Trade, Non-Homothetic Preferences, and the Impact of Country Size on Wages*

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November 10, 2014

Abstract

We study the impact of country size on wages in a two-country trade model with monopolistic competition and non-homothetic preferences. Since there is renewed interest in non-homothetic preferences and variable markups, we revisit standard results on wages obtained using homothetic (CES) preferences. The standard results are that, under free trade, wages are equal and that, with iceberg costs, the larger country has a higher wage. With non-homothetic preferences, these results do not necessarily hold and it is important to distinguish between the number of workers and the labor endowment per worker. Under free trade, the relative wage depends on the relative labor endowment per worker under certain conditions. With iceberg costs, the larger country does not necessarily get the higher wage unless there is a representative consumer.

Key Words: Markups, Country Size, Wages
JEL: F12, L11

*The authors would like to thank Tongzhe Li, Phil Wandschneider, and Xin Zhao for valuable comments.
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1 Introduction

Since the seminal work of Krugman (1980), most trade models based on monopolistic competition stick to the case of constant elasticity of substitution (CES) preferences. These preferences are homothetic and generate constant markups. Recently, the literature shows that additively separable (AS) preferences[1], which are non-homothetic and generate variable markups, imply alternative views of the gains from trade (Arkolakis et al., 2012), market efficiency (Dhingra and Morrow, 2014; Bilbiie, Ghironi and Melitz, 2008), and trade patterns (Mrázová and Neary, 2014; Bertoletti and Epifani, 2014; Zhelobodko et al., 2012). Most of these papers focus, however, on the symmetric countries case, so the impact of country size[2] on wages goes largely unanalyzed.[3] This paper fills this gap by analyzing the impact of country size on wages in a two-country trade model with AS preferences. The standard results with homothetic preferences are that, under free trade, wages are equal and that, with iceberg costs, the larger country has a higher wage. We show that, with non-homothetic preferences, these results do not necessarily hold.

When preferences are non-homothetic, it is important to distinguish between the number of workers and the labor endowment per worker. We consider a model in which two countries differ only along these two dimensions. Under free trade, the relative wage depends on the relative labor endowment per worker under certain conditions. With iceberg costs, the larger country does not necessarily have a higher wage. If there is one representative consumer, however, the larger country does have a higher wage. Our results imply that the impact of country size on wages is sensitive to the assumptions on preferences and labor endowments.

Kichko, Kokovin and Zhelobodko (2014) also study a model with two asymmetric countries and AS preferences, but their paper differs from this paper in that they use a homogeneous sector to equalize the wage rate between countries. Chen and Zeng (2013) show that the larger country gets the higher wage under constant absolute risk aversion (CARA) preferences[4]. In current paper, we use general AS preferences in which CES and CARA preferences are special cases.

[1]For AS preferences, see originally Dixit and Stiglitz (1977), and Krugman (1979).

[2]In this paper, country size is defined as the the total labor endowment of a country.

[3]In the literature, the impact of country size on wages is also referred as the home market effect in terms of wage, see Behrens et al. (2009) and Chen and Zeng (2013).

[4]CARA preferences are special cases of AS preferences. See Behrens and Murata (2007), and Bertoletti (2006) for discussions.

2 The Model

We consider a model of monopolistic competition in which there are two countries in the world: Home and Foreign. Variables for Foreign have an asterisk.

2.1 Consumer

In Home, each consumer chooses consumption of each domestic good, $c(i)$, and consumption of each foreign good, $c_x(i)$, to maximize

$$U = \int_0^N u(c(i))di + \int_0^{N^*} u(c_x(i))di,$$

where i represents the variety, N and N^* are endogenous measures of firms in Home and Foreign, respectively, and the sub-utility function u is strictly increasing and concave and is at least thrice continuously differentiable. The budget constraint for each consumer in Home is

$$\int_0^N p(i)c(i)di + \int_0^{N^*} p_x^*(i)c_x(i)di = wl,$$

where w is the wage and l is the labor endowment per worker (each consumer is also a worker). There is measure k of identical consumers in Home, so the total labor endowment (country size) is $L = kl$. In Foreign, there is measure k^* of identical consumers, each endowed with l^* units of labor. Foreign's country size is $L^* = k^*l^*$. The wage in Foreign is w^*. We normalize w^* and L^* to one so that Home's relative wage is w and relative size is L.

Firms are homogeneous, so we drop the variety indicator i when convenient. From the Home consumer's problem we obtain the inverse demand functions

$$p = u'(c)/\lambda \tag{1}$$

$$p_x^* = u'(c_x)/\lambda, \tag{2}$$

where $\lambda = (Nu'(c)c + N^*u'(c_x)c_x)/(wl)$. Similar results hold for Foreign. The inverse demand functions imply the following relationships between domestic and foreign consumption:

$$u'(c)/u'(c_x) = p/p_x^* \tag{3}$$

$$u'(c^*)/u'(c_x^*) = p^*/p_x. \tag{4}$$

The price elasticity of demand for each variety is

$$\varepsilon(x) = -u'(x)/(u''(x)x), \tag{5}$$

where $x \in \{c, c^*, c_x, c_x^*\}$.

2.2 Production

Each firm produces a unique variety using labor only. In order to produce, each firm must pay a fixed cost of f units of labor. Each firm has labor productivity φ. The iceberg cost of exporting to the other country is $\tau \geq 1$. Therefore, the profit function for each firm in Home is

$$\pi(i) = p(i)c(i)k + p_x(i)c_x^*(i)k^* - wf - w(c(i)k + \tau c_x^*(i)k^*)/\varphi,$$

where p is the domestic price, and p_x is the export price. Firms in Foreign have analogous profit functions.

Markets are segmented so, taking each consumers demand functions as given, each firm maximizes variable profits in the domestic and foreign markets separately. This leads to the following pricing rules:

$$p = m(c)w/\varphi \tag{6}$$

$$p^* = m(c^*)/\varphi \tag{7}$$

$$p_x = m(c_x^*)\tau w/\varphi \tag{8}$$

$$p_x^* = m(c_x)\tau/\varphi, \tag{9}$$

where the markup factor is $m(x) = \varepsilon(x)/(\varepsilon(x) - 1)$.

As Bertoletti and Epifani (2014) show, the reciprocal of the elasticity of marginal revenue in absolute value is

$$\eta(x) = -r'(x)/(r''(x)x), \tag{10}$$

where $r(x) = u'(x)x$, $r'(x) = u' + u''(x)x$, and $r''(x) = 2u''(x) + u'''(x)$. According to (1), we have $r(c) = \lambda pc$, and similar expressions hold for c_x, c^*, and c_x^*. To get a concave profit function, we need $r'(x)/\lambda > 0$ and $r''(x)/\lambda < 0$. Thus $\eta(x)$ is positive. This leads to three results. The first result is on what we call a good's total markup factor, $m(x)x$.

Lemma 1. *A good's total markup factor is increasing in consumption of that good.*

Proof. See the Appendix. □

The second result is that we can rewrite (3) and (4) as

$$r'(c)/r'(c_x) = w/\tau \tag{11}$$

$$r'(c^*)/r'(c_x^*) = w^{-1}\tau^{-1}. \tag{12}$$

The third result is we have the following result for the income elasticity of demand.

Lemma 2. *The income elasticity of demand is positive (the products are normal goods).*

Proof. See the Appendix. □

2.3 Equilibrium

To close the model, we specify the free-entry and labor-market-clearing conditions. The free-entry condition implies that each firm makes zero profits. Plugging in for prices, we get

$$(m(c) - 1)ck + (m(c_x^*) - 1)\tau c_x^* k^* = \varphi f \tag{13}$$

$$(m(c^*) - 1)c^* k^* + (m(c_x) - 1)\tau c_x k = \varphi f. \tag{14}$$

The above expressions and (11) and (12) are used to solve for the demand functions. In equilibrium, the labor demanded by firms is equal to the labor endowment:

$$N(f + ck/\varphi + \tau c_x^* k^*/\varphi) = L \tag{15}$$

$$N^*(f^* + c^* k^*/\varphi + \tau c_x k/\varphi) = 1. \tag{16}$$

These expressions are used to solve for the measures of firms.

3 Analyses

Based on the above conditions, we use the balanced-trade condition to pin down the relative wage. Plugging measures of firms and prices into the balanced-trade condition, we get the relative wage as follows:

$$w = \frac{1}{L}\frac{k}{\alpha^* k^* + k}\frac{\alpha k + k^*}{k^*}, \tag{17}$$

where $\alpha = m(c)c/(m(c_x^*)\tau(c_x^*))$, and $\alpha^* = m(c^*)c*/(m(c_x)\tau(c_x))$. We use this equation to analyze the impact of differences in labor endowments on the relative wage. We consider cases with free trade ($\tau = 1$) and iceberg costs ($\tau > 1$).

3.1 Free Trade

The following proposition documents the result when there are no transport costs.

Proposition 1. *Under free trade, homothetic AS preferences imply equal wages, while non-homothetic AS preferences do not necessarily imply equal wages. If $m'(x)/m(x) - \eta'(x)/\eta(x)$ is positive or negative for all x, then the relative wage depends on the labor endowment per worker.*

Proof. See the Appendix. □

With CES preferences, both $m(x)$ and $\eta(x)$ are constant, so $m'(x)/m(x) - \eta'(x)/\eta(x) = 0$. Contrary to the standard model, with AS preferences wage differences can emerge without transport costs. As the proof shows, if $m'(x)/m(x) - \eta'(x)/\eta(x)$ is positive or negative for all x, then the relative wage can be written as $w = l^*/l$. It implies that the assumption of preferences is very important to the implications derived from the model.

Two commonly used non-homothetic preferences satisfy this condition. When $u(x) = log(x + \theta), \theta > 0$ as in Simonovska (2014), we have $m'(x) = 1/\theta > 0$ and $\eta' = -\theta/(2x^2) < 0$. When $u(x) = 1 - exp(-\theta x), \theta > 0$ as in Behrens and Murata (2007), we have $m'(x) = \theta(1 - \theta x)^{-2} > 0$ and $\eta'(x) = -[(\theta x - 1)^2 + 1]\theta^{-1}x^{-2}(\theta x - 2)^{-2} < 0$.

3.2 Iceberg Costs

With iceberg costs, we consider wage differences under two cases: a representative consumer and many identical consumers. First we consider a representative consumer, so $k = k^* = 1$. Then we consider the case of many identical consumers, each with $l = l^* = 1$.

3.2.1 Representative Consumer

When there is one representative consumer in each country, labor endowments satisfy $l = L$ and $l^* = 1$. The balanced-trade condition can then be written as

$$\bar{B}(w, L) = (\alpha^* + 1)wL - (\alpha + 1) = 0. \tag{18}$$

We have the following result.

Proposition 2. *When there is one representative consumer in each country, under AS preferences and iceberg transport costs, the larger country gets the higher wage.*

Proof. See the Appendix. □

The keys behind Proposition 2 are increasing returns to scale, transport costs, and normal goods. Due to increasing returns to scale, the larger country attracts more firms because this saves on transportation costs. More firms will result in higher labor demand, and, therefore, higher wages in the larger country. Moreover, since products are normal goods, more firms are willing to operate in the larger country.

3.2.2 Many Identical Consumers

When there are many identical consumers, each endowed with one unit of labor, the measures of consumers are $k = L$ and $k^* = 1$. The balanced-trade condition can be written as

$$\tilde{B}(w, L) = (\alpha^* + L)w - (\alpha L + 1) = 0. \tag{19}$$

The following proposition documents the impact of country size on wages in this case.

Proposition 3. *When each consumer is endowed with only one unit of labor, the larger country does not necessarily get the higher wage.*

Proof. See the Appendix. □

This shows that the larger country gets the higher wage when parameters satisfy certain conditions. Compared to the representative consumer case, this implies that, under different assumptions on per-worker labor endowment, the model can generate different results. In summary, with iceberg costs, homothetic AS preferences imply that the larger country has the higher wage, while non-homothetic AS preferences do not necessarily imply this.

4 Conclusion

This paper analyzes the impact of country size on wages in a trade model with monopolistic competition and additively separable preferences. We show that, unlike the case with CES preferences, the assumption of how many units of labor each worker supplies matters. Without transport costs, under certain conditions, the relative wage depends on the units of labor each worker supplies. With iceberg transport costs, when there is single representative consumer, the larger country always gets the higher wage; however, this result does not always hold in the case where each worker supplies one unit of labor. We hope the results in this paper shed new light on non-homothetic preferences and variable markups in the international trade literature.

5 Appendix

5.1 Proof of Lemma 1

According to the definition of markups and price elasticity, we have

$$m(x) = \varepsilon(x)/(\varepsilon(x) - 1) = u'(x)/r'(x).$$

Hence, we have the following:

$$\partial(m(x)x)/\partial x = [u''(x)x + u'(x)]/r'(x) - u'(x)r''(x)r'(x)^{-2} = m(x)(1 - 1/\varepsilon(x) + 1/\eta(x)).$$

Combining the above two equations together, we get

$$\partial(m(x)x)/\partial x = m(x)/\eta(x) + 1 > 1.$$

Moreover, following similar steps, we can get

$$\partial[(m(x) - 1)x]/\partial x = m(x)/\eta(x) > 0.$$

5.2 Proof of Lemma 2

We show the income elasticity of c as an example; others follow the same logic. Plugging λ into (1), the Walrasian demand function $c(p, w)$ can be solved from

$$p(Nu'(c)c + N^*u'(c_x)c_x) - u'(c)wL = 0.$$

Taking the derivative of the left side with respect to c we get

$$\partial[p(Nu'(c)c + N^*u'(c_x)c_x) - u'(c)wL]/\partial c = pN(u''(c)c + u'(c)) - u''(c)wL > 0.$$

Taking the derivative of the above expression with respect to w, we get

$$\partial[p(Nu'(c)c + N^*u'(c_x)c_x) - u'(c)wL]/\partial w = -u'(c)l < 0.$$

Therefore, according to the implicit function theorem, Lemma 2 is proved.

5.3 Proof of Proposition 1

We first check the CES preferences case. Since CES preferences are homothetic, k and k^* do not matter. The results in Krugman (1980) still hold with different values of k and k^*. Now we turn to the general AS preferences case. Recall that relative wage rate is

$$w = \frac{1}{L}\frac{m(c_x)c_xk}{m(c^*)c^*k^* + m(c_x)c_xk}\frac{m(c)ck + m(c_x^*)c_x^*k^*}{m(c_x^*)c_x^*k^*}.$$

Profit maximization implies that marginal revenue equals marginal cost: $\partial(m(c)c)/\partial c = \partial(m(c_x^*)c_x^*)/\partial c_x^*$, and $\partial(m(c^*)c^*)/\partial c^* = \partial(m(c_x)c_x)/\partial c_x$. According to Lemma 1, $\partial(m(x)x)/\partial x > 0$. The second-order derivative of $m(x)x$ with respect to x is

$$\frac{\partial^2(m(x)x)}{\partial x^2} = \frac{m'(x)}{\eta(x)} - \frac{m(x)\eta'(x)}{\eta(x)^2} = \frac{m(x)}{\eta(x)}\left(\frac{m'(x)}{m(x)} - \frac{\eta'(x)}{\eta(x)}\right).$$

As long as the above equation is always positive or negative, the relative wage can be written as $w = l^*/l$.

5.4 Proof of Proposition 2

We want to show that $\partial w/\partial L > 0$. According to the implicit function theorem, it suffices to show that $\partial\bar{B}(w, L)/\partial w < 0$ and $\partial\bar{B}(w, L)/\partial L > 0$. Starting with the sign of $\partial\bar{B}(w, L)/\partial w$, we have

$$\partial\bar{B}(w, L)/\partial w = L(\alpha^* + 1 + w\partial\alpha^*/\partial w) - \partial\alpha/\partial w,$$

where

$$\begin{aligned}
\frac{\partial\alpha}{\partial w} &= \frac{m(c)/\eta(c) + 1}{m(c_x^*)\tau c_x^*}\frac{\partial c}{\partial w} - \alpha\frac{m(c_x^*)/\eta(c_x^*) + 1}{m(c_x^*)c_x^*}\frac{\partial c_x^*}{\partial w} \\
\frac{\partial\alpha^*}{\partial w} &= \frac{m(c^*)/\eta(c^*) + 1}{m(c_x)\tau c_x}\frac{\partial c^*}{\partial w} - \alpha^*\frac{m(c_x)/\eta(c_x) + 1}{m(c_x)c_x}\frac{\partial c_x}{\partial w}.
\end{aligned}$$

Conditions (11)-(14) provide a solution for the change in consumption with respect to the wage rate. By Lemma 2, we have the following:

$$\begin{aligned}
\frac{\partial c}{\partial w} &= -\frac{m(c_x^*)}{m(c)}\frac{\eta(c)}{\eta(c_x^*)}\tau\frac{\partial c_x^*}{\partial w} > 0 \\
\frac{\partial c^*}{\partial w} &= -\frac{m(c_x)}{m(c^*)}\frac{\eta(c^*)}{\eta(c_x)}\tau\frac{\partial c_x}{\partial w} < 0 \\
\frac{\partial c_x}{\partial w} &= -\frac{\eta(c_x)c_x}{\eta(c_x^*)c_x^*}\frac{1 + 1/\alpha}{1 + 1/\alpha^*}\frac{\partial c_x^*}{\partial w} > 0 \\
\frac{\partial c_x^*}{\partial w} &= \frac{\eta(c_x^*)c_x^*}{w}\frac{\alpha\alpha^* + \alpha}{1 - \alpha\alpha^*} < 0.
\end{aligned}$$

Hence we have $\partial\alpha/\partial w > 0$ and $\partial\alpha^*/\partial w < 0$. In addition, we have the following:

$$\begin{aligned}
\alpha^* + 1 + \frac{\partial\alpha^*}{\partial w}w &= \alpha^* + 1 + \left(\frac{m(c^*) + \eta(c^*)}{m(c^*)} + \alpha^*\frac{m(c_x) + \eta(c_x)}{m(c_x)}\right)\frac{\alpha\alpha^* + \alpha^*}{1 - \alpha\alpha^*} \\
&< \alpha^* + 1 + (1 + \alpha^*)\frac{\alpha\alpha^* + \alpha^*}{1 - \alpha\alpha^*} = (1 + \alpha^*)\frac{1 + \alpha^*}{1 - \alpha\alpha^*} < 0.
\end{aligned}$$

Therefore, $\partial B(w, L)/\partial w < 0$. With regard to the sign of $\partial\bar{B}(w, L)/\partial L$, we have

$$\partial\bar{B}(w, L)/\partial L = (\alpha^* + 1)w + wL\partial\alpha^*/\partial L - \partial\alpha/\partial L,$$

where

$$\frac{\partial\alpha}{\partial L} = \frac{m(c)/\eta(c)+1}{m(c_x^*)\tau c_x^*}\frac{\partial c}{\partial L} - \alpha\frac{m(c_x^*)/\eta(c_x^*)+1}{m(c_x^*)c_x^*}\frac{\partial c_x^*}{\partial L}$$
$$\frac{\partial\alpha^*}{\partial L} = \frac{m(c^*)/\eta(c^*)+1}{m(c_x)\tau c_x}\frac{\partial c^*}{\partial L} - \alpha^*\frac{m(c_x)/\eta(c_x)+1}{m(c_x)c_x}\frac{\partial c_x}{\partial L}.$$

According to (11)-(14), when $k = k^* = 1$, the demand functions do not change with respect to country size, so $\partial\alpha/\partial L = \partial\alpha^*/\partial L = 0$ and $\partial\bar{B}(w,L)/\partial\alpha > 0$.

5.5 Proof of Proposition 3

The proof of Proposition 3 follows the similar steps as in the proof of Proposition 2. In this case, we still have $\partial\tilde{B}(w,L)/\partial w < 0$. However, the sign of $\partial\tilde{B}(w,L)/\partial L$ is ambiguous because the demand functions are now related to country size, and the signs of $\partial x/\partial L$ are ambiguous.

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