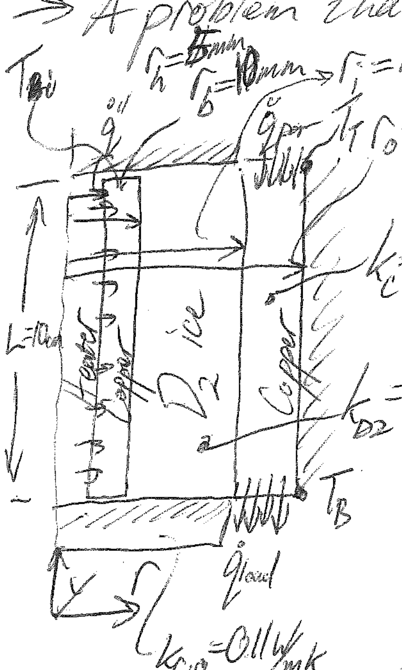
→ A problem that doesn't meet our checks;



→ D_2 thermal conductivity measurement. What is temperature profile along outer copper cap?

→ 1st question, are G10 ends really adiabatic?

$$B_i = \frac{\text{resistance to neglect}}{\text{resistance to consider}} \Rightarrow \frac{R_{G10}}{R_{radial}}$$

$$R_{G10} = \frac{\Delta x}{kA_c} \Rightarrow \frac{0.01m}{0.11(\pi .065_m^2)} \Rightarrow 128.6 \frac{K}{W}$$

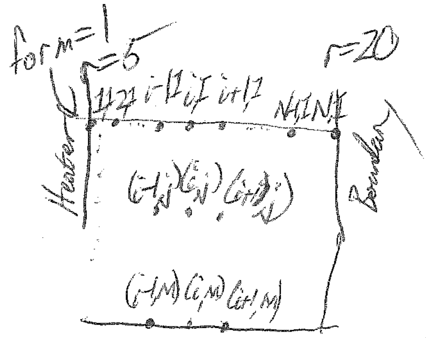
$B_i << 1 = 80$

$$R_{radial} = \frac{\ln\left(\frac{r_o}{r_{in}}\right)}{2\pi kL} = R_{c,i} + R_{D2} + R_{c,o} \Rightarrow 1.618 \frac{K}{W}$$

We can assume all HT is radial.

→ Lets follow our numerical solution steps outlined for 1-D problems

Step 1: Define & Distribute a nodal network, this time 2-D



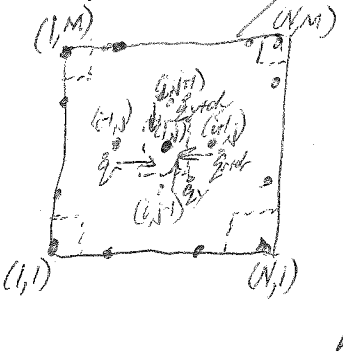
N Uniformly distributed nodes in r direction
× M Uniformly distributed rows in y direction

$$\Delta r = \frac{(r_o - r_h)}{(N-1)}, \quad \Delta y = \frac{L}{(M-1)}$$

& the positions are defined by

$$r_i = r_h + \Delta r(i-1), \quad y_j = (j-1)\Delta y \qquad \dot{q} = \frac{kA_c\Delta T}{\Delta x}$$

Step 2: Carry out energy balance on each CV



Internal Nodes: IN = OUT + STORED $\Rightarrow \dot{q}_{LHS} + \dot{q}_{RHS} + \dot{q}_{top} + \dot{q}_{bottom} = 0$

$$\dot{q}_{LHS_{i,j}} = \frac{2\pi k \Delta y}{\Delta r}\left(r_i - \frac{\Delta r}{2}\right)\left(T_{i-1,j} - T_{i,j}\right), \quad \dot{q}_{RHS,i,j} = \frac{2\pi k \Delta y}{\Delta r}\left(r_i + \frac{\Delta r}{2}\right)\left(T_{i+1,j} - T_{i,j}\right)$$

$$\dot{q}_{top_{i,j}} = \frac{k\left(\pi\left(r_i + \frac{\Delta r}{2}\right)^2 - \pi\left(r_i - \frac{\Delta r}{2}\right)^2\right)}{\Delta y}\left(T_{i,j+1} - T_{i,j}\right) \quad \dot{q}_{bottom_{i,j}} = \frac{k}{\Delta y}\left(\pi\left(r_i + \frac{\Delta r}{2}\right)^2 - \pi\left(r_i - \frac{\Delta r}{2}\right)^2\right)\left(T_{i,j-1} - T_{i,j}\right)$$

applicable for $i = 2 \ldots N-1$, $j = 2 \ldots M-1$

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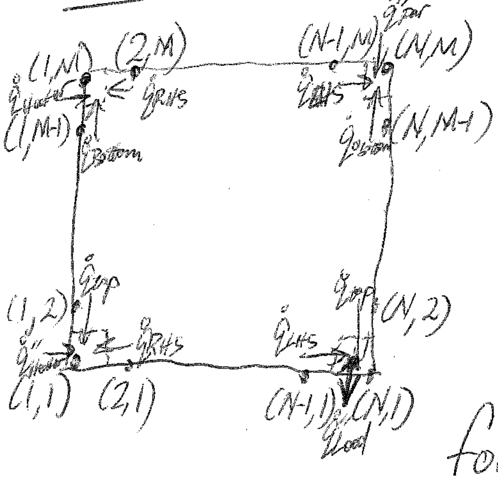
Corner Nodes: Energy Balance: IN = OUT + STORED



for (1,1) $\dot q_{flux[1]} + \dot q_{RHS1,1} + \dot q_{top1,1} = 0$

$\dot q_{flux[1]} = \dot q'' 2\pi r h \frac{\Delta y}{2}$ $\dot q_{RHS1,1} = \frac{\pi k \Delta y}{\Delta r}\left(r_i + \frac{\Delta r}{2}\right)\left(T_{2,1} - T_{1,1}\right)$

$\dot q_{top1,1} = \frac{k}{\Delta y}\left(\pi\left(r_i+\frac{\Delta r}{2}\right)^2 - \pi(r_i)^2\right)\left(T_{1,2}-T_{1,1}\right)$

for (N,1) $\dot q_{LHSN,1} + \dot q_{topN,1} = \dot q_{load}$ .....

$\dot q_{LHSN,1} = \frac{\pi k\Delta y}{\Delta r}\left(r_N - \frac{\Delta r}{2}\right)\left(T_{N-1,1}-T_{N,1}\right)$ $\dot q_{Load[N]} = \dot q''_{load}\left(\pi r_N^2 - \pi\left(r_N-\frac{\Delta r}{2}\right)^2\right)$

$\dot q_{topN,1} = \frac{k}{\Delta y}\left(\pi r_N^2 - \pi\left(r_N-\frac{\Delta r}{2}\right)^2\right)\left(T_{N,2}-T_{N,1}\right)$

for (1,M) $\dot q_{flux[M]} + \dot q_{Bottom1,M} + \dot q_{RHS2,M} = 0$

$\dot q_{flux[M]} = \dot q_{flux[1]}$, $\dot q_{Bottom1,M} = \frac{k}{\Delta y}\left(\pi\left(r_i+\frac{\Delta r}{2}\right)^2 - \pi(r_i)^2\right)\left(T_{1,M-1}-T_{1,M}\right)$

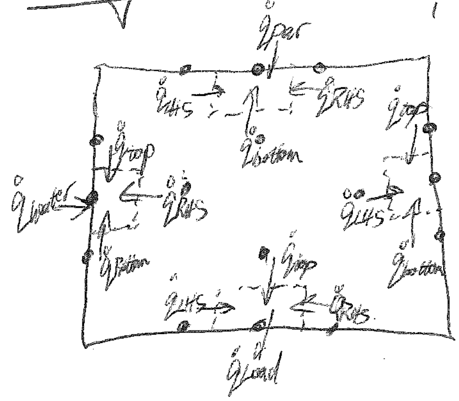
$\dot q_{RHS2,M} = \frac{\pi k\Delta y}{\Delta r}\left(r_i+\frac{\Delta r}{2}\right)\left(T_{2,M}-T_{1,M}\right)$

for (N,M) $\dot q_{par[N]} + \dot q_{LHS,N,M} + \dot q_{BottomN,M} = 0$

$\dot q_{par[N]} = \dot q''_{par}\left(\pi r_N^2 - \pi\left(r_N - \frac{\Delta r}{2}\right)^2\right)$, $\dot q_{LHSN,M} = \frac{\pi k \Delta y}{\Delta r}\left(r_N - \frac{\Delta r}{2}\right)\left(T_{N-1,M}-T_{N,M}\right)$

$\dot q_{bottom} = \frac{k}{\Delta y}\left(\pi r_N^2 - \pi\left(r_N-\frac{\Delta r}{2}\right)^2\right)\left(T_{N,M-1}-T_{N,M}\right)$

Edge Nodes for $i=2\rightarrow N-1$, $j=2\rightarrow M-1$ respectively



Bottom edge: IN=OUT+STORED

$$\mathring{q}_{LHS,i,1}+\mathring{q}_{RHS,i,1}+\mathring{q}_{top,i,1}=\mathring{q}_{Load,i}$$

$$\mathring{q}_{LHS,i,1}=\frac{\pi k\Delta y}{\Delta r}\left(r_i-\frac{\Delta r}{2}\right)\left(T_{i-1,1}-T_{i,1}\right)$$

$$\mathring{q}_{RHS,i,1}=\frac{\pi k\Delta y}{\Delta r}\left(r_i+\frac{\Delta r}{2}\right)\left(T_{i+1,1}-T_{i,1}\right)$$

$$\mathring{q}_{top,i,1}=\frac{k}{\Delta y}\left(\pi\left(r_i+\frac{\Delta r}{2}\right)^2-\pi\left(r_i-\frac{\Delta r}{2}\right)^2\right)\left(T_{i,2}-T_{i,1}\right),\quad \mathring{q}_{Load,i}=q''_{Load,i}\left(\pi\left(r_i+\frac{\Delta r}{2}\right)^2+\pi\left(r_i-\frac{\Delta r}{2}\right)^2\right)$$

Top edge: $\mathring{q}_{LHS,i,M}+\mathring{q}_{RHS,i,M}+\mathring{q}_{bottom,i,M}+\mathring{q}_{par,i}=0$

$$\mathring{q}_{LHS,i,M}=\frac{\pi k\Delta y}{\Delta r}\left(r_i-\frac{\Delta r}{2}\right)\left(T_{i-1,M}-T_{i,M}\right),\quad \mathring{q}_{RHS,i,M}=\frac{\pi k\Delta y}{\Delta r}\left(r_i+\frac{\Delta r}{2}\right)\left(T_{i+1,M}-T_{i,M}\right)$$

$$\mathring{q}_{Bottom,i,M}=\frac{k}{\Delta y}\left(\pi\left(r_i+\frac{\Delta r}{2}\right)^2-\pi\left(r_i-\frac{\Delta r}{2}\right)^2\right)\left(T_{i,M-1}-T_{i,M}\right),\quad \mathring{q}_{par,i}=\mathring{q}_{Par,i}\left(\pi\left(r_i+\frac{\Delta r}{2}\right)^2+\pi\left(r_i-\frac{\Delta r}{2}\right)^2\right)$$

Left edge: $\mathring{q}_{heater,j}+\mathring{q}_{top,1,j}+\mathring{q}_{bottom,1,j}+\mathring{q}_{RHS,1,j}=0$

$$\mathring{q}_{heater,j}=q''_{heater}\,2\pi r_1\Delta y,\quad \mathring{q}_{top,1,j}=\frac{k}{\Delta y}\left(\pi\left(r_1+\frac{\Delta r}{2}\right)^2-\pi(r_1)^2\right)\left(T_{1,j+1}-T_{1,j}\right)$$

$$\mathring{q}_{bottom,1,j}=\frac{k}{\Delta y}\left(\pi\left(r_1+\frac{\Delta r}{2}\right)^2-\pi(r_1)^2\right)\left(T_{1,j-1}-T_{1,j}\right)$$

$$\mathring{q}_{RHS,1,j}=2\pi k\frac{\Delta y}{\Delta r}\left(r_1+\frac{\Delta r}{2}\right)\left(T_{2,j}-T_{1,j}\right)$$

Right edge: $\mathring{q}_{top,N,j}+\mathring{q}_{Bottom,N,j}+\mathring{q}_{LHS,N,j}=0$

$$\mathring{q}_{top,N,j}=\frac{k}{\Delta y}\left(\pi(r_N)^2-\pi\left(r_N-\frac{\Delta r}{2}\right)^2\right)\left(T_{N,j+1}-T_{N,j}\right)$$

$$\mathring{q}_{bottom,N,j}=\frac{k}{\Delta y}\left(\pi r_N^2-\pi\left(r_N-\frac{\Delta r}{2}\right)^2\right)\left(T_{N,j-1}-T_{N,j}\right)$$

$$\mathring{q}_{LHS,N,j}=2\pi k\frac{\Delta y}{\Delta r}\left(r_N-\frac{\Delta r}{2}\right)\left(T_{N-1,j}-T_{N,j}\right)$$