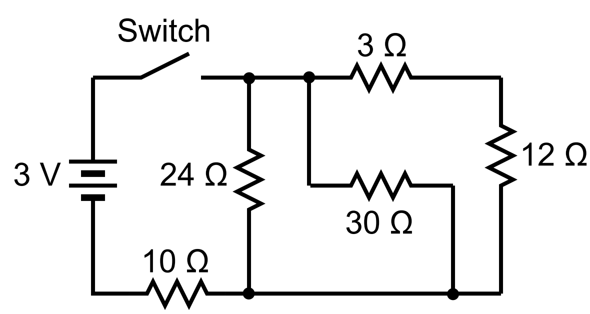


Figure 5.2. Diagram of Circuit 3.

Calculate, then measure the potential differences across and currents through each component in Circuit 3.

Before you leave the lab please:

Turn off the power to all the equipment.

Please put all leads and small components in the plastic tray provided.

Report any problems or suggest improvements to your TA.

Resistor Color Code

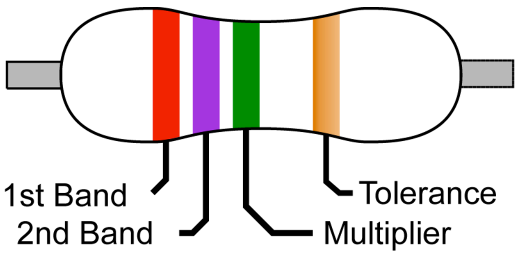


Figure 5.3. Resistor with labeled bands. To read the bands in order, orient the resistor so that the tolerance band (which is all by itself) is on the right. If the first band is red (2), the second violet (7), and the third green (10^5), the resistance is 27×10^5 ohms or 2.7 MΩ. If the tolerance band is gold, the actual resistance of a new resistor may differ from the indicated value by ±5%. Exceeding the current rating of a resistor can destroy it or change its resistance permanently. Image courtesy of Wikipedia (public domain).

Color Blank	Band 1 First Digit	Band 2 Second Digit	Band 3 Third Digit	Band 4 Tolerance
Black	0	0	$10^0 = 1$	
Brown	1	1	10^1	
Red	2	2	10^2	
Orange	3	3	10^3	
Yellow	4	4	10^4	
Green	5	5	10^5	
Blue	6	6	10^6	
Violet	7	7	10^7	
Gray	8	8	10^8	
White	9	9	10^9	
Gold			10^{-1}	±5%
Silver			10^{-2}	±10%
No Color				±20%

Series and Parallel Resistors Data Sheet

Circuit 1 — Series Resistors

		Calculated	Measured	%Difference	Power (W)
$R_{total} =$ ___ Ω	ΔV_{total} (V)				
	I_{total} (A)				
$R_1 = 10\,\Omega$	ΔV_1 (V)				
	I_1 (A)				
$R_2 = 3\,\Omega$	ΔV_2 (V)				
	I_2 (A)				
$R_3 = 15\,\Omega$	ΔV_3 (V)				
	I_3 (A)				

Circuit 2 — Parallel Resistors

		Calculated	Measured	%Difference	Power (W)
$R_{total} =$ ___ Ω	ΔV_{total} (V)				
	I_{total} (A)				
$R_1 = 12\,\Omega$	ΔV_1 (V)				
	I_1 (A)				
$R_2 = 30\,\Omega$	ΔV_2 (V)				
	I_2 (A)				
$R_3 = 20\,\Omega$	ΔV_3 (V)				
	I_3 (A)				

Circuit 3 — Combined Series and Parallel Resistors

		Calculated	Measured	%Difference	Power (W)
$R_{total} =$ ___ Ω	ΔV_{total} (V)				
	I_{total} (A)				
$R_1 = 10\,\Omega$	ΔV_1 (V)				
	I_1 (A)				
$R_2 = 24\,\Omega$	ΔV_2 (V)				
	I_2 (A)				
$R_3 = 30\,\Omega$	ΔV_3 (V)				
	I_3 (A)				
$R_4 = 3\,\Omega$	ΔV_4 (V)				
	I_4 (A)				
$R_5 = 12\,\Omega$	ΔV_5 (V)				
	I_5 (A)				

Lab 6. RC Circuits

Goals

- To appreciate the capacitor as a charge storage device.
- To measure the voltage across a capacitor as it discharges through a resistor, and to compare the result with the expected, theoretical behavior.
- To use a semilogarithmic graph to verify that experimental data is well described by an exponential decay and to determine the decay parameters.
- To determine the apparent internal resistance of a digital multimeter.

Introduction

A diagram of a simple resistor-capacitor (RC) circuit appears in Figure 6.1. A power supply is used to charge the capacitor. During this process, charge is transferred from one side of the capacitor to the other. A digital multimeter set on a voltage scale behaves in a circuit like a large (in ohms) resistor. When the power supply is disconnected from the capacitor, charge "leaks" from one side of the capacitor, through this resistor, back to the other side of the capacitor, until no voltage appears across the terminals of the capacitor.

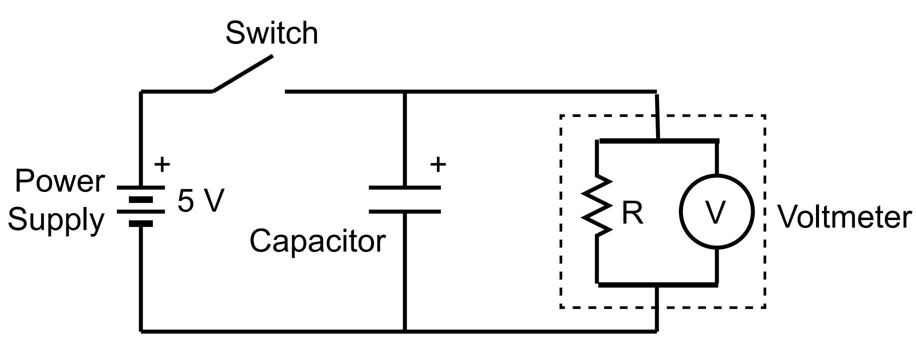


Figure 6.1. Diagram of RC circuit and power supply.

The power supply in Figure 6.1 is represented by a battery. Note that the positive output of the power supply is connected to the plate of the capacitor marked with a plus sign. The capacitors used in this experiment make use of a thin layer of dielectric material that forms by a chemical (electrolytic) reaction when the appropriate voltage is placed across the capacitor. These layers can be quite thin and uniform. Electrolytic capacitors can be made inexpensively and are widely used in power supplies. As you may remember from chemistry, the sign of the voltage is critical in electrolytic reactions. Make sure that the plus end of the capacitor is connected to the plus output of the power supply in your circuit.

The voltmeter in Figure 6.1 is enclosed by a dashed line. The voltage sensing circuit is represented by a circle with a "V" inside. All voltmeters have resistance, and this resistance is represented by the resistor symbol inside the box. Our goal is to measure the value of this resistance, R.

Theory

We plan to monitor the voltage across the capacitor as a function of time after the switch is opened. The functional form of this dependence can be derived by circuit analysis using Kirchhoff's loop law. A simplified diagram of the circuit after the switch is opened is shown in Figure 6.2. For the purposes of analysis, we indicate the positive direction of current by an arrow. This choice defines the sign of positive charge, Q on the capacitor. (Q is positive when the arrow points toward the plate with positive charge.) It also defines the positive direction of ΔV. (ΔV is positive when the arrow points in the direction of *increasing* potential.)

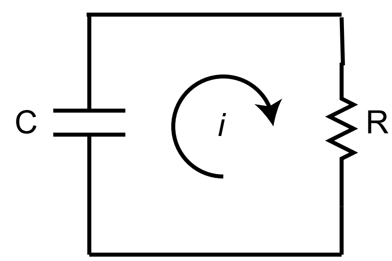


Figure 6.2. Diagram of RC circuit and power supply.

Because our circuit contains no source of emf, the only potential differences in the circuit appear across the capacitor and across the resistor. By Kirchhoff's loop rule, the total potential change as you go all the way around the loop (ΔV_{loop}) must be zero. Let the potential difference across the capacitor be ΔV_C and the potential difference across the resistor be ΔV_R. Then

$$\Delta V_{loop} = \Delta V_C + \Delta V_R = 0 \quad . \tag{6.1}$$

In the presence of a positive charge Q on the capacitor, ΔV_C must be negative, as the potential drops as one moves from a positively charged plate to a negatively charged plate. The capacitance, C, of a capacitor is defined so that the magnitude of ΔV_C is Q/C. Therefore $\Delta V_C = -Q/C$. Likewise, potential drops as charge passes through a resistor in the direction of positive current, I. The magnitude of this drop is given by Ohm's law, so that $\Delta V_R = -IR$. Substituting these relations into Kirchhoff's loop rule yields

$$\Delta V_C + \Delta V_R = -\frac{Q}{C} - IR = 0 \quad \text{or} \quad I = -\frac{Q}{RC} \tag{6.2}$$

When the switch in Figure 6.1 is closed, a positive charge $Q = \Delta V \times C$, where $\Delta V = 5$ V, is on the top plate of the capacitor. According to our choice of positive direction, both Q and ΔV_C are initially negative. While negative charges and potential differences may appear to be inconvenient, they make no difference as far as the math is concerned. It is safe to choose the direction of positive current arbitrarily and work from there. With a positive negative charge on the top plate of the capacitor, a positive current I will flow through the resistor. In this case, a positive current will *decrease* the magnitude of Q, but since Q is initially negative, the corresponding dQ/dt is positive. Therefore $I = dQ/dt$. One of the handy features of Kirchhoff's loop rule is that I always equals dQ/dt if you set it up correctly. This is not always true for other approaches to circuit analysis. This relation allows us to reduce Kirchhoff's loop rule to a simple equation with one derivative.

$$\frac{Q}{C} = -R\frac{dQ}{dt} \quad \text{or} \quad \frac{1}{Q}\frac{dQ}{dt} = -\frac{1}{RC} \tag{6.3}$$

Equation 6.3 is a simple differential equation. The expression on the right hand side of Equation 6.3 is easily integrated, but its solution depends on the initial charge across the capacitor. If we start with an initial charge Q_0 on the capacitor, the charge as a function of time, $Q(t)$ is given by

$$Q(t) = Q_0 \exp\left(-\frac{t}{RC}\right) \tag{6.4}$$

Since the voltage across the capacitor is directly proportional to the charge stored on it at any instant of time, the voltage difference ΔV_C can be written as

$$\Delta V_C(t) = \frac{Q_0}{C}\exp\left(-\frac{t}{RC}\right) = \Delta V_0 \exp\left(-\frac{t}{RC}\right) \tag{6.5}$$

where ΔV_0 is the initial voltage across the capacitor. The voltmeter measures this voltage directly. When t/RC equals one (that is, when $t = RC$), the voltage has decayed to $1/e$ of its original value. The quantity RC is called the time constant of the decay process. When R and C are expressed in the SI units of ohms and farads, respectively, the RC time constant has units of seconds.

Before proceeding, verify that the expression for $Q(t)$ given above is really a solution to the differential equation preceding it. Include this verification in your lab notes.

Experiment

Set up the circuit shown in the diagram using the Fluke voltmeter. Have your TA check it before continuing. With the power supply set to 5.0 V, close the switch and charge the capacitor. When the switch is opened, the voltage begins to decrease. Try it! Now read the initial voltage, then open the switch and read the voltmeter at 10-second intervals until the voltage is less than 10% of its original value. Repeat this process two more times, making sure that the initial voltage is the same for all three trials.

Analysis

To determine the resistance of the voltmeter, make a table in Excel listing the observed voltages and times for your three data sets. (Enter the time data only once.) Add an additional column in the spreadsheet to average the three voltage readings corresponding to each time. Then calculate the standard deviation of the mean of these three values for each time. (Refer to the Uncertainty/Graphical Analysis Supplement at the back of the lab manual for additional details.) The standard deviation of the mean gives an estimate of the uncertainty in the individual voltage measurements.

Plot the average voltage values as a function of time with error bars. Your error bars should look like the one in Figure 6.3. The circle in Figure 2 marks the calculated average value for one data point, y_{avg}. The top and bottom bars mark the maximum and minimum values (y_{max} and y_{min}) on either end of the range of y-values within one standard deviation (σ) of y_{avg}. Thus the top bar is located at $y_{max} = y_{avg} + \sigma$, while the bottom bar is located at $y_{min} = y_{avg} - \sigma$. Get help from your TA if you aren't sure how to plot error bars in Excel.

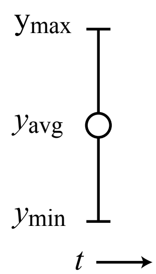


Figure 6.3. Diagram of a data point with upper and lower error bars.

Clearly this is not a linear graph. To determine the value of RC, one could perform an exponential curve fit using Excel. However, Excel's curve fit function does not provide the uncertainty estimate we need. Excel's Regression function will provide an uncertainty, but it requires a linear function. Taking the natural logarithm of both sides of Equation 6.5 will produce the linear equation we need.

$$\ln \Delta V_C(t) = ln(\Delta V_0) - \frac{t}{RC} \tag{6.6}$$

A plot of $\ln[\Delta V_C(t)]$ vs t should produce a line of slope $-t/RC$ and that equals $\ln(\Delta V_0)$ at time $t = 0$. The intercept is not very useful this case, except to confirm our knowledge of ΔV_0. The slope, however, gives us $1/RC$. Assuming that the value of C marked on the capacitor is reasonably accurate, we can calculate R, the internal resistance of the meter. In practice, the uncertainty in the marked value of C is ±20%. With this and the uncertainty (standard error) in the slope given by Excel's Regression feature, we can estimate the uncertainty in R.

A graph with the logarithm of one quantity on one axis versus a non-logarithmic quantity on the other axis is called a semilog graph. (The logarithm appears on only one of the two axes.) Plot a semilog graph of your data. Again include the error bars with each plotted point. Does your graph support the hypothesis that the relationship between the voltage and time is an exponential function? Using the value of C marked on your capacitor, compute the value of the R of the voltmeter and compare it to the value from the manufacturer's specification.

Internal resistance of an inexpensive voltmeter

Repeat your measurements of $\Delta V_C(t)$ versus time using the relatively inexpensive (smaller, red or black) digital voltmeter at your lab station. Repeat the analysis above to determine its internal resistance. How does it compare with the internal resistance of the relatively expensive Fluke digital voltmeter?

The internal resistance of a voltmeter is one measure of its quality. To measure the potential difference across a component with a high resistance, the internal resistance of your voltmeter should be much higher than the resistance of the component. A voltmeter with a high internal resistance can be used in applications where the measurement error of a meter with a low internal resistance would be unacceptably high.

Summary

Begin by "filling in the blanks" of the argument for a simple exponential function being a straight line when plotted semi-logarithmically. Then state your findings clearly, succinctly, and completely.

Before you leave the lab please:

Turn off the power to all the equipment, including the battery-powered digital voltmeters.
Please put all leads and small components in the plastic tray provided.
Report any problems or suggest improvements to your TA.

Lab 7. Magnetic Fields

Goals

- To visualize the magnetic fields produced by several different configurations of simple bar magnets using iron filings.
- To use small magnetic compasses to trace out the magnetic field lines of a single bar magnet on a large sheet of paper.
- To calculate the magnetic flux passing through the bar magnet by determining the locations of the points where the magnetic fields of the Earth and the bar magnet sum to zero.

Introduction

A magnetic field exerts forces on a compass needle such that the needle tends to align itself with the direction of the field. If the magnetic field is strong enough and additional non-magnetic forces (gravity, etc.) are negligible, then the compass needle points for all practical purposes in the direction of the field. In this lab the magnetic fields surrounding bar magnets are mapped out using a compass and iron filings.

The end of your compass needle that points toward the magnetic pole of the Earth in the northern hemisphere (when it is far away from other magnets and magnetic materials) is by definition a N (north-seeking) pole. Therefore Earth's magnetic pole in northern Canada is actually an S pole, since the N pole of the compass points to it and unlike poles attract. The N pole of the compass needle points toward the S pole of your magnet. The magnetic poles of all magnets can thus be labeled by means of a compass and the definition of an N pole.

Mapping magnetic fields with iron filings

In the presence of a magnetic field, iron filings act like many small compass needles. By spreading them out on the paper above the magnet a "picture" of the magnetic field is produced. At your lab station you have a piece of particle board with some grooves in it to hold the bar magnets.

Do not pick up iron filings with the magnet. The filings are difficult to remove from the magnet. Place the jar on a clean piece of paper and open the lid. Filings often will spill out from under the lid. Gently lift the paper with filings off of the magnet. Let the paper sag to make a funnel of sorts, and then pour the filings into the jar. Then replace the jar cover.

Sketch field lines for isolated bar magnet

Draw a full scale outline of the bar magnet on fresh piece of paper and label the N and S poles. Place the bar magnet in the middle groove of the particle board and cover it with a second piece of white paper. Sprinkle iron filings around on the surface of this second sheet of paper. Gently tapping the board will often make the pattern of field lines more clear. Now on the first sheet of paper, with the outline of the bar magnet already drawn, make a careful free hand sketch of the magnetic field lines shown by the iron filings. **On your sketch include the direction of the field lines by means of arrows. By convention the field lines outside the magnet itself go from the N pole to the S pole.** Each member of your lab group is expected to draw their own sketch.

Sketch field lines for more complex configurations

Now repeat this process for the following configurations of bar magnets. In each case sketch the magnetic field lines and indicate the direction of the field lines everywhere on your sketch.

1. Place two bar magnets end-to-end in the same groove along the middle of the particle board with their N poles several centimeters apart.
2. Place two bar magnets side by side in parallel grooves with either like poles near or unlike poles near each other.
3. Pick another configuration of your choice.

Analyze your drawings

1. Describe the general characteristics of the fields that you observe.
2. On your sketches label the regions where the magnetic field is especially strong and where it is especially weak for each configuration. Are there any points where the field is essentially zero? Identify these locations clearly as well. Include the reasoning you use to identify these regions of strong and weak fields.
3. Can you find any places where the magnetic field lines cross? If there were a point in space where two field lines crossed, what would the direction of the field be at that point? If magnetic fields from two difference sources are present at some point in space—for instance, the magnetic fields of Earth and the bar magnet—will some iron filings feel forces from one field and other filings feel forces from the other field, or will all filings feel forces from both fields simultaneously? Discuss/explain.

Mapping a magnetic field with a compass

Equipment set up

1. Tape a large sheet of paper to the hardboard sheet (area about 1 m^2) located at your lab station. Orient a bar magnet at the center of the sheet as directed by your TA.

2. Carefully outline the bar magnet and mark the orientation of its magnetic poles on the sheet of paper.

Map the field

1. You can start your map anywhere in principle, but let's start with a point about 10 cm from the center of the magnet. Place the compass on your paper. Use a non-magnetic pencil (Check this carefully!) to put dots on the paper at the tip and tail of the arrow of the compass.
2. Now move the compass (approximately one diameter) so that the tail of the arrow is at the point where the tip was previously. Put a dot at the location of the tip of the arrow. Repeat this procedure until you move off the edge of the paper or run into the magnet itself.
3. To complete the field line in the other direction go back to the initial position, but this time move the compass so that the tip of the arrow is where the tail was previously. This time put a dot at the location of the tail of the arrow and repeat.
4. Connect all the dots with a smooth curve. This now constitutes one magnetic field line. Before proceeding put arrows on the line to indicate which way the magnetic field is pointing.
5. Choose a new starting point and repeat the procedure until you have filled your paper with field lines. Check with your TA to make sure that you have sufficiently mapped the field.

Analyze your map

1. Are there any regions on the map that the field lines seem to avoid? What is the magnetic field at these points? Explain your reasoning. How many such points are there on your map?
2. Look at the magnetic field maps drawn by the other lab groups in your lab section. Each map has been made with the bar magnet in a different orientation. Sketch simple halfpage diagrams of these other map configurations to include with your lab notes. Do these other maps have any features in common with your map? How do they differ from your map? Explain.

Calculating the magnetic flux of the magnet

When a magnet is immersed in the Earth's magnetic field, the resulting field is the vector sum of the magnet's field and Earth's field. In regions where the magnet's field is larger than Earth's field, a compass aligns itself more with the magnet's field. In regions where Earth's field dominates, a compass aligns more with Earth's field.

You should be able to see this effect on your magnetic field map. As you move away from the bar magnet and its field gets weaker, Earth's field, which is essentially constant everywhere on your map, begins to dominate. Use your knowledge of the magnetic field due to a bar magnet alone to predict the direction of the field due to only the bar magnet at the "special" point(s) that field lines have avoided. Note the direction of Earth's magnetic field at this same "special" point. This result suggests that the sum of the fields from the bar magnet and the Earth cancel at this point, summing to zero net field. Look at the other map configurations to determine whether this seems to be a general result.

Magnetic field lines exit the N pole of the magnet, circle around, enter the S pole of the magnet, and return through the magnet to the N pole. Since magnetic charges have never been observed, we can safely assume that every field line observed outside the magnet passes through the magnet itself. A useful measure of the strength of a magnet is the magnitude of the magnetic flux, Φ_{BAR}, passing through the magnet. This flux equals the product of the average magnetic field inside the magnet and its cross sectional area.

The pattern of magnetic field lines *outside* a magnet looks much like the pattern of electric fields lines from an electric dipole. That is, the vector sum of a radially outward field from a N pole and a radially inward field into a S pole will circle around from the N pole to the S pole outside the magnet, as observed. The critical difference between magnetic and electric dipoles is that the magnetic field lines complete the circuit through the magnet, running from the S pole to the N pole. In addition, the N and S poles are not right at the ends of the physical magnet. A sketch of the relation between a "fat" physical magnet and the ideal, thin magnet used to model it is shown in Figure 7.1.[1] Although the magnetic field is only approximated by the dipole field, the approximation is quite good at positions far from the magnet.

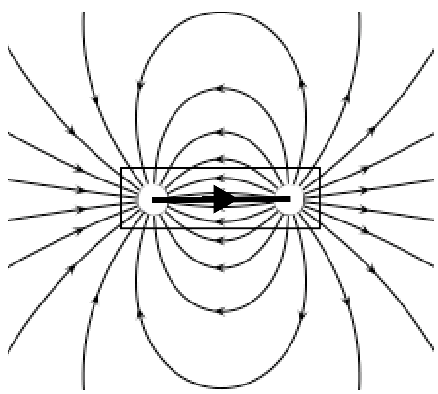


Figure 7.1. Sketch of the magnetic field due to an ideal, infinitely thin permanent magnet. The thick dark line represents the ideal magnet, while the dotted line outlines the corresponding physical magnet.

We will use this approximation to determine the magnetic flux of the bar magnet. That is, we will treat the magnetic field *outside* the magnet, $\mathbf{B_{BAR}}$, as the sum of a vector directed away from the N pole, $\mathbf{B_N}$, and another vector directed into the S pole, $\mathbf{B_S}$.

$$\mathbf{B_{BAR}} = \mathbf{B_N} + \mathbf{B_S} \quad , \tag{7.1}$$

where $\mathbf{B_N}$ and $\mathbf{B_S}$ vary with distance like electric fields (Coulomb's law). In the case of magnets, however, the source of these fields are the magnetic fluxes leaving the N pole and entering the S pole. At distances far from the poles, the equation for the magnetic field due to one pole can be

[1]The image of the field lines, without the magnets, was supplied by the Wikimedia Commons.

obtained from Coulomb's law by replacing q/ε_0 with Φ_{BAR}. (The total electric flux from a positive point charge is q/ε_0 by Gauss's Law.)

$$\begin{aligned} \mathbf{B_N} &= \frac{\Phi_{BAR}}{4\pi r_N^2} \qquad \{\text{Pointing radially away from the north pole}\}, \text{ and} \\ \mathbf{B_S} &= \frac{\Phi_{BAR}}{4\pi r_S^2} \qquad \{\text{Pointing radially away from the south pole}\}. \end{aligned} \tag{7.2}$$

Figure 7.2 shows a typical null point and the vectors $\mathbf{B_N}$ and $\mathbf{B_S}$ showing the contribution of the magnet's N and S poles to the magnetic field at the null point. In the equations and the diagram, r_N is the distance from the N pole of the magnet to the null point and r_S is the distance from the S pole of the magnet to the null point. Since the magnetic field is a vector quantity we must be careful to add the fields associated with the N and S poles as vectors.

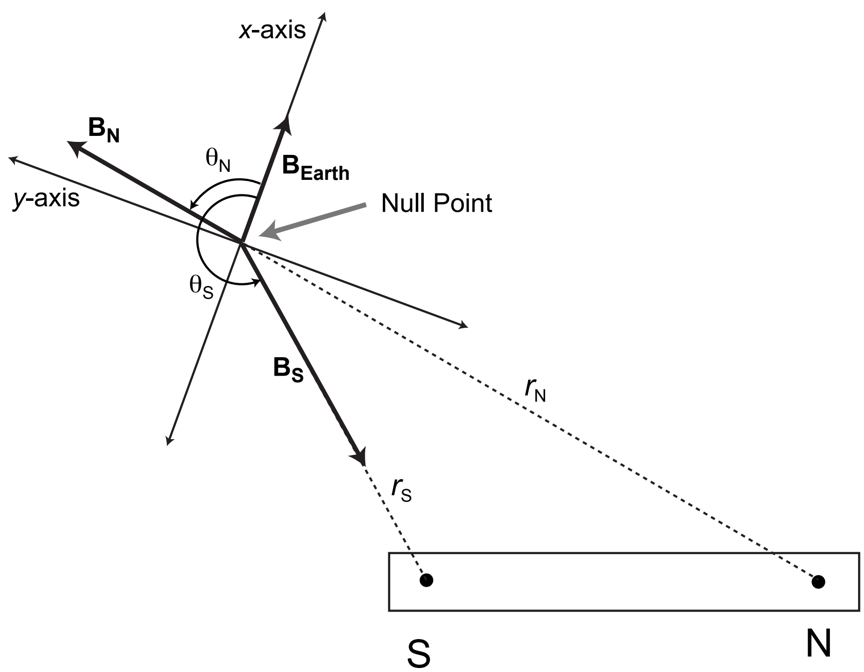


Figure 7.2. Diagram of a coordinate system with its origin at a null point and with its $+x$-axis pointing in the direction of the Earth's magnetic field vector, $\mathbf{B_{Earth}}$. Also shown are $\mathbf{B_N}$ and $\mathbf{B_S}$, vectors that mathematically represent the contribution of the N and S poles of the magnet to the magnetic field at the null point. The distances from null point to the N and S poles of the magnet are labeled r_N and r_S, respectively.

On the magnetic field map you made with a compass, choose one of the special "null" points where the magnetic fields of Earth and the bar magnet cancel one another. Earth's magnetic field actually points downward at an angle of about 70° relative to the surface of Earth at the latitude of Pullman, but the magnetic field map you have drawn lies only in a horizontal plane. Further, our compasses are constrained to rotate only about a vertical axis, so they respond only to the horizontal (parallel to Earth's surface) component of Earth's magnetic field. In other words the magnetic field of the bar magnet cancels only the horizontal component of Earth's field at a null point. That is,

$$\mathbf{B_N} + \mathbf{B_S} + \mathbf{B_{Earth}} = \mathbf{0} \qquad \{\text{horizontal component only}\} \tag{7.3}$$

at a null point. The magnitude of the horizontal component of Earth's field is 1.9×10^{-5} T here at Pullman. Show the direction of Earth's field on your map at your null point. Now you know the horizontal component of $\mathbf{B_{Earth}}$ (both direction and magnitude) at the null point. Define a coordinate system with its origin at the null point and with the positive x-axis in the direction of Earth's magnetic field at the null point, as shown in Figure 7.2. Draw this coordinate system directly on your field map. This choice of coordinate system simplifies the equations so that we only need to look at the x-components of $\mathbf{B_N}$ and $\mathbf{B_S}$. Then you can draw radial lines from the N and S poles of the bar magnet to the null point. The lengths of these lines give you r_N and r_S. After measuring the angles θ_N and θ_S (shown in Figure 7.2), you can calculate the x-components of the magnetic fields associated with $\mathbf{B_N}$ and $\mathbf{B_S}$ in terms of Φ_{BAR}. Since Φ_{BAR} is the only remaining unknown, you can complete the solution.

Before you leave the lab please:

- Return the bar magnet(s) to the TA Table.
- Put your rulers, compasses, and iron filings in the basket at your workstation.
- Straighten up your lab station.
- Report any problems or suggest improvements to your TA.

Lab 8. Current Balance

Goals

- To explore and verify the right-hand rule governing the force on a current-carrying wire immersed in a magnetic field.
- To determine how the force on a current-carrying wire depends on its length, the strength of the magnetic field, and the magnitude of the current flowing in the wire, and to display the relationships graphically.

Introduction

Electric charges can experience a force when they move through a region of nonzero magnetic field. Stationary charges experience no force. Since currents are just electric charges in motion, current carrying wires can also experience forces when immersed in magnetic fields. The magnitude of the force F on a straight wire of length L carrying a current I in the presence of a uniform magnetic field of strength B is given by

$$F = ILB\sin\theta \tag{8.1}$$

where θ is the angle between the direction of positive current flow and the magnetic field. The direction of the resulting force is determined by applying the "right-hand rule" as shown in your textbook. In this experiment the angle θ between the wire and the magnetic field is always 90° so that $\sin\theta = 1$.

The purpose of this experiment is to measure the force on a current carrying wire in the presence of a magnetic field and to determine how this force depends on magnetic field strength, current, and wire length. You should also be able to apply the right hand rule to predict the direction of the force on a current carrying wire in a magnetic field.

Caution: The load limit for these electronic balances is 200 grams. Use appropriate care to make sure that this limit is not exceeded.

Force versus wire length

Equipment set up

1. Using all six of the small magnets, place the magnets and magnet holder on the electronic balance and tare the balance. The "red ends" of the small magnets are N poles. The "white ends" are S poles. It is a good idea to check the direction of the magnetic field just above the gap with a compass. Make certain that all the magnets are oriented with the same polarity so that the magnetic field is maximized.
2. Plug circuit sf37 into the ends of the shiny metal bars of the current balance apparatus mounted on the stand. (sf37 is the manufacturer's designation and has no other purpose than to identify it.)
3. With the power supply off, connect the red and black jacks on the front of the power supply to the current balance apparatus using the holes provided on the tops of the metal bars of the apparatus.
4. Before turning on the power supply, adjust the "Coarse" voltage knob and the "Current" knob to their full counter-clockwise positions. Adjust the "Fine" voltage knob to the middle of its range, with the white mark pointing vertically upward. Set the current switch to the "Hi" position. In this position, the ammeter on the front of the power supply reads on the 0–3 A scale.

Analysis of forces on wire and balance

1. Draw a free-body diagram of the magnets and magnet holder in equilibrium on the balance with no current flowing through the circuit.
2. Draw another free-body diagram of the magnets and magnet holder in equilibrium when current is present in the wire that is between the poles of the magnet. You must apply the right-hand rule in conjunction with the magnetic force equation given earlier to determine the direction of the magnetic force on the wire. Make sure that your diagram and explanation are very clear here. Remember that, by convention, the magnetic field outside the magnet itself points from the N pole to the S pole. Also recall that current flows out of the red (+) terminal of the power supply and into the black (–) terminal.
3. On the basis of your free-body diagrams predict whether the electronic balance will read a positive value or a negative value.

Force measurements

1. Position the bottom of the U-shaped "wire" on sf37 so that it is centered between the poles of the magnet sitting on the electronic balance. Align sf37 carefully so that it is not touching the magnet holder anywhere. You may need to tare the balance again at this point before turning on the power supply.

2. Turn on the power supply and adjust the current knob clockwise until the ammeter reads 2 A. Check this from time to time during the rest of this exercise since the current sometimes can drift small amounts as the power supply warms up.

3. Compare and comment on the sign of the reading on the balance. If you didn't get it right the first time, go back and rethink it. Explain in your report how you went wrong and give a corrected explanation.

4. Record the balance readings for sf37, sf38, sf41, and sf42 keeping the current set at 2 A.

5. For sf42 only, reverse the direction of the current by switching the connections to the black and red terminals on the power supply. What happens to the reading given by the electronic balance? What did you expect to happen? Explain.

Data analysis

Convert all the balance readings from mass units to forces in newtons. For each of the circuits, sf37, sf38, sf41, and sf42, measure the effective length of the wire that was immersed in the magnetic field and produced a net force on the magnet. Plot the force on the magnet as a function of the length of the wire immersed in the magnetic field. If appropriate, fit a straight line to the data and calculate the magnetic field in tesla (T) for all six magnets. Refer back to the force law described above for help here.

Force versus strength of magnetic field

Equipment set up

1. Plug circuit sf41 into the ends of the current balance apparatus.

2. The manufacturer assures us that the magnetic field between the poles of the magnet is directly proportional to the number of small magnets used. You have already made a measurement with sf41 and six small magnets. Now remove one of the small magnets, leaving five. Center the five magnets relative to the magnet poles.

3. Align the wire of sf41 relative to the magnet poles as done previously.

Force measurements

1. Set the power supply current to 2 A.

2. Record the balance reading when current is passed through the wire. Be sure to tare the electronic balance appropriately.

3. Remove one magnet at a time and repeat the measurement. You should have six data points counting your measurement with sf41 during your study of force versus wire length.

Data analysis

Make a graph of the magnetic force as a function of the number of magnets. Based on your graph what can you say about the relationship between the force and the value of the magnetic field? If it is linear, find the slope of the graph and calculate the magnetic field of all six magnets again. Remember that the field of all six magnets is simply six times greater than the field of a single magnet.

Force versus current

Equipment set up

1. Replace all the magnets, making sure that all the red poles and white poles are aligned correctly.
2. Plug sf42 into the ends of the current balance apparatus.
3. Set the current from the power supply at 3 A.

Force measurements

1. Record the balance reading when current is passed through the wire between the poles of the magnet.
2. Lower the current to 2.5 A and repeat the measurement.
3. Continue reducing the current in 0.5 A increments until you reach 0.5 A. Record the balance reading in each case.

Data Analysis

Plot the magnetic force on sf42 as a function of the current. What can you say about the relationship between force and current? From this analysis you should be able to calculate the magnetic field with all the small magnets present. This calculated magnetic field should agree with the magnetic field value calculated from your measurements of force versus wire length and force versus magnetic field strength. Does it? Compare, discuss, and explain.

Conclusion

The fundamental magnetic force law for current carrying wires in magnetic fields given in the Introduction makes certain predictions about the dependence of the force on the current, wire length, and the magnetic field. Are your findings in harmony with the force law as formulated? Be very specific here and speak to the results of each set of measurements. If not in harmony, explain specifically in what way your results differ.

It is important to remember that the force law as formulated actually was induced from experiments like those you have done today. Thus the law as stated just characterizes how nature behaves; it doesn't prescribe beforehand how nature must behave. Nature behaves however she wishes, and we can only hope to characterize that behavior in simple ways from time to time. Of course, we often express these characterizations in mathematical terms, the shorthand of science.

Before you leave the lab please:

- Turn off the power to all the equipment.
- Disconnect the power supply.
- Make sure that all six of the small magnets are accounted for.
- Straighten up your lab station.
- Report any problems or suggest improvements to your TA.

Lab 9. Electromagnetic Induction

Goals

- To understand what it means to have magnetic flux through a loop or coil in a circuit.
- To understand and apply Lenz's law and the right hand rule for magnetic fields produced by currents to correctly predict the direction of currents produced by changing magnetic fields.
- To explain the steps in the induction process precisely through words and pictures for several different cases.

Introduction

Magnetic flux can be thought of as the number of magnetic field lines passing through a given area. According to Faraday's Law a change of the magnetic flux through an area bounded by closed circuit induces a voltage that drives the flow of current around the circuit. This is simply the induction process. Lenz's Law is an abbreviated, text version of Faraday's Law that gives the direction of the emf (potential change) as one moves around the circuit loop:

The polarity of the induced emf (or voltage) is such that it tends to produce a current that will create a magnetic flux to oppose the change in magnetic flux which is causing the emf.

In this experiment you are supplied with a coil of wire, a bar magnet, and a sensitive ammeter—also called a galvanometer. Remember that the ammeter reads a positive value of current when the current enters the positive (+) input terminal and leaves through the negative (–) or common terminal.

Move the bar magnet in to, out of, or through the coil of wire. Using the galvanometer, you can demonstrate that an electrical current flows when you do this.

Remember that, by convention, the magnetic field lines external to a bar magnet go from the N pole to the S pole. Since magnetic field lines are continuous, that is, they do not start or end anywhere, the field lines inside the bar magnet must necessarily go from the S pole to the N pole. All the field lines outside the magnet must be squeezed together as they pass through inside, going the opposite direction. If this is confusing, draw a simple diagram of a bar magnet, and add field lines to your drawing both inside and outside the magnet, indicating the directions of the fields with arrows.

Just a reminder that electric and magnetic fields differ significantly in this regard. Electric fields do begin and end somewhere, namely on electric charges. At this point scientists have yet to discover a single magnetic “charge” existing by itself, with magnetic field lines emanating from it radially analogous to the electric field of a point electric charge.

Be sure to check the pole designation of your bar magnet with a compass using the Earth’s magnetic field as a reference before beginning this experiment. Bar magnets can be remagnetized in strange ways by bringing them close to another magnet, so this check is important. It is not hard to do!

Prediction

Imagine pushing the bar magnet N-pole first into the right-hand end of the wire coil. Predict which way the galvanometer needle will deflect based on your knowledge of the magnetic fields of bar magnets, the magnetic fields due to currents in wires, the configuration of the wire windings of the coil, the right-hand rule, and the connection of the ammeter. Illustrate your method of prediction with a series of simple, annotated cartoons: pictures with words of explanation. Your TA will have some important suggestions for making simple, accurate drawings, particularly of the coil itself. Your cartoons must clearly show:

- The position of the ammeter and coil in your circuit. Clearly label the positive terminal of the ammeter.
- How the direction of the current (clockwise or counterclockwise) around the solenoid is related to the direction of its flow (from left-to-right or from right-to-left) along the coil.
- The initial position of the magnet relative to the coil and the direction of magnet motion. Clearly label the N and S poles of the magnet.
- The dominant direction of the magnetic field of the magnet at points inside the coil.

In notes below these cartoons, draw arrows and additional annotated sketches to show:

- The direction of increasing magnetic field inside the coil.
- The direction of the induced magnetic field required by Lenz’s Law. Refer to Lenz’s Law in this step.
- The direction of current in the coil required to produce this induced magnetic field. Specify both direction (left-to-right) and sense (clockwise or counterclockwise).
- You will need the right-hand rule. Draw a simple right hand. The direction of the current at the positive terminal of the ammeter. Clearly indicate the direction of the initial motion of the needle.

The required cartoons and notes will occupy most of a page in your lab notebook.

The process of prediction is important for two reasons. First, prediction is the true test of whether we understand a phenomenon. When we know the answer ahead of time, we often settle for a partial explanation with missing or incorrect steps. Second, we remember what we observe better if we make a prediction before observing it.[1] This is true whether our prediction is correct or incorrect. In the end, prediction is much better test of understanding than explanation.

Experiment

Now perform the experiment. Did the ammeter deflect in the predicted direction? Do not erase or throw away your cartoons in any case. Go over them carefully and identify any mistakes. Make a note in the margin near the mistaken text or drawings, then redraw or rewrite the mistaken material below your original prediction or on a subsequent page. **This is the only acceptable way of correcting lab notes when an error has been made.**

Predictions and experiments for other geometries

Magnet starting at rest in coil with N pole to right—move to right

Position the bar magnet inside the wire coil with the N pole on the right and S pole on the left. Predict the direction of the current when you pull the magnet out the right-hand end of the coil—drawing another set of annotated cartoons. Then do the experiment and draw corrected cartoons as required. Make sure that your explanation above is consistent with your explanation here.

Magnet starting left of the coil with S pole to right—move into coil

Push the bar magnet S-pole first into the left-hand end of the coil. Predict/observe.

Magnet starting at rest in coil with N pole to right—move to left

Starting with the bar magnet at rest inside the wire coil, with the N pole on the right and S pole on the left, pull the magnet out the left-hand end of the coil. Predict/observe.

What does it take to induce a current in an ammeter?

Perform additional experiments to answer the following questions:

What effect does varying the speed with which you insert or remove the magnet from the coil have? Explain your observations using Faraday's Law.

Under what conditions does a current flow in response to a magnetic field? For instance, how about when the magnet is at rest in the coil? Explain.

Can you cause a current to flow in the coil by moving the bar magnet along the outside of the the coil rather than inside the coil? If so, are certain orientations of the magnet more effective than others for inducing this current? Observe and explain.

[1] Kelly Miller, Nathaniel Lasry, Kelvin Chu, and Eric Mazur, "Role of physics lecture demonstrations in conceptual learning," Phys. Rev. ST Phys. Educ. Res. **9**, 020113 (2013).

Summary

Be as precise as possible in presenting your experimental results. Don't make such broad sweeping statements that they are meaningless. State all your conclusions clearly in a summary (maybe even a table) at the end of the report.

Before you leave the lab please:
    Straighten up your lab station.
    Report any problems or suggest improvements to your TA.

Lab 10. AC Circuits

Goals

- To show that AC voltages cannot generally be added without accounting for their phase relationships. That is, one must account for how they vary in time with respect to one another.
- To understand the use of “root mean square” (rms) voltages and currents.
- To learn how to view and interpret AC voltage and current waveforms using the “scope” function of the Capstone software.
- To learn how to measure the phase between sinusoidal voltage waves displayed with Capstone.
- To understand how to use phasor diagrams (analogous to diagrams of vector addition) as a technique for adding AC voltages or currents with various phases.
- To observe electrical resonance (analogous to mechanical resonance in a vibrating string) in an RLC circuit.

Introduction

While DC (direct current) circuits employ constant voltages and currents, AC (alternating current) circuits employ sinusoidally varying voltages and currents. It may seem strange that sinusoidal quantities should be so common, but rotating devices generate most of the electrical power in the world. In a natural way, this produces voltages and currents that vary in a sinusoidal fashion.

A complicating factor in AC circuits is that inductors and capacitors introduce phase shifts. That is, the voltages across some components can peak well before or after the currents flowing through them. At any one instant in time, the voltage across each component in a series circuit will indeed sum to zero—but the voltage peak for each component (proportional to the amplitude) will be reached at different times. Under these conditions, the sum of the voltage amplitudes in a circuit containing inductors and capacitors will not in general be zero—in apparent violation of Kirchhoff’s Voltage or Loop Rule. In this experiment you will explore the relationships between voltages and currents for inductors, capacitors, and resistors. This will include determining their phase relationships and how they depend on frequency. For this study, we consider a simple circuit consisting of a resistor, a capacitor, and an inductor connected in series with a sinusoidal voltage source.

A brief review of theory

A diagram of a typical RLC circuit is shown in Figure 10.1. Normally the current (which must be equal at all points along a series circuit) is used as a reference signal in AC circuits. Although the current flows back and force, one direction is designated the positive direction. This defines the direction of positive voltage differences as well. The positive end of each component in Figure 10.1 is marked. Note that the positive end of the power supply is is chosen so that positive current flows out of it. This is consistent with the convention for batteries and DC power supplies.

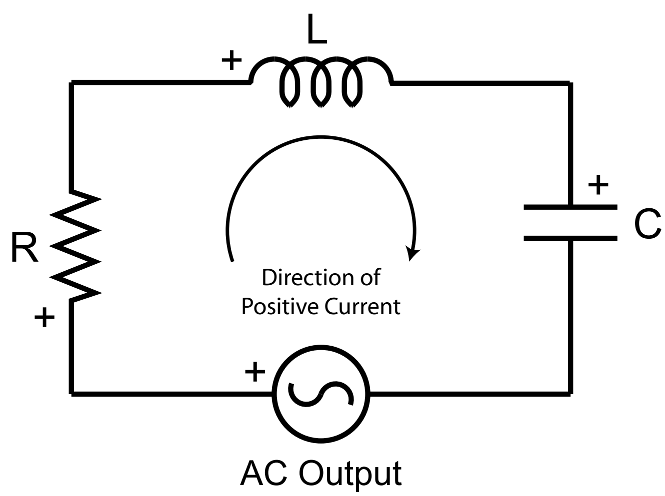


Figure 10.1. Diagram of a resistor, inductor, and capacitor connected in series.

Potential differences for RLC circuit

With a DC power supply, the sum of the voltages across the inductor V_L, the resistor, V_R, and the capacitor, V_C in Figure 10.1 would equal the output of the power supply, V_{Out}. That is

$$V_L + V_R + V_C = V_{Out} \tag{10.1}$$

When the voltages are changing in time, we still expect that the sum of the voltage drops across the three components $V_L + V_R + V_C$ equals the output voltage V_{Out} at each instant of time. When the output of the power supply is sinusoidal, the steady state voltages across each of the components will also be sinusoidal. However, each voltage in the circuit will have its own phase. That is

$$V_{L(0\text{-}p)} \cos(\omega t + \phi_L) + V_{R(0\text{-}p)} \cos(\omega t + \phi_R) + V_{C(0\text{-}p)} \cos(\omega t + \phi_C) = V_{Out(0\text{-}p)} \cos(\omega t + \phi_{Out}) \tag{10.2}$$

where the $(0\text{-}p)$ subscript in $V_{L(0\text{-}p)}$ and the other voltage amplitudes refers to their "zero-to-peak" values. When multiplied by the proper sine or cosine function, the zero-to-peak amplitude gives the actual measured value of voltage as a function of time. The non-zero phase angles, denoted by ϕ in Equation 10.2, complicate the analysis of AC circuits.

The phase of the potential difference across a capacitor

Capacitors are essentially two conducting sheets or plates separated by some insulating material that may include air or a vacuum. When a voltage is applied between the two plates of the capacitor, charge is transferred from one plate to the other. Thus a current flows through the voltage source and the connecting wires. As the voltage increases and more charge collects on the plates, adding more charge becomes increasingly more difficult, because like charges repel. Therefore the current flowing into the capacitor is greatest when the plates begin to charge. The current drops to zero when the charge build-up reaches a maximum. If a sinusoidally varying voltage source (one that oscillates positively and negatively in time with the shape of a sine function), is connected across the capacitor, the voltage across the capacitor "lags" the current by 90° in phase, meaning that the voltage peaks occur one-fourth of an oscillation period after the current peaks. This relationship is illustrated in Figure 10.2, where the voltage across the resistor (V_R) shows the variation of current during a single cycle. The oscillation period of the signal in Figure 10.2 is 1 s, and the voltage across the capacitor (V_C) peaks 0.25 s (one-fourth of a cycle) *after* the peak in V_R. We say that the phase of the voltage across an ideal capacitor is shifted 90° (360°/4) relative to the current. The sign is chosen so that if I and V_R are proportion to $cos(\omega t)$, V_C is proportional to $\cos(\omega t + \phi)$, where $\phi = -90°$.

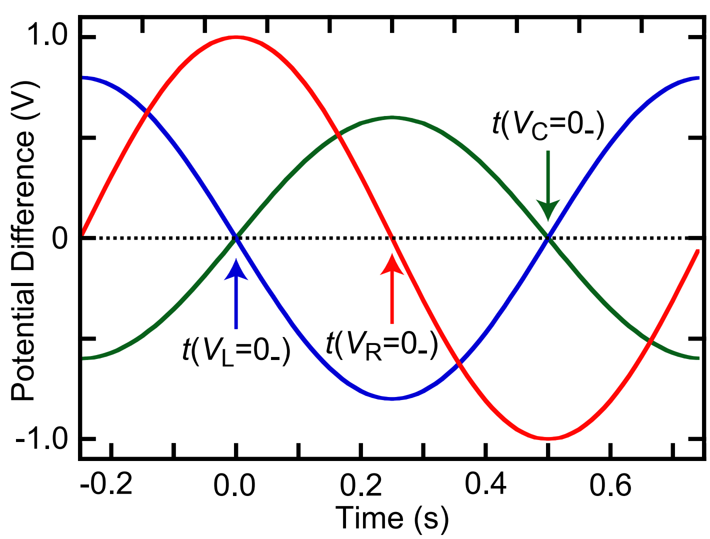


Figure 10.2. Voltages across an ideal induction, an ideal resistor, and an ideal capacitor in an RLC circuit. The times as which the three voltages cross zero (during the falling portion of the cycle) are labeled $t(V_L = 0_-)$, $t(V_R = 0_-)$, and $t(V_C = 0_-)$, respectively. The components in your experiment are not ideal, so the phases will be different.

The phase of the potential difference across an inductor

An inductor is usually takes the form of a coil of wire with many loops. When a time-varying electrical current passes through the loops, the resulting time-varying magnetic field induces a voltage in the coil. According to Lenz's law (and energy conservation) this induced voltage opposes the source voltage, making the current small. When sinusoidally driven, the voltage across and ideal inductor peaks one-fourth of an oscillation period *before* the current peaks. That is, the voltage "leads" the current by 90° in an ideal inductor. We say that the voltage experiences a +90° phase shift relative to the current in an ideal inductor. This relationship is illustrated in Figure 10.2, where the voltage across the inductor, V_L, peaks 0.25 s (one-fourth of a cycle) before the peak in V_R. Again, the sign is chosen so that if I and V_R are proportion to $\cos(\omega t)$, V_L is proportional to $\cos(\omega t + \phi)$, where $\phi = +90°$.

In practice, it is difficult to determine the position of the peak in a sinusoidal signal precisely, because voltage changes slowly near the peak. Measuring the time at which the voltage crosses zero, where the voltage changes rapidly, gives more precise results. Because the voltage crosses zero twice per cycle, it is important to be consistent about which zero crossing is used. The arrows in Figure 10.2 show the zero crossings for V_L, V_R, and V_C where the voltage is falling, that is, where the voltage crosses zero from above.

To derive an equation for the phase angle ϕ for a given voltage signal, one observed that 360° of phase corresponds to one oscillation period T,

$$\phi = \frac{[t(V_R = 0_-) - t(V = 0_-)] \times 360°}{T} \tag{10.3}$$

The order of terms in Equation 10.3 is chosen so that a voltage signal that lags V_R has a negative phase, as required by the sine and cosine functions.

Using phasors to represent AC voltages

The AC voltages across an AC power supply, an inductor, a capacitor, and a resistor, all connected in series, can be added much like vectors. The length of each vector, or phasor, represents the measured voltage amplitude of the corresponding circuit element, and the angle between each phasor and the resistor phasor (which points in the same orientation as the current phasor) equals the phase difference between the AC voltage across that circuit element and the AC voltage across the resistor. These relationships are illustrated in Figure 10.3. To represent the time-varying voltages in an AC circuit, all four phasors are rotated at angular velocity of ωt. The measured voltage across each circuit element at time t is equal to the horizontal component of that element's phasor at that time.

Phasors are used to represent the various time-varying voltages in more complex AC circuits. They are also used to represent the addition of other quantities that vary sinusoidally in time. For instance, the electric fields in monochromatic electromagnetic waves (laser beams) vary sinusoidally in time. Phasors are often used to account for phase differences in single-slit diffraction.

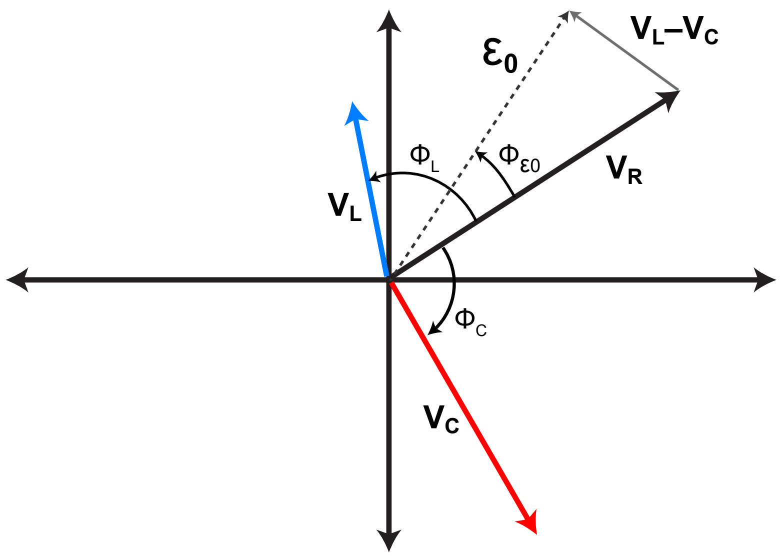


Figure 10.3. Phasor diagram of the voltages across an inductor, a resistor, a capacitor, and the output in an *RLC* circuit. The current phasor is not shown, but is proportional to the resistor's phasor. The phases Φ_L, Φ_C, and $\Phi_{\varepsilon o}$ are measured with respect to the phase of the voltage across the resistor (or equivalently, the phase of the current signal). The measured voltage across each component is equal to the projection of its vector onto the horizontal axis. As a function of time, each vector rotates about the origin with angular velocity ωt.

Expressing AC voltages in terms of their root-mean-square (rms) values

AC voltages are often expressed in terms of their root mean square (abbreviated rms) values. In DC circuits the product of the current and voltage gives the power. It is convenient to use a similar formula for the average power dissipated in AC circuits when the current and voltage are in phase. However, the product of the raw voltage and current amplitudes (the zero-to-peak voltages and zero-to-peak currents), is twice the actual average power. To correct for this, we use rms voltages and currents. The rms voltage is the zero-to-peak voltage divided by $\sqrt{2}$, and the rms current is the zero-to-peak current divided by $\sqrt{2}$. When these are multiplied, the factor of 2 in the denominator yields the correct average power. (This procedure yields the average power only when the voltage and current have the same phase.) Most AC voltmeters and ammeters display rms volts and rms amps, respectively. The voltage at a wall plug in the United States is 120 V rms. The corresponding zero-to-peak voltage is about 170 V.

Equipment set up

The Pasco Scientific RLC Circuit (Model CI-6512) is already configured with a series combination of resistor, inductor, and capacitor. Choose the 10 Ω resistor, the 8.2 mH inductor, and the 100 μF capacitor. They are already connected in series. (You can see the connections on the bottom of the circuit board.) The analog inputs of the interface unit (Channels A, B and C) can be employed to measure the voltage difference across each of the three components using the three patch cords supplied with the circuit board.

Since switching the red and black leads across a component reverses the sign of the detected voltage difference, it is important to connect the red and black ends of each patch cord to the three components in a consistent fashion. This requires that you define one current direction to be positive, and use this direction to identify the positive end of each component. The positive end of each component is labeled (+) in Figure 1 for the choice of positive direction shown in the figure. Attach the red lead of the patch cord for the resistor, for instance, to the positive end of the resistor, and the black lead to the negative end of the resistor. Attach the patch cords used to measure the voltage differences across the inductor and the capacitor in the same fashion, being careful of sign.

To take the data, you will need to tell Capstone that you want to connect voltage sensors to Channels A, B, and C, and that you wish to use the output from the interface unit as the voltage source for the circuit. The output jacks are to the right of Channel C. You will need to add a “scope” display so all of this can be viewed. Then you can drag and drop the voltages for Channels A, B, and C. Capstone automatically records the output voltage, V_{Out}, and current, I. Drag and drop the icons for these signals to the same scope display. You want to show all five signals on the same display. Your TA can be helpful here.

If a waveform appears choppy, like a series of connected straight lines, you probably need to increase the data sampling rate. For best results, the sampling rate should be about 50 times the frequency of the wave that you want to observe. Adjusting the time per division on the horizontal scale of the scope display will automatically change the sampling rate and may solve this problem. Otherwise, you can manually change the sampling rate on the Control Palette along the bottom of Capstone’s Display Area.

Phase and voltage measurements

Set the sinusoidal output voltage amplitude to 4.0 V at a frequency of 10 Hz. Now individually measure the zero-to-peak voltages across the resistor, inductor, and capacitor and the zero-to- peak current. (Why is one current reading enough?) A table is a good way to record all this information. Convert all the peak voltages and currents to rms values. Record the zero-crossing times for all four voltages and and the current, and compute their phases with respect to the phase of the voltage across the resistor.

Repeat the voltage and phase measurements for each component at 100 Hz and 1000 Hz.

Adding AC voltages

From Figure 10.1, we expect that the sum of the voltage drops across the three components $V_L + V_R + V_C$ equals the output voltage V_{output} at each instant of time. In the absence of time variation, the voltages would add like DC voltages. In terms of rms voltages, we would expect

$$V_{Lrms} + V_{Rrms} + V_{Crms} = V_{Outrms} \tag{10.4}$$

However, each voltage in the circuit varies in time with its own phase. Expressing Equation 10.2

in terms of rms voltages yields

$$V_{Lrms}\cos(\omega t+\phi_L)+V_{Rrms}\cos(\omega t+\phi_R)+V_{Crms}\cos(\omega t+\phi_C)=V_{Outrms}\cos(\omega t+\phi_{Out}) \quad (10.5)$$

To verify that the voltages do add this way, it is sufficient to show that the equation holds at two times. Two times are needed to resolve the ambiguity associated with the phases of the voltage signals in Figure 10.2. At most times, it is not enough to know the voltage reading alone. One must also know whether the voltage is rising or falling.) The times $\omega t = 0$ and $\omega t = -90°$ make for simple expressions. Then

$$\begin{aligned} V_{Lrms}\cos(\phi_L)+V_{Rrms}\cos(\phi_R)+V_{Crms}\cos(\phi_C)=V_{Outrms}\cos(\phi_{Out}) \\ V_{Lrms}\sin(\phi_L)+V_{Rrms}\sin(\phi_R)+V_{Crms}\sin(\phi_C)=V_{Outrms}\sin(\phi_{Out}) \end{aligned} \quad (10.6)$$

Ideal components constitute an important special case. For ideal components, $\phi_L = +90°$ and $\phi_C = -90°$. By convention, $\phi_R = 0°$. For ideal components, these relations reduce to

$$\sqrt{(V_{Crms}-V_{Lrms})^2+V_{Rrms}^2}=V_{Outrms} \quad (10.7)$$

At each frequency check to see whether the voltages across the resistor, inductor, and capacitor obey Equations 10.4, 10.6 and 10.7. Tabulate all these results clearly. Is ignoring the phase a good idea?

For future reference, it is worth comparing the measured phases for V_L and V_C to their ideal values. Capacitors are usually pretty close to ideal. This observation can help you on an exam.

RLC circuit at resonance

By trial and error, adjust the frequency of the sine wave output of the interface unit until the output voltage and current, which drive the circuit, are in phase. Do this carefully. When the current and voltage are in phase as required, look at the inductor voltage and the capacitor voltage. What relationship do they now have with respect to one another? Since real inductors have both resistance and inductance, the phase shift for the real inductor does not equal the +90° phase shift for an ideal inductor. You will need to compare the vertical component of the voltage across the real inductor with the vertical component of the voltage across the capacitor. (Real capacitors are well-represented by ideal capacitors, so the capacitor vector should be nearly vertical anyway.) This particular state of the system is called "resonance." What is the overall effect on the circuit of the inductance and capacitance at resonance? Resonant circuits are useful in filtering out certain frequencies. Radio tuning dials work on this principle.

Summary

Summarize all of your results clearly and concisely. Refer to appropriate data tables liberally.

Before you leave the lab please:

Disconnect the Pasco interface and the RLC Circuit.

Put all the connecting wires neatly in the tray provided at your lab table.

Close the Capstone software and any other software you have been using.

Report any problems or suggest improvements to your TA.

Lab 11. Interference of Light

Goals

- To observe the interference patterns for laser light passing through a single narrow slit, through two closely spaced slits, and through multiple closely spaced slits, noting the similarities and differences.
- To determine by graphical techniques the wavelength of the laser light based on the observed interference patterns for single, double, and multiple slits.
- To compare the calculated values of wavelength with the accepted value for a red helium-neon laser.
- To "measure" the diameter of a human hair by observing and analyzing the interference pattern created when it is placed in the center of laser beam.

Introduction

Two waves that have the same frequency can "interfere" constructively when the peaks coincide or destructively when a peak of one wave coincides with a valley of the other. When speaking of peaks and valleys, water waves are a useful example. With sound waves the peaks and valleys correspond to regions of high and low pressure. With electromagnetic waves, such as light, the peaks and valleys correspond to regions of positive and negative electric and magnetic field vectors. Constructive interference of light rays produces regions of high intensity or brightness. Destructive interference produces regions of low intensity or darkness.

Double slit interference

The simplest example of interference takes place when monochromatic light passes through two nearby, parallel slits (narrow openings for the light to come through). Laser light is nearly monochromatic (all of the same frequency and wavelength). The diagram in Figure 11.1 shows the path of a laser beam, traveling from left to right, incident on two slits at an incident angle of 0°. This configuration assures that the phase of the waves at each of the slits is the same. In other words the peak of the wave in one slit is synchronized with the peak of the wave in the other slit. Let d be the center-to-center spacing between the slits. The light intensity is observed at a distance y from the center of the slit pattern. For constructive interference to take place at the point y, the difference in the distances from the point y to the individual slits, $r_2 - r_1$, must be equal to some integer multiple of the wavelength λ of the light. This can be expressed as

$$r_2 - r_1 = m\lambda \qquad \text{where } m \text{ is an integer } (... -2, -1, 0, +1, +2, ...) \quad . \tag{11.1}$$

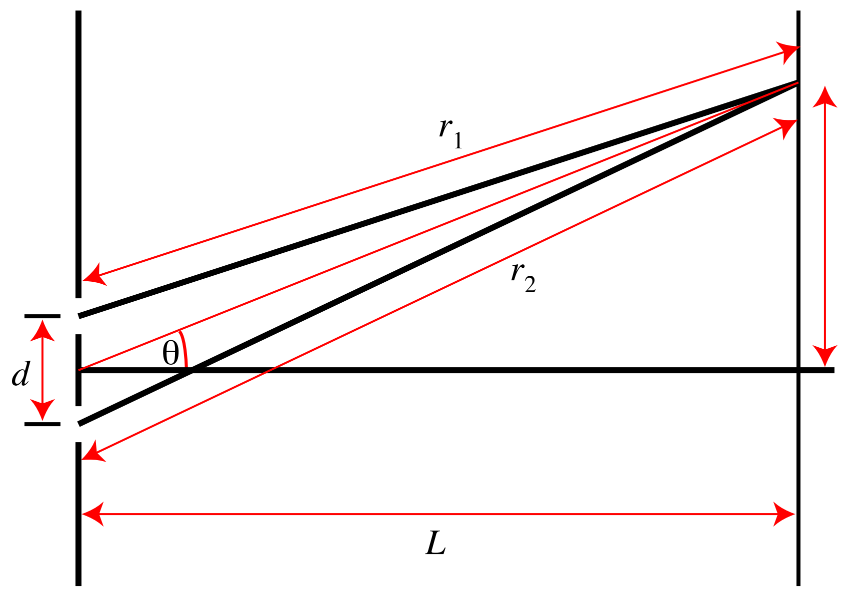


Figure 11.1. Geometry for determining the condition for constructive interference for a double slit.

When the distance L from the viewing screen to the slits is much larger than the distance between the slits d, the lines denoting the distances r_2 and r_1 are essentially parallel, like the edges of a very tall skinny triangle. For this limiting case the difference in the distances can be written to a good approximation as $d\sin\theta$. Then condition for constructive interference then becomes

$$d\sin\theta = m\lambda \qquad \text{where } m \text{ is an integer } (... -2, -1, 0, +1, +2, ...) \quad . \tag{11.2}$$

This equation defines the angles for maximum intensity on the screen.

Interference patterns from double slits can be used to find the spacing between the two sources of light if the wavelength of the light being used is known. In other words, from the measured positions of the intensity maxima on the viewing screen, one can calculate the angles corresponding to the various values of m and determine the unknown d. On the other hand, if d is known, then the wavelength can be determined. Historically the wavelengths of light were difficult to measure until good quality slits became available about 100 years ago.

Single slit diffraction

A narrow aperture such as a single slit will interact with a narrow beam of light such a way that some of the light appears to be “bent” from its original direction of travel. The term diffraction refers to this apparent change of direction. This behavior is due to interference between parts of the light wave that pass through the slit at different points within the slit. Thus diffraction can be thought of—not as some new phenomenon—but as another manifestation of the interference of waves. For a single slit of width a the relationship that describes the locations of the minima of intensity on the viewing screen is given by

$$a \sin\theta = n\lambda \tag{11.3}$$

where n is an integer excluding zero, that is, (... –2, –1, +1, +2, ...) Note that zero is missing from the list!

This expression looks a great deal like Equation 11.2, which describes intensity maxima for a double slit arrangement. Remember the important differences!

Multiple slit (more than two slits) interference

When more than two equally spaced slits are present, the explanation proceeds in exactly the same way as it does for the double slit arrangement. In fact the condition for making light from adjacent slits interfere constructively on the viewing screen is sufficient to ensure that the light from all of the slits will interfere constructively on the screen. Thus Equation 11.2 also prescribes the conditions to be met for intensity maxima when more than two equally spaced slits are present.

Determining the wavelength of light from a helium-neon laser

Never look directly into the beam or at reflections of the beam. Don't point the laser at anything other than the screen. Failure to follow these instructions may lead to a zero for the lab.

If you need to locate the laser beam, insert a piece of paper into the beam path. Minimize reflections by positioning the slide with the slits close to the exit aperture of the laser, which directs the reflected beam back toward the laser. If a laser is powered up but the beam is not visible, make sure the aperture at the front of the laser is open.

Using single slit diffraction

While the physics of double slit interference is relatively simple, the resulting diffraction patterns are relatively complicated. This is because each member of a double slit pair is also a source of single slit diffraction; in the double slit geometry, both patterns are observed together. The two effects are easier to disentangle after you have characterized the simpler, single slit pattern.

Use the single slit from Column 1, Row (e). (This slit has the same width of each of the double slits on your slide.) Again mark maxima or minima as appropriate and calculate the wavelength of the light from this data. Does your calculated value agree with the accepted value within the limits of error? From your data, does the slit width have to be less than the laser wavelength in order to produce a diffraction pattern?

Using double slit interference

On the viewing screen observe and mark the locations of the maxima or minima of intensity, as appropriate, for a double slit. The glass slide with the green tape on the edges contains the various slit arrangements. Use one of the double slits from Column 5, either (b) or (c). From this information calculate the wavelength of the laser light. Consider an appropriate graph. Most students will find Excel helpful. You should compare your calculated wavelength to the accepted value listed for He-Ne lasers in your textbook or a handbook. Does your calculated value agree with the accepted value within the limits of the expected uncertainties?

Using multiple slits

Choose a multiple slit from Column 3, either (b), (c), or (d), and calculate the wavelength from the resulting data. Does your calculated value agree with the accepted value within the limits of the expected uncertainties?

Measuring the diameter of a human hair with laser light

Mount a human hair so that it can be placed in front of the laser beam and look at the resulting light pattern. Does it most closely resemble the pattern of a single slit, a double slit, or multiple slits? Look at it carefully and note the pattern of bright and dark regions, particularly their spacing with respect to the center of the pattern. Then mark intensity maxima or minima as appropriate on the viewing screen. Using the textbook value for the wavelength of the laser light, calculate the diameter of the hair. Compare this value of the diameter to that obtained with a micrometer. Machinists use micrometers to make precise length measurements. Do the measurements agree within their expected uncertainties?

Before you leave the lab please:

- Straighten up your lab station.
- Report any problems or suggest improvements to your TA.

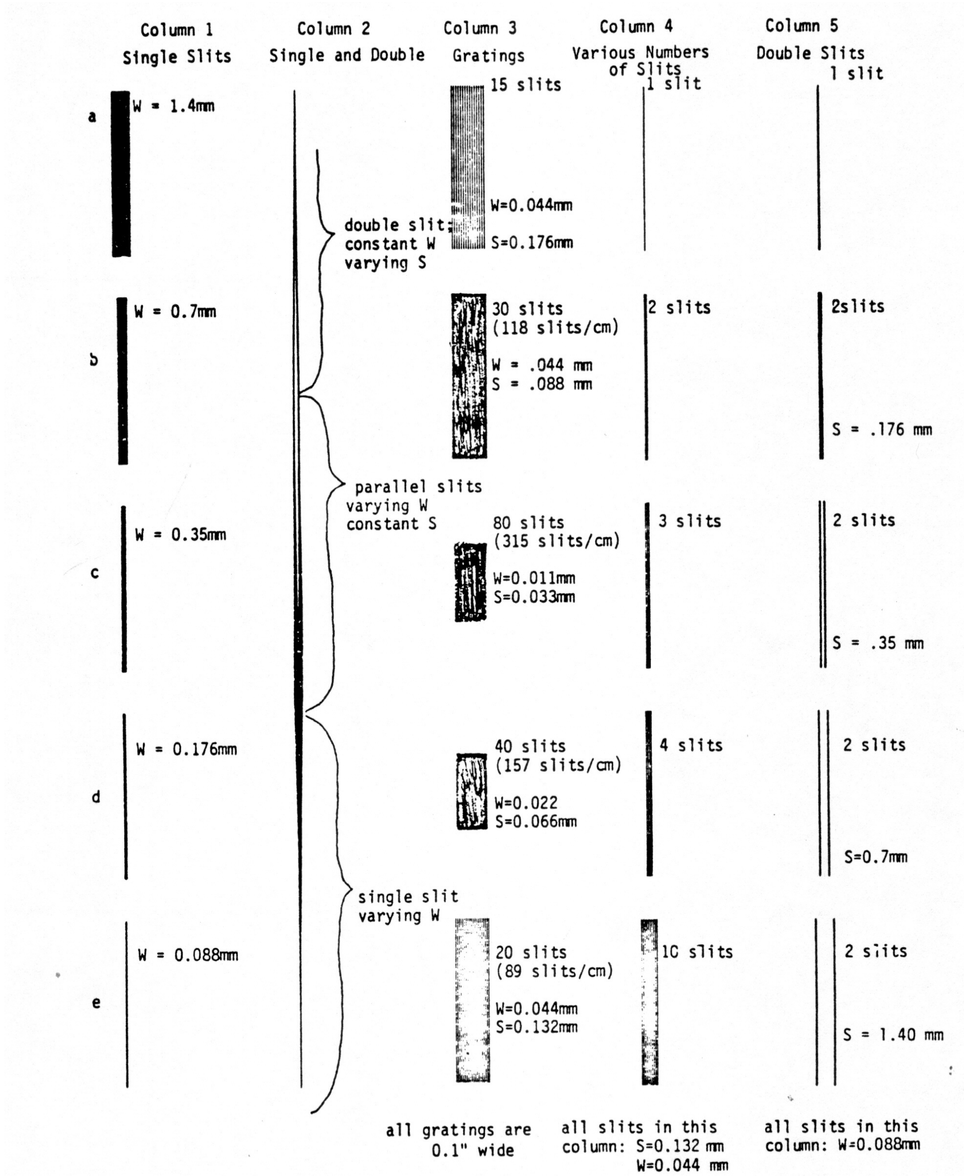


Fig. 2 Arrangement of slits and gratings on the green slide. The dark lines in the figure represent transparent regions on the slide.

W = width of an individual slit

S = spacing between centers of neighboring slits

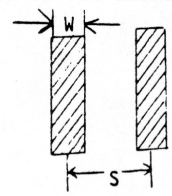


Figure 11.2. Arrangement of slits and gratings on black slide.

Lab 12. Images with Thin Lenses

Goals

- To learn experimental techniques for determining the focal lengths of positive (converging) and negative (diverging) lenses in conjunction with the thin-lens equation.
- To learn how to make a scale “ray diagram” for a combination of a positive and negative lens using three principle rays for each lens and interpret it.
- To understand the specific meaning of the term “magnification” as applied to optical systems and to determine its value by three methods: (a) direct measurement, (b) calculation using the thin lens equation, and (c) using a ray diagram.

Introduction

For a simple focusing element with focal length f, it can be shown that

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \tag{12.1}$$

where s and s' are the object and image distances respectively. This is called the thin-lens equation. The object distance is measured from the light source to the center of the lens, and the image distance is measured from the viewing screen, where the real image is displayed, to the center of the lens.

An optical bench with a metric length scale attached to it, two lenses and holders, a light source, and a viewing screen are provided. Some ray box light sources are provided with crossed arrows that serve as the object to be imaged. When one has a clear view the light bulb in the light source, the filament of the light bulb can also serve as the object to be imaged. For clearer view of the image, hang a clean sheet of paper over the glass viewing screen on the side facing toward the ray box.

Caution: Always secure (but not too tightly) the optical mounts on the optics bench so that the optical elements do not fall and break. Handle the lenses using the edges only. Your TA will demonstrate how to put a lens in the lens holder.

Determining the focal length of a converging lens

Use the optical bench with the light source and the viewing screen to determine the focal lengths of the two lenses provided, one a converging lens (positive focal length) and the other a diverging lens (negative focal length). You should be able to tell which is which by looking at their cross-sections. The focal length of the converging lens should be determined first. This can be done experimentally by finding pairs of object and image distances that give clear real images of the light source on the viewing screen. (A “real” image can be projected onto a screen.) Then use the thin lens equation to calculate the focal length. Repeat this several times using significantly different values of s and s'.

Find the mean value of the focal length and compute its standard deviation. If you do not know the how to compute a standard deviation, consult the Uncertainty/Graphical Analysis supplement to the lab manual.

What happens when you try the same procedure for the negative (diverging) lens?

Determining the focal length of a diverging lens

A diverging lens forms a real image only when used in conjunction with a converging lens. Using both lenses (place the converging lens nearest the light source), find lens and screen positions that yield clear images. In this configuration we can measure only the object distance of the converging lens and the image distance of the diverging lens. Knowing the focal length of the converging lens from your measurements above, the thin lens equation can be used to find the location of the image formed by the converging lens. Then treat the image of the converging lens as an object (be careful of the sign of the object distance) for the diverging lens. Apply the thin lens equation again to find the focal length of the diverging lens. Note that the sign conventions used in the thin lens equation demand that the focal length for a diverging lens be a negative number. Repeat this process for several significantly different lens and viewing screen positions. Calculate the mean focal length and the corresponding standard deviation.

Drawing a ray diagram for a two-lens system

Pick one configuration of lenses and viewing screen from your measurements on the diverging lens and draw a complete ray diagram to scale showing the formation of the intermediate image from the converging lens and the final image of the diverging lens. Ray diagrams for single converging and diverging lenses are shown in your textbook. Trace the rays for the lens closest to the light source first; then use the resulting image as the object for the second lens. Use your experimental values of focal lengths as given values on your diagram. Does your ray diagram predict the correct location for the final image? Compare the result to your experimental value using the diverging lens.

Magnification

Magnification is defined as the ratio of the size of the image to the size of the object being imaged. When the image is upside down, the magnification is negative. If the image is upright, having the same orientation as the object, the magnification is positive. Using ray diagrams, one can show that the magnification (sometimes called the transverse magnification), m, is equal to $-s'/s$ for both positive and negative lenses. Compare the heights of the object and image in your ray diagram to determine magnification of the two-lens combination. Compare this value with the magnification calculated using the thin-lens equation for the same lens configuration, knowing the focal lengths and positions of the lenses relative to the object and the final image.

Before you leave the lab please:

Remove the lenses from the lens holders and place them in the plastic tray provided.
Straighten up your lab station.
Report any problems or suggest improvements to your TA.

Uncertainty and Graphical Analysis

Introduction

Two measures of the quality of an experimental result are its accuracy and its precision. An accurate result is consistent with some ideal, “true” value, perhaps a commonly accepted value from the scientific literature. When a literature value is not available, we often perform an additional measurement by other methods. Different methods are usually prone to different errors. We can hope that, if two or three different methods yield consistent results, our errors are small. However, measurements made by different methods never agree exactly. If the discrepancy is small enough, we claim that the results are consistent and accurate. Most of our work with uncertainties will address the question, “How small is small enough?”

Precision refers to the reproducibility of a result made using a particular experimental method. When random variations are large, the precision is low, and vise versa. While we should work hard to reduce the size of random effects, they cannot be entirely eliminated. When we claim that two measurements are consistent, we are claiming that their difference (the discrepancy) is smaller than these random variations. Since many quantities of interest are calculated from measured values, we also need to know how random variations in measured quantities affect the results of these calculations.

Measurements in the presence of random deviations

Mean and standard deviation of the mean[1]

In the presence of random variations, the best estimate of a physical quantity is generally given by the average, or mean. The average value of a set of N measurements of x, $(x_1, x_2, x_3, \ldots x_N)$, is given by

$$x_{avg} = \frac{x_1 + x_2 + x_3 + \ldots + x_N}{N} = \frac{1}{N}\sum_{i=1}^{N} x_i \tag{13.1}$$

[1] A good reference for much of the information in this section is John R. Taylor, *An Introduction to Error Analysis—The Study of Uncertainties in Physical Measurements, 2nd Edition* (University Science Books, Herndon, Virginia, USA, 1987), especially Chapter 5.

The individual measurements of x will generally deviate from x_{avg} due to random errors. The standard deviation of x, denoted $\sigma(x)$, indicates how far a typical measurement deviates from the mean. The value of $\sigma(x)$ reflects the size of random errors.

$$\sigma(x) = \sqrt{\frac{(x_1 - x_{avg})^2 + (x_2 - x_{avg})^2 + (x_3 - x_{avg})^2 + (x_4 - x_{avg})^2 + \ldots (x_N - x_{avg})^2}{N-1}}$$

$$= \frac{1}{\sqrt{(N-1)}} \left[\sum_{i=1}^{N} (x_1 - x_{avg})^2 \right]^{1/2} \tag{13.2}$$

A small standard deviation indicates that the measurements (x-values) are clustered closely around the average value, while a large standard deviation indicates that the measurements scatter widely relative to the average value. Thus a small standard deviation indicates that this particular quantity is very reproducible—that is, the measurement is very precise. Note that the units of the standard deviation are the same as the units of the individual measurements, x_i.

The relation between the standard deviation to the deviation of the data from its average value is illustrated in Figure 13.1. Figure 13.1 is a histogram of 100 scores, chosen from a set of over 1000 random scores with an average was 85 and a standard deviation of 7.5. Because of their random distribution, the average of the 100 scores is not exactly 85, and their standard deviation is not exactly 7.5. Because we cannot take an infinite number of measurements, Equations 13.1 and 13.2 are only approximations to the true average and standard deviation. On average, the approximations improve as the number of measurements, N, increases.

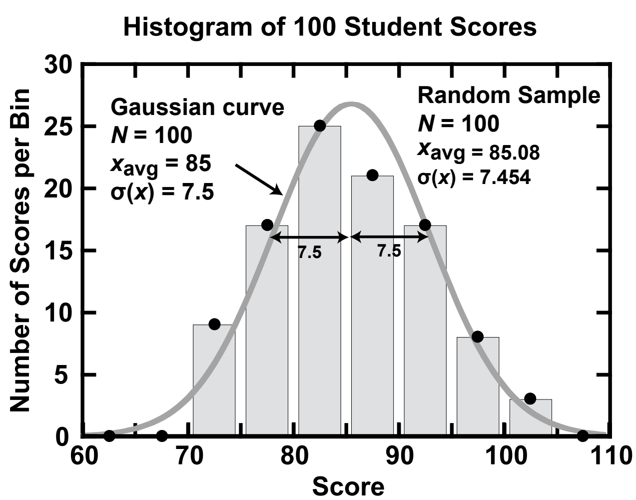


Figure 13.1. Histogram of 100 scores with an average of 85 and a standard deviation of 7.5. The smooth curve is the Gaussian function corresponding to the same number of measurements, average, and standard deviation.

The Gaussian function, $G(x)$, corresponding to 100 scores with an average of exactly 85 and a standard deviation of exactly 7.5 is also shown in Figure 13.1. According to the Central Limit Theorem of statistics, the Gaussian function represents the ideal distribution of scores for a given N, x_{avg}, and $\sigma(x) = \sigma$ *if* the scores have a finite average and the measurements are statistically independent. These conditions apply to most of the measurements made in lab. (Important exceptions are found in the stock market, among other things.)

$$G(x) = \frac{N}{2\pi\sigma}\exp\left[\frac{-(x-x_{avg})^2}{2\sigma^2}\right] \tag{13.3}$$

The value of the standard deviation in the context of uncertainties is that the probability of finding a score at some distance from the average falls in a predictable way as the distance increases. For an ideal Gaussian distribution, 68% of the measurements lie within one standard deviation of the mean (x_{avg}). In Figure 13.1, 63 scores (63% of 100) lie within 7.5 points of 85. Ideally, 95% of the scores lie within two standard deviations (here, ± 15 points) of the average. Ideally, one would expect 99.7% of the points to lie within three standard deviations (here, ± 22.5 points) of the average. No score in Figure 13.1, is more than three standard deviations from the average. (All of the scores lie between $x_{avg} - 3\sigma = 62.5$ and $x_{avg} + 3\sigma = 107.5$.) Unless the total number of scores is very high, the probability of finding a score more than 3σ from the average is quite low.

Since the standard deviation characterizes random errors, we can pretty much rule out random errors as the source of any difference greater than 3σ. We will make this assumption in the physics labs, although the precise probabilities will usually differ from those given by the ideal Gaussian function. For instance, when the number of measurements is small, our estimates of x_{avg} and $\sigma(x)$ may be poor. In more advanced work, it can be important to correct for this lower precision.[2] When one is attempting to show that one measurement out of a large number differs significantly from the others, a higher threshold for significance (4σ or 5σ) may be necessary.

Since the result of an experiment is generally an average value, we need a measure of the precision of the average. This is called the "standard deviation of the mean," $\sigma(x_{avg})$. Although one can repeat the entire set of N measurements several times to compute $\sigma(x_{avg})$, statistics allows us to estimate $\sigma(x_{avg})$ using the original N measurements alone:

$$\sigma(x_{avg}) = \frac{1}{\sqrt{N(N-1)}}\left[\sum_{i=1}^{N}(x_1 - x_{avg})^2\right]^{1/2} = \frac{\sigma(x)}{\sqrt{N}} \tag{13.4}$$

The standard deviation function of most spreadsheet programs (Excel, OpenOffice), Capstone, and calculators gives $\sigma(x)$, from Equation 13.2. To calculate the standard deviation of the mean from this number, you must divide by the square root of N, the number of measurements.

On the other hand, spreadsheet Regression functions and Capstone's curve fit function provide the standard deviation of the mean, $\sigma(x_{avg})$ from Equation 13.4.

[2]Student's t-test is used to make this adjustment in more advanced work. This is described at the end of Chapter 5 in John R. Taylor, *op. cit.*, and in many statistics books.

Other methods for estimating the effect of random errors

When several measured quantities are used in a calculation, a relatively crude measurement of one quantity may contribute little to the overall uncertainty. If so, there is little point in improving the measurement. To demonstrate that the uncertainty is small, we must provide an upper bound on the uncertainty and show that the effect of this uncertainty is indeed relatively small.

Smallest division

Most measuring devises have a a smallest division that can be read. In this case, one can use the size of the smallest division as an upper bound on the uncertainty. In some cases, it is appropriate to to use one-half of this smallest division. For instance the smallest division displayed on a meter stick is usually 1 mm. The distance d is read to the nearest mark. Suppose, for example, you look at the meter stick a few times and read d = 85 mm each time. Because you never measured 84 or 86 mm, you are confident that $84.5 \leq d \leq 85.5$. That is, the magnitude of the uncertainty in d is less than 0.5 mm. This is a useful upper bound. You must use your judgement in cases where the measurement cannot be practically made with this precision. For instance, your precision can be much worse if you don't have a clear view of the ruler.

Interpolation

If the uncertainty in such a measurement is not small relative to the other uncertainties in an experiment, a better estimate of the uncertainty is needed. In this case, taking the standard deviation of the mean of multiple measurements is necessary. For instance, you can estimate d to one-tenth of a mm using a meter stick. (Estimating values between the marks is called interpolation.) In this case, repeated estimates, made with care, will disagree, and you can calculate the standard deviation of their mean.

Manufacturer's specification

The user manuals for many instruments (electronic ones in particular) often include the manufacturer's specification as to the "guaranteed" reliability of the readings. For example, the last digit on the right of digital voltmeters and ammeters is notoriously inaccurate. In this case, it makes sense to use the manufacturer's specification as a simple upper bound.

Terminology—Uncertainty and significant digits

Because the standard deviation is not the only measure of random variation, it helps to have another name and symbol for this quantity. We will call the the expected effect of random variation on x_{avg} its uncertainty, and represented it by the symbol $u(x_{avg})$. If the average and standard deviation of x are available, the best estimate of x is x_{avg}, and the best estimate of the uncertainty of x_{avg} is the standard deviation of its mean, $\sigma(x_{avg})$. Then $u(x_{avg}) = \sigma(x_{avg})$. The uncertainty is often indicated by a $\pm$ sign after the average value. For instance, you might specify a length measurement as "1.05 $\pm$ 0.02 mm. Because there is more than one way to estimate the uncertainty, you must also specify how your estimate was made. For instance, the result of a length measurement may be reported as "1.05 $\pm$ 0.02 mm, where the uncertainty is the standard deviation of the mean of five length readings;" or "24 ± 1 mm, where the uncertainty is the distance between marks on the meter stick."

With or without a formal uncertainty estimate, you are expected to have a general idea of the uncertainties of the numbers you use. These uncertainties are communicated by the number of significant digits you provide with the number. For instance, a length written as 3.14 mm has an implied uncertainty of less than 0.1 mm; the inclusion of a digit in the second decimal place means that you have some knowledge of it. In your lab notebook and reports, you should not use more significant digits than are justified by your knowledge. Since rounding operations slightly increase the uncertainty in the last decimal place, it is appropriate to keep one extra significant digit in each step of a calculation. However, the final result must be rounded to an appropriate number of significant digits. Most physics texts include a discussion of significant figures.

Uncertainties in calculated quantities—the Derivative Method[3]

Derivatives can be used to estimate the uncertainty associated with a function of the measured quantity, $f(x)$, due to uncertainty in the measured variable, x. We normally have an experimental value of x_{avg}. To see how the uncertainty in x affects $f(x_{avg})$, we can plot $f(x)$ as shown in Figure 13.2. The change due to small variation in x is given by $\Delta f \approx f'(x)\Delta x$, where $f'(x)$ is the slope (and the derivative) of $f(x)$ at x_{avg}.

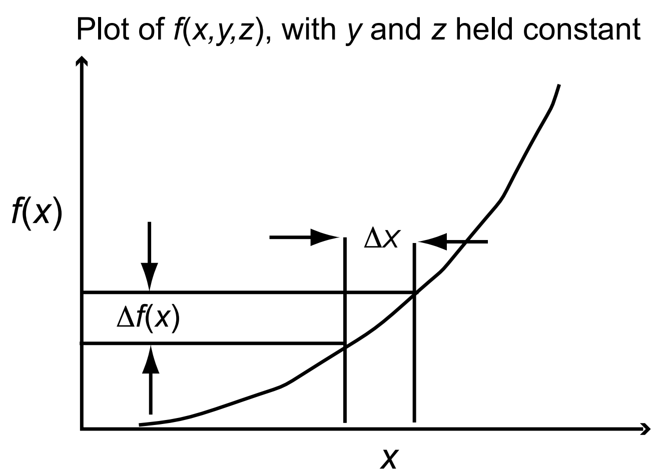


Figure 13.2. Diagram relating the uncertainty in $y = f(x)$ due to the uncertainty in x.

For the simple function $f(x) = 1/x$, with, $x_{avg} = 2.0$ and $u(x_{avg}) = 0.1$, the uncertainty in $f(x)$, $u[f(x)]$, is

$$u[f(x)] = \sqrt{\left[\frac{df}{dx}u(x_{avg})\right]^2} = \sqrt{\left[-\frac{1}{x^2}u(x_{avg})\right]^2} = \sqrt{\left(\frac{1}{4.0}\right)^2 (0.1)^2} = 0.25 \qquad (13.5)$$

[3]Good references on the propagation of uncertainties include John R. Taylor, *op. cit.*, Section 3.11, "A general formula for error propagation," and Douglas C. Montgomery, George C. Runger, and Norma Faris Hubele, Section 3.12.3, "Nonlinear functions of independent random variables," *Engineering Statistics*, Fifth Edition (John Wiley, Hoboken, New Jersey, 2012).

If f is a function of more than one variable, say (x,y,z), where x, y, and z represent three measured quantities, the uncertainty in $f(x,y,z)$ is found by computing uncertainties for each variable alone and adding them in quadrature, as explained below. The uncertainty due to x is computed by treating $f(x,y,z)$ as a function of x only. Then from Equation 13.5

$$u[f(x)] = \sqrt{\left[\frac{\partial f}{\partial x}u(x_{avg})\right]^2} \tag{13.6}$$

where we introduce the ∂ symbol to indicate that the y and z variables are being treated as constants when the derivative is taken. This is equivalent to assuming that the variables are *independent*; that is, none of the variables are completely determined by any subset of the others. Likewise:

$$u[f(y)] = \sqrt{\left[\frac{\partial f}{\partial y}u(y_{avg})\right]^2} \qquad u[f(z)] = \sqrt{\left[\frac{\partial f}{\partial z}u(z_{avg})\right]^2} \tag{13.7}$$

where $u[f(x,y,z)]$ is the estimated uncertainty in $f(x,y,z)$; $u(x_{avg})$, $u(y_{avg})$, and $u(z_{avg})$ are the uncertainties in the measured values of x_{avg}, y_{avg}, and z_{avg}, respectively, all evaluated at $(x_{avg}, y_{avg}, z_{avg})$. Again, the ∂ symbols indicate that x and z are treated as constants when the derivative with respect to y is taken; likewise x and y are treated as constants when the derivative with respect to z is taken.

If you draw a two dimensional version of Figure 13.2, the Pythagorean theorem can be used to show that the uncertainties add like the edges of a right triangle, that is, in “quadrature.” (This is how individual deviations add when a standard deviation is calculated.) For a function of three variables, the uncertainties add in the same way:

$$u[f(x,y,z)] = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 u(x_{avg})^2 + \left(\frac{\partial f}{\partial y}\right)^2 u(y_{avg})^2 + \left(\frac{\partial f}{\partial z}\right)^2 u(z_{avg})^2} \tag{13.8}$$

This technique can be generalized to account for as many measured parameters as necessary. When uncertainties from difference sources are added in this way, the result is called the “combined standard uncertainty,”[4] or the “standard uncertainty.”[5]

Consider the function $f(x,y,z) = x^{1/2}y^2\sin(z)$. To illustrate the difference between the derivatives used to calculate uncertainties, considerthe regular (or total) derivative of $f(x,y,z)$ with respect to x, calculated using the product rule for derivatives.

$$\frac{df}{dx} = \frac{y^2\sin(z)}{2x^{1/2}} + x^{1/2}2y\sin(z)\frac{dy}{dx} + x^{1/2}y^2\cos(z)\frac{dz}{dx} \tag{13.9}$$

[4]BIPM, IEC, IFCC, ISO, IUPAC, IUPAP and OIML, “GUM: Guide to the Expression of Uncertainty in Measurement,” Organization for Standardization (2008).

[5]Barry N. Taylor and Chris E. Kuyatt, “Guidelines for Evaluating and Expressing the Uncertainty of NIST Measurement Results,” NIST Technical Note 1297, 1996 edition (National Institute of Standards and Technology, Gaithersburg, Maryland, 1994).

However, in an experiment, x, y, and z are independent variables. Therefore we expect $dy/dx = dz/dx = 0$. For the purposes of calculating the contribution of $u(x_{avg})$ to the uncertainty of f, y and z might as well be constants. The three required derivatives of $f(x,y,z)$ from Equation 13.8 are:

$$\frac{\partial f}{\partial x} = \frac{y^2 \sin(z)}{2x^{1/2}} \qquad \frac{\partial f}{\partial y} = 2x^{1/2} y \sin(z) \qquad \frac{\partial f}{\partial z} = x^{1/2} y^2 \cos(z) \tag{13.10}$$

In practice, taking derivatives can be a lot of work. However, many calculations involve products, which are simplified by starting with the natural logarithm of the calculated quantity. Since

$$\frac{\partial}{\partial x}[\ln(f)] = \frac{1}{f}\frac{\partial f}{\partial x} \quad , \tag{13.11}$$

we can calculate the derivatives we need from the derivatives of the logarithm. Since the logarithm function splits our function into terms with simple (partial) derivatives, they are easy to compute. In our example, $\ln(f) = (1/2)\ln(x) + 2\ln(y) + \ln(\sin z)$, so

$$\begin{aligned}
\frac{\partial}{\partial x}[\ln(f)] &= \frac{\partial}{\partial x}\left[\frac{\ln(x)}{2}\right] = \frac{1}{2x} \\
\frac{\partial}{\partial y}[\ln(f)] &= \frac{\partial}{\partial y}[2\ln(y)] = \frac{2}{y} \\
\frac{\partial}{\partial z}[\ln(f)] &= \frac{\partial}{\partial y}[2\ln(\sin z)] = \frac{\cos z}{\sin z} = \cot(z)
\end{aligned} \tag{13.12}$$

Substituting these partial derivatives into Equation 13.8 yields

$$\frac{u[f(x,y,z)]}{f(x,y,z)} = \sqrt{\left[\frac{u(x_{avg})}{2x_{avg}}\right]^2 + \left[\frac{2u(y_{avg})}{y_{avg}}\right]^2 + [\cot(z_{avg})u(z_{avg})]^2} \tag{13.13}$$

While this expression is not pretty, it is much simpler than the one obtained by substituting the derivatives of Equation 13.10 directly into Equation 13.8. For simplicity, the uncertainty is in Equation 13.13 is expressed as a fraction of the value of $f(x,y,z)$. This is called the "relative uncertainty," or more completely, the "relative combined standard uncertainty."

Using uncertainties to compare measurements or calculations

Suppose you have measured a cart's mass, $m_{F/a}$, from force and acceleration measurements and Newton's Second Law, $F = ma$. To check for systematic errors, you have also measured the cart's mass using an electronic balance, with the result m_{bal}.

A straightforward way to determine whether these two measurements is to compare the discrepancy between the two measurements, say $\Delta = \left|m_{F/a} - m_{bal}\right|$, with the expected uncertainty of Δ, that is $u(\Delta)$. As illustrated in Figure 13.1, the probability of Δ being more than three standard deviations from the mean because of random errors alone is quite small. Therefore, if $\Delta > 3u(\Delta)$ most of the discrepancy is almost certainly due to systematic problems. In this case, we say that the measurements of $m_{F/a}$ and m_{bal} are not consistent.

The ratio between the discrepancy and its combined standard uncertainty is a useful measure of the seriousness of a discrepancy. Because this ratio is similar to the t-statistic of classical statistics, we call it the t'-score.[6] In this example,

$$t' = \frac{\Delta}{u(\Delta)} = \frac{\Delta}{\sqrt{u(m_{F/a})^2 + u(m_{bal})^2}} \tag{13.14}$$

When you compare experimental results and find $t' > 3$, you should carefully review your calculations and measurement procedures for errors. If systematic errors appear to be significant, and you know what they might be, you should describe them in your lab notes. If time permits, repeating a portion of the experiment is in order. Whatever your conclusion, your lab notes must indicate how you estimated your uncertainties.

In the United States, the general authority on the reporting of uncertainties is the National Institute of Standards and Technology.[7] These standards have been developed in consultation with international standards bodies. When the potential consequences of a decision are critical or when the data are unusual in some way, one should consult a statistician.[8]

Determining functional relationships from graphs

Linear relations are simple to identify visually after graphing and are easy to analyze because straight lines are described by simple mathematical functions. It is often instructive to plot quantities with unknown relationships on a graph to determine how they relate to one another. Since data points have not only measurement uncertainties but also plotting uncertainties (especially when drawn by hand), slopes and such should not be determined by using individual data points but by using a "best-fit line" that appears to fit the data most closely as determined visually. If graphing software is used, then the slope of the line can usually be determined by a computer using a "least squares" technique. We won't go into detail about these methods here.

Linear functions $(y = mx + b)$

If x and y are related by a simple linear function such as $y = mx + b$ (where m and b are constants), then a graph of y (on the vertical axis) versus x (on the horizontal axis) will be a straight line whose slope ("rise" over "run") is equal to m and whose y-axis intercept is b. Both m and b can be

[6] N. T. Holmes and D. A. Bonn, "Quantitative comparisons to promote inquiry in the introductory physics lab," Phys. Teach. **53**(7), 352 (2015). DOI: 10.1119/1.4928350

[7] Ibid, Barry N. Taylor and Chris E. Kuyatt.

[8] W. Edwards Deming, *Out of the Crisis* (MIT Press, Cambridge, Massachusetts, 1982). Some authors attribute the ability of Japanese automakers to break into the U.S. market to their skillful application of the principles of statistical quality control popularized by W. Edwards Deming and Joseph Juran.

determined once the graph is made and the "best-fit" line through the data is drawn. if $x = 0$ does not appear on your graph, b can be found by determining m and finding a point (x, y) lying on the "best-fit" line; then equation $y = mx + b$ can be solved for b.

Simple power functions ($y = ax^n$)

In nature we often find that quantities are related by simple power functions with $n = \pm 0.5, \pm 1, \pm 1.5, \pm 2$, etc., where a is a constant. Except for $n = +1$, making a simple graph of y (vertical axis) and x (horizontal axis) for simple power functions will yield a curved line rather than a straight line. From the curve it is difficult to determine what the actual functional dependence is. Fortunately it is possible to plot simple power functions in such a way that they become linear.

Starting with the equation $y = ax^n$, we take the natural logarithm of each side to show

$$\ln(y) = \ln(ax^n) = \ln(a) + \ln(x^n) = \ln(a) + n\ln(x) \tag{13.15}$$

If $\ln(y)$ is plotted on the vertical axis of a graph with $\ln(x)$ plotted on the horizontal axis (This is often called a doubly logarithmic, or log-log graph.), then Equation 13.15 leads us to expect that the result is a straight line with a slope equal to n and a vertical axis intercept equal to $\ln(a)$. If the relationship between y and x is a simple power law function, then a graph of $\ln(y)$ as a function of $\ln(x)$ will be linear, where the slope is n, the power of x, and the intercept is the natural logarithm of the coefficient a. This is quite useful, because it is easy to determine whether a graph is linear. If we suspect a simple power function relationship between two quantities, we can make a log-log graph. If the graph turns out to be linear, then we are correct in thinking that it should be a simple power function and can characterize the relationship by finding values for n and a.

Exponential functions ($y = ae^{bx}$)

Radioactive decay, the temperature of a hot object as it cools, and chemical reaction rates are often exponential in character. However, plotting a simple graph of y (on the vertical axis) and x (on the horizontal axis) does not generate a straight line and therefore will not be readily recognizable. A simple graphical method remedies this problem. Starting with an equation for the exponential function, ($y = ae^{bx}$). We can take the natural logarithm of each side to show

$$\ln(y) = \ln(ae^{bx}) = \ln(a) + \ln(e^{bx}) = \ln(a) + bx \tag{13.16}$$

If $\ln(y)$ is plotted on the vertical axis and x is plotted on the horizontal axis (This is called a semi-log graph.), Equation 13.16 takes the form of a straight line with a slope equal to b and a vertical axis intercept equal to $\ln(a)$. Thus any relationship between two variables of this simple exponential form will appear as a straight line on a semi-log graph. We can test functions to check whether they are exponential by making a semi-log graph and seeing whether it is a straight line when plotted this way. If so, the values of a and b that characterize the relationship can be found.

Using error bars to indicate uncertainties on a graph

When plotting points (x, y) with known uncertainties on a graph, we plot the average, or mean, value of each point and indicate its uncertainty by means of "error bars." If for example the uncertainty is primarily in the y quantity, we indicate the upper limit of expected values by drawing a bar at a position y_{max} above y_{avg}, that is, at position $y_{max} = y_{avg} + u(y_{avg})$. Similarly, we indicate the lower limit of expected values by drawing a bar at position $y_{min} = y_{avg} - u(y_{avg})$. Figure 13.3 shows how the upper error bar at y_{max} and the lower error bar at y_{min} are plotted. If the quantity x also has significant uncertainty, one adds horizontal error bars (a vertical error bar rotated 90°) with the rightmost error bar at position x_{max} and the leftmost error bar at position x_{min}.

Occasionally one encounters systems where the upper and lower error bars have different lengths. In this case, the upper uncertainty, $u_+(y_{avg})$ does not equal the lower uncertainty, $u_-(y_{avg})$.

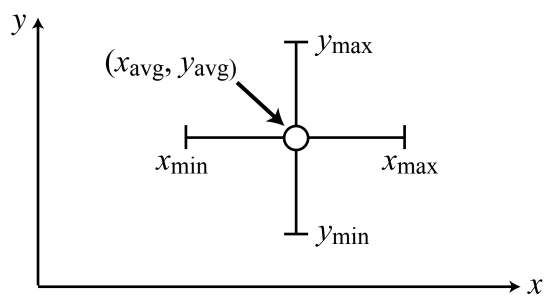


Figure 13.3. Diagram of error bars showing uncertainties in the value of the x- and y-coordinates for point (x_{avg}, y_{avg}) When you print a graph in lab, the labels are omitted.

Excel Spreadsheets and Graphs

Spreadsheets are useful for making tables and graphs and for doing repeated calculations on a set of data. A blank spreadsheet consists of a number of cells (just blank spaces surrounded by lines to make a little "box"). The cell rows are labeled with numbers while the columns are labeled with letters of the alphabet. Thus Cell A6 is the "box" in Row 6 of Column A, which is the first column. Text, numbers, and formulas of various kinds can be entered in each cell.

Tables

Making a table of, say, the force exerted by a spring as its length is changed requires entering the force values in the cells of one column and the length values in the corresponding rows of an adjacent column. Adding some explanatory text in the cells above each column can complete the table. It is sometimes useful or necessary to adjust row heights and/or column widths to accommodate more or less "stuff" in the cells. Clicking on "Help" in the main toolbar at the top of the screen opens a small window where you can type in your question. In this case type in the words "column width" (without the quotation marks) and click on "Search." Several options will be displayed, including "Changing column width and row height." Click on it and get detailed instructions how to make the desired changes. Don't be afraid to use the help screens in Excel. Most of the time you can find answers to your questions fairly quickly.

Graphs

To make a graph in Excel, first select the data to be graphed by clicking on the upper-left cell of the x-data and dragging the cursor down to the lower-right cell of the y-data. A box should appear around your data and the selected cells will change color. Then select the Insert tab on the main toolbar, click on the Scatter icon, and select the "Scatter with Only Markers" icon from the pull down menu that appears. This icon appears first in the list and shows dots for data points, with no lines joining them. This choice is almost always the best choice for the graphs we make in lab. A graph of the data should appear on the worksheet. In addition, the "Chart Tools" ribbon should appear in the main toolbar. (If your x-values are not adjacent to your y-values, you will need to use the "Select Data" option to add data points to your blank graph. This option appears in the "Chart Tools" ribbon after clicking on the graph.)

If you do something unwanted, immediately stop the operation and click on "Undo" icon near the top-left corner of the Excel window. This icon is a blue arrow that curves to the left. Usually you can escape your predicament and try again.

Now you can add a descriptive title ("Graph 1" or "Exercise 1" is not sufficiently descriptive) to the graph and label the quantities (with their units!) plotted on the horizontal and vertical axes. Clicking on the "Layout" tab in the Chart Tools ribbon at the top of the Excel window will bring up icons labeled "Chart Title" and "Axis Labels", among others. For the chart title, select the "Above Chart" option. A text box for the title will appear. Move your cursor to the text box and type your title. To label the horizontal axis, move your cursor to the "Axis Labels" icon and choose the "Title Below Axis" option for the "Primary Horizontal Axis Title". To label the vertical axis, choose the "Rotated Title" option for the "Primary Vertical Axis Title." In each case, a text box will appear in which you can type the axis label with units. For instance, if a cart velocity is plotted along the *y*-axis, you would want a label like "Velocity (m/s)". The velocity units should be indicated parentheses after the main label.

You may wish to add other features to your graph, such as legends, gridlines, best-fit curves to match the plotted data, different axis labels, etc. Even the size and aspect ratio of the graph can be changed. Some of these options appear when you right-click on an axis. Others can be accessed from icons under the Design, Layout, and Format tabs in the Chart Tools ribbon. Your best approach is to do some exploring. Only a few of the options will likely be useful to you on a regular basis, but you need to find where they are.

When you print a graph, don't print the whole spreadsheet. Move the cursor over the graph and click it to highlight the graph. Then using the "Print" command in the drop-down menu under the "File" tab on the main toolbar will print just the graph. Selecting "Landscape Orientation" under "Settings" will make the graph as large as possible while still fitting on one page. Graphs printed for you lab notes should be printed in the landscape orientation. Excel will display a preview that shows exactly how the graph will appear on the paper when it is printed. Make any necessary adjustments, then print the graph by clicking on the printer icon in the top-left hand corner of the print window.

Making calculations on a set of data

For example let us say that you have data values in Cells A1 and B1 and you wish to take the product of these two numbers and put the result in Cell C1. In Cell C1 type =A1*B1 (the * symbol indicates multiplication). The "equal" sign tells Excel that a formula is to follow. When you hit "enter," the calculation will be performed and the product displayed in Cell C1. The formula for calculating the number in the cell is still present but hidden behind the number in a sense. If you now change the number in Cell A1, as soon as you enter it, the number in Cell C1 will also change as it re-computes the product with the new number. Suppose that we have one set of numbers in Column A, Rows 1–10, another set of numbers in Column B, Rows 1–10, and that we want to calculate the following products, A1*B1, A2*B2, ..., A10*B10. After typing the product formula into Cell C1, we can click on Cell C1, making a dark outline appear around it. Move the cursor to the bottom right corner of Cell C1 until the cursor morphs into a little + sign. Click and drag down to Cell C10 copying the product formula to successive cells along the way. When you release the click button, the desired products should be displayed in Column C, Rows 1–10.

The symbols used for various mathematical functions are:

* = multiplication / = divide

+ = addition

- = subtraction

ˆ = powers (need not be integer values)

Use parentheses to make it perfectly clear to Excel what you want to do. The formula =A1+B1/C1 is computed as =A1+(B1/C1). If you wish to sum A1 and B1, then divide by C1, you need to write it as =(A1+B1)/C1. The operations of multiplying, dividing, and taking powers are done first before adding and subtracting.

Some other useful functions in Excel are:

SUM(A1:A9) = sums the numbers in Cells A1–A9.

AVERAGE(A1:A9) = calculates the average (mean) of the numbers in Cells A1–A9.

STDEV(A1:A9) = calculates the standard deviation, $\sigma(x)$, of the numbers in Cells A1–A9.

SIN(A3) = assumes that A3 is in radians and calculates the sine of the angle.

COS(A3) = assumes that A3 is in radians and calculates the cosine of the angle.

TAN(A3) = assumes that A3 is in radians and calculates the tangent of the angle.

ASIN(A6) = calculates the angle in radians whose sine is the number in Cell A6.

ACOS(A6) = calculates the angle in radians whose cosine is the number in Cell A6.

ATAN(A6) = calculates the angle in radians whose tangent is the number in Cell A6.

SQRT(A11) = square root of the number in A11.

LN(A7) = natural logarithm of the number in A7.

Note: These functions must be preceded by the "equal" sign in order to be treated as a formula and do a calculation. For example, =SQRT(B9) typed into Cell C12 will calculate the square root of the number in cell B9 and record it in Cell C12. If the functions are part of a more complicated formula, then only the leading "equal" sign is required. For example, =A2+SIN(A4) typed in Cell B8 will add the number in Cell A2 and the sine of the number in Cell A4 and record it in Cell B8.

Fitting data with straight lines–only if the data are linear!

Often in physics the dependence of one variable on another is characterized by a linear relationship, meaning that the variables are related to one another through the equation of a straight line of the form $y = mx + b$, with m being the slope and b the y-intercept of the graph. The slope and intercept often can be quantities of interest. When several data points, (x, y), are related linearly, how can we calculate the best values of the slope and intercept of the relationship? "Least squares" methods minimize the sum of the squares of the deviations of the fitted line from each of the data points and thus give the "best" values for the slope and intercept of the line.

Excel is capable of doing these kinds of fits quite easily. If you have a graph that appears to be quite linear and thus suitable for fitting with a straight line, you can add a “Trendline” to the graph by moving the cursor over the symbol for one data point on the graph and right clicking on it. A drop-down menu should appear with “Add Trendline” as one of the options. Click on it and choose “Linear”. In the same small window click on the “Options” tab near the top and mark the little box for “Display equation on chart.” Clicking on “OK” will display the “best-fit” line on the graph and give the equation of the line as well on the graph. You can move the equation with your cursor by clicking and dragging if it obscures some of the data points. You can also add or subtract digits of precision to the numbers given for the slope and intercept by right clicking on the equation after highlighting it with the cursor. In spite of its applications in other disciplines, the *R* or *R*-squared value is seldom useful in the physical sciences and should not be displayed on the graph.

Finding the “standard error” (basically the standard deviation of the mean) for the slope and intercept values, respectively, is also important, because it gives information regarding how precisely we know the slope and intercept values. Excel can do this using the more advanced Regression feature of least-squares fitting. (In OpenOffice and LibreOffice, the LINEST function performs the same regression.) In Excel, the following steps are required:

1. Click on the “Data” tab in the Chart Tools ribbon and click on the “Data Analysis” icon in the “Analysis” group on the right.

2. In the pull-down menu that appears, scroll down to the “Regression” option and click on it to highlight it. After choosing OK, the Regression window should appear.

3. To input the y-values in the first blank text box, move the cursor to the box and click in it. Now move the cursor to the top of the y-data column in your spreadsheet and click and drag down to select the whole set of y-values. The corresponding cell numbers should appear in the y-value box in the Regression window. Now move the cursor to the box for inputting the x-values in the Regression window. Click and drag over the column of x-values in your spreadsheet and these cell numbers should appear in the x-value box in the Regression window.

4. In the Regression window under “Output options” mark the circle for “Output range.” Move the cursor into the blank space just to the right of “Output range” and click it.

5. Now move the cursor to an empty cell in the leftmost column of your spreadsheet near the bottom and click it. The corresponding cell number will appear in the box. This tells Excel where to put the results of the regression analysis.

6. Now you are ready to click OK in the Regression window. Excel will do the appropriate calculations and display them below and to the right of the cell that you chose for the Output range. The values of interest are displayed in the lower-left corner of the stuff displayed, just to the right of labels, “Intercept” and “X Variable.” The first column to the right of the word “Intercept” shows the value of the y-intercept. This value should equal the value in the trendline equation on the graph—a nice check! The next column to the right shows the “standard error”, or uncertainty of the intercept value. In other words, the intercept will have a plus/minus uncertainty given by this standard error. Similarly the first column to the right of “X Variable” shows the value of the slope (which should equal the slope in the trendline

equation) and the next column shows the plus/minus uncertainty of the slope value. How does Excel get from X Variable to slope? If you look carefully at the regression output, Excel is calling "slope" the coefficient of the X Variable, which is true in the equation of a straight line. A little awkward, but it works.

It is important to avoid fitting a straight line to data that is definitely curved. In this case, your eye is telling you that your model does not fit the data. Such fits are misleading at best. It is often acceptable to select part of your data that does appear to lie on a straight line and fit those points to a straight line.

Formal Lab Report Instructions

The following eight pages of instructions are formatted like your formal lab report. The format is deliberately plain to the point of being ugly. Reports generally undergo substantial revision after submission and before approval no matter where you work. The double spaced text allows room for edits. A uniform, plain look encourages the editor to focus on the presented information and logic. It is also designed to fit smoothly into the institution's publishing workflow.

If you intend to include your report in your Junior Writing Portfolio, please follow the instructions with care. Avoid copying fragments of text from the lab manual or other sources—especially material from the goals and introduction sections. Those who evaluate the physics submissions often have experience as physics teaching assistants, and are likely to identify the material as plagiarized.

Formal Lab Report Instructions—Title of Lab Here

Authors names here. You will be the first author, with your lab partner's name following.
Author address(es) here. Write your Course and Lab Section Numbers here in lieu of an address.

Put your abstract on the first page. Do not label it "Abstract." It will be obvious that it is an abstract. The abstract is a brief summary of your report, including your results and conclusions. Normally, the length of your abstract should be about 5% the length of your report. Your abstract should make it implicitly clear what your report includes and what it omits. By implicit, I mean that you don't say, "This report includes *x* and omits *y*." You don't even say that it is a report. A good summary of your results and conclusions will do the job implicitly. Readers use these summaries to decide whether to read your report. If they notice discrepancies between your abstract and what actually appears in your report, they feel cheated.

1. Introduction

What to include in the introduction. Like the abstract, your introduction should describe the subject of your paper. An introduction is longer than an abstract, so the subject is described in more detail. One includes the purpose of the experiment and describes its scope. For instance, you will often specify which parameters were explored and how much they were varied. Any background information that the reader needs to understand the rest of your introduction is also included. For the purposes of this exercise, assume that your reader is an introductory physics student like yourself who has not performed this particular experiment.

The introduction is usually written after the main body of your report is complete. Paradoxically, it is seldom clear exactly what you are going to write until you actually write it.

Characteristics of good technical writing.[1] Good technical English is unified, coherent, clear and concise. Unity is achieved by enforcing a theme to the paper as a whole. Subjects that are not encompassed by the theme should be left out. The *choice* of theme is critical to the success of the writing operation. The theme is not stated explicitly. (Don't write, "The theme of this paper is…") The abstract of the work should make it clear what belongs in the report and what does not. A good abstract helps the author maintain unity. Ideally, your theme should include everything you intend to write and exclude everything else. After writing the abstract, you may decide to add or delete material as appropriate to make the report a unified whole.

Coherence is achieved by providing logical transitions between the parts of your paper. The order of topics in your paper has a major effect on coherence. If you find yourself repeating ideas in different parts of the paper, you may have failed to order your topics appropriately. Cause and effect is a major part of technical writing. Be sure you state cause and effect relation-ships clearly.

Clarity is achieved by removing potential sources of ambiguity. Avoid text that can be interpreted inappropriately. In general, your statements should be as specific as possible. The goal is to communicate as much information as possible. Do not hide information that should be available to the reader.

Conciseness is generally achieved by good editing. All other things being equal, you should use as few words as necessary to communicate what you have to say. Sentences that start with “There are” or “It is” can often be shortened by making an appropriate noun the subject of the sentence. This often resolves unintended ambiguities as to what “it” refers to. Verbs and adjectives with more specific meanings can shorten sentences and improve readability. Active verbs are better than passive verbs unless they shift your focus inappropriately. Your report is not about you. Similarly, do not write about your report in your report. Focus on your subject.

Formatting technical reports is mostly a matter of achieving conformity. Creative formats are not rewarded. (The nail that sticks up gets beaten down.) Your reader must focus on the content of your work, not the details of presentation. Any deviation from standard formatting must be well justified as an improvement (more clear, more concise, etc.). Although these standards are to a large degree arbitrary, many are related to the need for good-looking copy when reproduced. For the purposes of this assignment, the formatting requirements will follow those of the American Institute of Physics.[2]

With the exception of figures and equations, this assignment must be printed from a word processor. Use a 12 point serif font, preferably Times. (A point is about 1/72 of an inch. In this context, the font size refers to the intended spacing between single-spaced lines, not the size of the letters themselves.) Set the line spacing to exactly 24 points (not double-spaced), with no extra space before or after paragraphs. (Exceptions include lines with equations or figures, which often need more room. These usually need to be single-spaced, that is, with a line space of “at least 12 points”. Lines with equations and figures should be the only lines in your paper that are single-spaced.) Use one-inch margins on all four sides of the paper. For regular paragraphs, justify the text along both left and right margins. The first line of every paragraph should be indented 0.5 inches. Disable automatic formatting options like “format the next paragraph like the one before it." They often cause formatting faults and complicate inserting equations and figures.

Use a spell checker, but keep a dictionary at hand for unusual words.

2. Experiment

Title this section Experiment, not Experimental. Titles normally function as nouns, and "Experimental" is not appropriate in this role. Put an extra line break (24 point) before and after numbered headings (as well as figures and equations).

What to include in the experiment section. The experiment section should not include all the procedures that appear in your lab notes. Assume that your reader is familiar with the equipment. Omit most of the information that would normally be found in equipment manuals. Do include the manufacturer's name and model numbers of any equipment with special features that might not be easily duplicated. Also include any details that might be necessary to the replication of the experiment but would not be clear from reading the manuals. For instance, it is often important to include sample rates for data collection, but not important to specify the units employed when acquiring data.

The experiment section is not the best place to describe some experimental details. Details that apply to only one section of the results can (and usually should) be included in the appropriate part of the results section. This reduces the strain on the reader's memory and eliminates the temptation to repeat these details unnecessarily. Details that apply to more than one section of the results are generally included in an experiment section. The goal is to avoid repetition, not to collect all the experimental details in one place.

Equations and math. Any equations in your paper should be numbered in sequence on the right hand margin. The number should be in parentheses. Position the equation itself near the middle of the page (left and right). In a word processor, this is achieved by right-justifying the line to position the equation number [(1), (2), etc.] on the right hand margin, then inserting tabs to center the equation. Equations are normally type-set using an equation editor. If necessary, hand-write your equations. Computer type-set equations must generally be inserted into lines that are single-spaced. For instance, the magnitude of the gravitational force of the earth on the moon, $|F_{EonM}|$, can be calculated using the equation:[3]

$$|F_{EonM}| = \frac{Gm_E m_M}{R_{EM}^2} \quad , \tag{1}$$

where G is the Universal Gravitational Constant (6.67 × 10^{-11} N-m^2/kg^2), m_E is the mass of the earth (5.97 × 10^{24} kg), m_M is the mass of the moon (7.35 × 10^{22} kg), and R_{EM} is the distance between the earth and the moon (average 3.84 × 10^8 m).[4] In text, symbols for variables and constants are italicized. If experimental uncertainties are available, specify them as well (for example, 1.01 ± 0.01 g).

All mathematical variables must be defined in the text immediately before or after the first time they are used, except for numbers like π and *e*. (Similarly, acronyms must be defined the first time they are used.) If you define a variable in Equation (1), and the same variable is used in Equation (2), use the same symbol in both equations and define this variable only once, with Equation (1). In the text that follows an equation, refer to it as Equation (1) or Equation (2), etc. Equation can be abbreviated "Eq.", except at the beginning of a sentence. Do not abbreviate the first word of a sentence. The first word of each sentence should be completely spelled out.

3. Results and Discussion

Descriptive titles. Papers of modest length do not need numbered subsections. Subheadings are useful. Mark a new subsection by placing a bold title at the beginning of the first paragraph of that section. Do not include the exercise number. Readers quickly loose count.

Short *descriptive* titles are a great help. When sections become longer than a few double-spaced pages, numbered subsections are appropriate.

For emphasis, use *italic*, not **bold** or underlined characters.

What to include in the Results and Discussion section. The general principle is to include all the data needed to support your conclusions, with enough discussion to convince the reader of the truth of your conclusions. If some of your data can be interpreted in more than one way, for instance, you will want to present data and/or explanations that support your interpretation. Although we must structure the labs so that they make maximum use of your data to teach physics, your report should be more focused. Everything needed to support your conclusions must be included, and everything that is not related to those conclusions must be excluded. A great deal depends upon what you choose to conclude. Conclusions that are overly broad or too narrow can ruin your entire report. Consider your conclusions carefully.

Unless each data point is of special interest to the reader, tables of data are generally inappropriate. (Data tables are important in your lab notes.) If you need to display your data, use a format that communicates not only the data, but any important relationships. In most cases, figures are the best way to display data. If a table is necessary, they should be numbered with Roman numerals (Table I, Table II, etc.) and provided with descriptive titles. Double lines run across the top and bottom, and a single line separates the column headings from the data. No other lines should appear. A properly formatted example appears below as Table I.

Table I. Power loss versus frequency.

Frequency (Hz)	Power Loss (W)
10	0.24
100	1.75
1000	0.68

Figure formats. In your lab notes, your figures generally should be as large as possible. You may, for instance, want to add handwritten notes or slope calculations. In a formal report, you want them to fit comfortably with the text. Figures are normally less than 3.2 inches across, including all labels. If a figure has many parts that can be arranged in two columns, you can double the width. Large figures must fit on a single page with their captions while maintaining the normal one-inch margins. In reports, titles are optional, but captions are mandatory. Figure labels should normally use the same font as the text or a sans serif font like Arial. The figure and caption must contain sufficient information so that the reader does not need to refer to the text to understand what is being presented. Figure captions start with a phrase that serves as a title. This introductory phrase is not a complete sentence. The text that follows consists of complete sentences. Captions should not normally include a discussion of the data. The implications of your data should appear in the text.

All figures must be described in the text. Figures must appear as soon as possible *after* they are mentioned in the text. The word Figure may be abbreviated (Fig.) in the middle of a sentence, but never at the beginning.

If possible, embed your figures as high resolution bitmap files—at least 300 dots per inch (dpi). TIF files (Tagged Image Files) are compatible with many word processes. To ensure that your graphics files are readable, all lines should be at least 1 point (about 0.014 inch) thick. The smallest letter (including superscripts and subscripts) should be at least 1 mm high. This rules out most superscript and subscript fonts unless you can manually control the size. Do not use open symbols (○) for data points; always use closed symbols (●). Remove all grids and backgrounds. (The background should be transparent.) Center your figure left and right on the page on a single-spaced line. Not long ago, figures were traced by hand for publication. You may trace your figures and label them neatly by hand, if the size requirements are met.

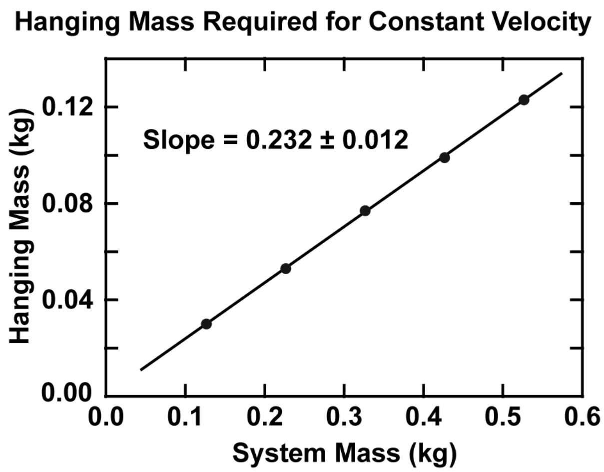


Fig. 1. Hanging mass required to move a pine block across a clean aluminum surface at a constant velocity of 0.2–0.3 m/s as a function system mass (the sum of the mass of the block and any added masses). The slope of the graph corresponds to the kinetic coefficient of friction.[5]

Discussion Section. Relatively short papers do not need a separate discussion section. (Section 3 is then a "Results and Discussion" section, as above.) Otherwise, the third section is titled "**3. Results**" and the fourth section is titled "**4. Discussion**." In your report, a separate discussion section is probably not warranted. Discuss your results as they are presented in the results section.

Generally, it is inappropriate to answer questions for further discussion in a formal lab report. Although these questions are designed to help you learn, your report must be more focused. You should include everything necessary to support your conclusions and nothing more. If the answers form a logical part of your report, and you have data to back them up, they are probably appropriate. If this is true, it would be superfluous to have a subsection entitled, "Questions for further discussion." You would need to provide other, more descriptive, titles.

Traceability. As a rule, formal reports do not contain the details needed to fully verify whether your conclusions are valid. The reader will assume a reasonable level of competence on the part of the authors. If questions arise, the reader will need access to your lab notes. Do not put anything in your report that is not supported in your lab notes. Your lab notes must in general be recorded at the same time the work was performed. That is, notes about experiment details must be made during the course of the experiment. Notes about data analysis must be made when you analyze the data. Notes about conclusions should be made when you are prepared to conclude. Notes made after the fact are not reliable records. Turn in your lab notes along with your lab report. You teaching assistant should be able to support your conclusions from your report's Results and Discussion section, which in turn must be supported by the data in your lab notes. This is called traceability.

4. Conclusion

Never conclude anything you don't discuss. Do not discuss anything that does not relate to your results. Raising new issues in the conclusion is a bad idea. Conclusions must be supported by your work, not merely be related to it. Medical misinformation, for instance, is often presented in conclusions that are not supported in the rest of the report.[6] As noted above, your conclusions determine what you choose to put in the rest of your report.

Acknowledgments

Acknowledgments are optional. If someone or some organization has supported your work financially or provided significant assistance, say so here. Example:

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[5] Charles A. Maney, "Experimental study of sliding friction," Am. J. Phys. **20**(4), 203-210 (1952).

[6] J. P. A. Ioannidis, "Why most published research findings are false," PLoS Med **2**(8), e124. (2005) doi:10.1371/journal.pmed.0020124.

[7] Some publications permit paragraph-length notes in the references section. Few publications require article titles in their citation lists, but they help the reader. Please include them. If you do not know the official abbreviation of a journal title, write out the entire journal title. Article and section titles are enclosed in quotes. Book titles are italic. Journal volume numbers are bold. Journal issue numbers, if provided, should be in parentheses. Page numbers follow, with the year of publication in parentheses. Normally one does not cite URL's (Uniform Resource Locators for web-based material) unless the links are permanent. To address this problem, many publishers provide each article with a Digital Object Identifier (DOI). One can often locate an article on the web by searching on its DOI at http://www.doi.org. If you know the DOI, provide it. Do not cite unpublished work unless absolutely necessary.