Lab 10. AC Circuits

Goals

• To show that AC voltages cannot generally be added without accounting for their phase relationships. That is, one must account for how they vary in time with respect to one another.

• To understand the use of “root mean square” (rms) voltages and currents.

• To learn how to view and interpret AC voltage and current waveforms using the “scope” function of the Capstone software.

• To learn how to measure the phase between sinusoidal voltage waves displayed with Capstone.

• To understand how to use phasor diagrams (analogous to diagrams of vector addition) as a technique for adding AC voltages or currents with various phases.

• To observe electrical resonance (analogous to mechanical resonance in a vibrating string) in an RLC circuit.

Introduction

While DC (direct current) circuits employ constant voltages and currents, AC (alternating current) circuits employ sinusoidally varying voltages and currents. It may seem strange that sinusoidal quantities should be so common, but rotating devices generate most of the electrical power in the world. In a natural way, this produces voltages and currents that vary in a sinusoidal fashion.

A complicating factor in AC circuits is that inductors and capacitors introduce phase shifts. That is, the voltages across some components can peak well before or after the currents flowing through them. At any one instant in time, the voltage across each component in a series circuit will indeed sum to zero—but the voltage peak for each component (proportional to the amplitude) will be reached at different times. Under these conditions, the sum of the voltage amplitudes in a circuit containing inductors and capacitors will not in general be zero—in apparent violation of Kirchhoff’s Voltage or Loop Rule. In this experiment you will explore the relationships between voltages and currents for inductors, capacitors, and resistors. This will include determining their phase relationships and how they depend on frequency. For this study, we consider a simple circuit
consisting of a resistor, a capacitor, and an inductor connected in series with a sinusoidal voltage source.

**A brief review of theory**

A diagram of a typical RLC circuit is shown in Figure 10.1. Normally the current (which must be equal at all points along a series circuit) is used as a reference signal in AC circuits. Although the current flows back and forth, one direction is designated the positive direction. This defines the direction of positive voltage differences as well. The positive end of each component in Figure 10.1 is marked. Note that the positive end of the power supply is chosen so that positive current flows out of it. This is consistent with the convention for batteries and DC power supplies.

![Figure 10.1. Diagram of a resistor, inductor, and capacitor connected in series.](image)

**Potential differences for RLC circuit**

With a DC power supply, the sum of the voltages across the inductor $V_L$, the resistor, $V_R$, and the capacitor, $V_C$ in Figure 10.1 would equal the output of the power supply, $V_{Out}$. That is

$$V_L + V_R + V_C = V_{Out} \quad (10.1)$$

When the voltages are changing in time, we still expect that the sum of the voltage drops across the three components $V_L + V_R + V_C$ equals the output voltage $V_{Out}$ at each instant of time. When the output of the power supply is sinusoidal, the steady state voltages across each of the components will also be sinusoidal. However, each voltage in the circuit will have its own phase. That is

$$V_{L(0-p)} \cos(\omega t + \phi_0) + V_{R(0-p)} \cos(\omega t + \phi_R) + V_{C(0-p)} \cos(\omega t + \phi_C) = V_{Out(0-p)} \cos(\omega t + \phi_{Out}) \quad (10.2)$$
where the \((0-p)\) subscript in \(V_L(0-p)\) and the other voltage amplitudes refers to their “zero-to-peak” values. When multiplied by the proper sine or cosine function, the zero-to-peak amplitude gives the actual measured value of voltage as a function of time. The non-zero phase angles, denoted by \(\phi\) in Equation 10.2, complicate the analysis of AC circuits.

The phase of the potential difference across a capacitor

Capacitors are essentially two conducting sheets or plates separated by some insulating material that may include air or a vacuum. When a voltage is applied between the two plates of the capacitor, charge is transferred from one plate to the other. Thus a current flows through the voltage source and the connecting wires. As the voltage increases and more charge collects on the plates, adding more charge becomes increasingly more difficult, because like charges repel. Therefore the current flowing into the capacitor is greatest when the plates begin to charge. The current drops to zero when the charge build-up reaches a maximum. If a sinusoidally varying voltage source (one that oscillates positively and negatively in time with the shape of a sine function), is connected across the capacitor, the voltage across the capacitor “lags” the current by 90° in phase, meaning that the voltage peaks occur one-fourth of an oscillation period after the current peaks. This relationship is illustrated in Figure 10.2, where the voltage across the resistor \((V_R)\) shows the variation of current during a single cycle. The oscillation period of the signal in Figure 10.2 is 1 s, and the voltage across the capacitor \((V_C)\) peaks 0.25 s (one-fourth of a cycle) after the peak in \(V_R\). We say that the phase of the voltage across an ideal capacitor is shifted 90° \((360°/4)\) relative to the current. The sign is chosen so that if \(I\) and \(V_R\) are proportion to \(\cos(\omega t)\), \(V_C\) is proportional to \(\cos(\omega t + \phi)\), where \(\phi = -90°\).

![Figure 10.2](image-url)

Figure 10.2. Voltages across an ideal induction, an ideal resistor, and an ideal capacitor in an RLC circuit. The times as which the three voltages cross zero (during the falling portion of the cycle) are labeled \(t(V_L = 0-)\), \(t(V_R = 0-)\), and \(t(V_C = 0-)\), respectively. The components in your experiment are not ideal, so the phases will be different.
The phase of the potential difference across an inductor

An inductor is usually takes the form of a coil of wire with many loops. When a time-varying electrical current passes through the loops, the resulting time-varying magnetic field induces a voltage in the coil. According to Lenz’s law (and energy conservation) this induced voltage opposes the source voltage, making the current small. When sinusoidally driven, the voltage across and ideal inductor peaks one-fourth of an oscillation period before the current peaks. That is, the voltage “leads” the current by 90° in an ideal inductor. We say that the voltage experiences a +90° phase shift relative to the current in an ideal inductor. This relationship is illustrated in Figure 10.2, where the voltage across the inductor, \( V_L \), peaks 0.25 s (one-fourth of a cycle) before the peak in \( V_R \). Again, the sign is chosen so that if \( I \) and \( V_R \) are proportion to \( \cos(\omega t) \), \( V_L \) is proportional to \( \cos(\omega t + \phi) \), where \( \phi = +90° \).

In practice, it is difficult to determine the position of the peak in a sinusoidal signal precisely, because voltage changes slowly near the peak. Measuring the time at which the voltage crosses zero, where the voltage changes rapidly, gives more precise results. Because the voltage crosses zero twice per cycle, it is important to be consistent about which zero crossing is used. The arrows in Figure 10.2 show the zero crossings for \( V_L \), \( V_R \), and \( V_C \) where the voltage is falling, that is, where the voltage crosses zero from above.

To derive an equation for the phase angle \( \phi \) for a given voltage signal, one observed that 360° of phase corresponds to one oscillation period \( T \),

\[
\phi = \frac{[t(V_R = 0) - t(V = 0)] \times 360°}{T} \tag{10.3}
\]

The order of terms in Equation 10.3 is chosen so that a voltage signal that lags \( V_R \) has a negative phase, as required by the sine and cosine functions.

Using phasors to represent AC voltages

The AC voltages across an AC power supply, an inductor, a capacitor, and a resistor, all connected in series, can be added much like vectors. The length of each vector, or phasor, represents the measured voltage amplitude of the corresponding circuit element, and the angle between each phasor and the resistor phasor (which points in the same orientation as the current phasor) equals the phase difference between the AC voltage across that circuit element and the AC voltage across the resistor. These relationships are illustrated in Figure 10.3. To represent the time-varying voltages in an AC circuit, all four phasors are rotated at angular velocity of \( \omega t \). The measured voltage across each circuit element at time \( t \) is equal to the horizontal component of that element’s phasor at that time.

Phasors are used to represent the various time-varying voltages in more complex AC circuits. They are also used to represent the addition of other quantities that vary sinusoidally in time. For instance, the electric fields in monochromatic electromagnetic waves (laser beams) vary sinusoidally in time. Phasors are often used to account for phase differences in single-slit diffraction.
Figure 10.3. Phasor diagram of the voltages across an inductor, a resistor, a capacitor, and the output in an RLC circuit. The current phasor is not shown, but is proportional to the resistor’s phasor. The phases $\Phi_L$, $\Phi_C$, and $\Phi_\varepsilon$ are measured with respect to the phase of the voltage across the resistor (or equivalently, the phase of the current signal). The measured voltage across each component is equal to the projection of its vector onto the horizontal axis. As a function of time, each vector rotates about the origin with angular velocity $\omega t$.

Expressing AC voltages in terms of their root-mean-square (rms) values

AC voltages are often expressed in terms of their root mean square (abbreviated rms) values. In DC circuits the product of the current and voltage gives the power. It is convenient to use a similar formula for the average power dissipated in AC circuits when the current and voltage are in phase. However, the product of the raw voltage and current amplitudes (the zero-to-peak voltages and zero-to-peak currents), is twice the actual average power. To correct for this, we use rms voltages and currents. The rms voltage is the zero-to-peak voltage divided by $\sqrt{2}$, and the rms current is the zero-to-peak current divided by $\sqrt{2}$. When these are multiplied, the factor of 2 in the denominator yields the correct average power. (This procedure yields the average power only when the voltage and current have the same phase.) Most AC voltmeters and ammeters display rms volts and rms amps, respectively. The voltage at a wall plug in the United States is 120 V rms. The corresponding zero-to-peak voltage is about 170 V.

Equipment set up

The Pasco Scientific RLC Circuit (Model CI-6512) is already configured with a series combination of resistor, inductor, and capacitor. Choose the 10 $\Omega$ resistor, the 8.2 mH inductor, and the 100 $\mu F$ capacitor. They are already connected in series. (You can see the connections on the bottom of the circuit board.) The analog inputs of the interface unit (Channels A, B and C) can be employed to measure the voltage difference across each of the three components using the three patch cords supplied with the circuit board.
Since switching the red and black leads across a component reverses the sign of the detected voltage difference, it is important to connect the red and black ends of each patch cord to the three components in a consistent fashion. This requires that you define one current direction to be positive, and use this direction to identify the positive end of each component. The positive end of each component is labeled (+) in Figure 1 for the choice of positive direction shown in the figure. Attach the red lead of the patch cord for the resistor, for instance, to the positive end of the resistor, and the black lead to the negative end of the resistor. Attach the patch cords used to measure the voltage differences across the inductor and the capacitor in the same fashion, being careful of sign.

To take the data, you will need to tell Capstone that you want to connect voltage sensors to Channels A, B, and C, and that you wish to use the output from the interface unit as the voltage source for the circuit. The output jacks are to the right of Channel C. You will need to add a “scope” display so all of this can be viewed. Then you can drag and drop the voltages for Channels A, B, and C. Capstone automatically records the output voltage, $V_{out}$, and current, $I$. Drag and drop the icons for these signals to the same scope display. You want to show all five signals on the same display. Your TA can be helpful here.

If a waveform appears choppy, like a series of connected straight lines, you probably need to increase the data sampling rate. For best results, the sampling rate should be about 50 times the frequency of the wave that you want to observe. Adjusting the time per division on the horizontal scale of the scope display will automatically change the sampling rate and may solve this problem. Otherwise, you can manually change the sampling rate on the Control Palette along the bottom of Capstone’s Display Area.

**Phase and voltage measurements**

Set the sinusoidal output voltage amplitude to 4.0 V at a frequency of 10 Hz. Now individually measure the zero-to-peak voltages across the resistor, inductor, and capacitor and the zero-to-peak current. (Why is one current reading enough?) A table is a good way to record all this information. Convert all the peak voltages and currents to rms values. Record the zero-crossing times for all four voltages and and the current, and compute their phases with respect to the phase of the voltage across the resistor.

Repeat the voltage and phase measurements for each component at 100 Hz and 1000 Hz.

**Adding AC voltages**

From Figure [10.1](#), we expect that the sum of the voltage drops across the three components $V_L + V_R + V_C$ equals the output voltage $V_{output}$ at each instant of time. In the absence of time variation, the voltages would add like DC voltages. In terms of rms voltages, we would expect

$$V_{Lrms} + V_{Rrms} + V_{Crms} = V_{Outrms}$$ (10.4)

However, each voltage in the circuit varies in time with its own phase. Expressing Equation [10.2](#)
in terms of rms voltages yields

\[ V_{L_{\text{rms}}} \cos(\omega t + \phi_L) + V_{R_{\text{rms}}} \cos(\omega t + \phi_R) + V_{C_{\text{rms}}} \cos(\omega t + \phi_C) = V_{\text{Out}_{\text{rms}}} \cos(\omega t + \phi_{\text{Out}}) \]  

(10.5)

To verify that the voltages do add this way, it is sufficient to show that the equation holds at two times. Two times are needed to resolve the ambiguity associated with the phases of the voltage signals in Figure 10.2. At most times, it is not enough to know the voltage reading alone. One must also know whether the voltage is rising or falling.) The times \( \omega t = 0 \) and \( \omega t = -90^\circ \) make for simple expressions. Then

\[ V_{L_{\text{rms}}} \cos(\phi_L) + V_{R_{\text{rms}}} \cos(\phi_R) + V_{C_{\text{rms}}} \cos(\phi_C) = V_{\text{Out}_{\text{rms}}} \cos(\phi_{\text{Out}}) \]

\[ V_{L_{\text{rms}}} \sin(\phi_L) + V_{R_{\text{rms}}} \sin(\phi_R) + V_{C_{\text{rms}}} \sin(\phi_C) = V_{\text{Out}_{\text{rms}}} \sin(\phi_{\text{Out}}) \]  

(10.6)

Ideal components constitute an important special case. For ideal components, \( \phi_L = +90^\circ \) and \( \phi_C = -90^\circ \). By convention, \( \phi_R = 0^\circ \). For ideal components, these relations reduce to

\[ \sqrt{(V_{C_{\text{rms}}} - V_{L_{\text{rms}}})^2 + V_{R_{\text{rms}}}^2} = V_{\text{Out}_{\text{rms}}} \]  

(10.7)

At each frequency check to see whether the voltages across the resistor, inductor, and capacitor obey Equations 10.4, 10.6, and 10.7. Tabulate all these results clearly. Is ignoring the phase a good idea?

For future reference, it is worth comparing the measured phases for \( V_L \) and \( V_C \) to their ideal values. Capacitors are usually pretty close to ideal. This observation can help you on an exam.

**RLC circuit at resonance**

By trial and error, adjust the frequency of the sine wave output of the interface unit until the output voltage and current, which drive the circuit, are in phase. Do this carefully. When the current and voltage are in phase as required, look at the inductor voltage and the capacitor voltage. What relationship do they now have with respect to one another? Since real inductors have both resistance and inductance, the phase shift for the real inductor does not equal the \( +90^\circ \) phase shift for an ideal inductor. You will need to compare the vertical component of the voltage across the real inductor with the vertical component of the voltage across the capacitor. (Real capacitors are well-represented by ideal capacitors, so the capacitor vector should be nearly vertical anyway.) This particular state of the system is called “resonance.” What is the overall effect on the circuit of the inductance and capacitance at resonance? Resonant circuits are useful in filtering out certain frequencies. Radio tuning dials work on this principle.
Summary

Summarize all of your results clearly and concisely. Refer to appropriate data tables liberally.

Before you leave the lab please:
- Disconnect the Pasco interface and the RLC Circuit.
- Put all the connecting wires neatly in the tray provided at your lab table.
- Close the Capstone software and any other software you have been using.
- Report any problems or suggest improvements to your TA.