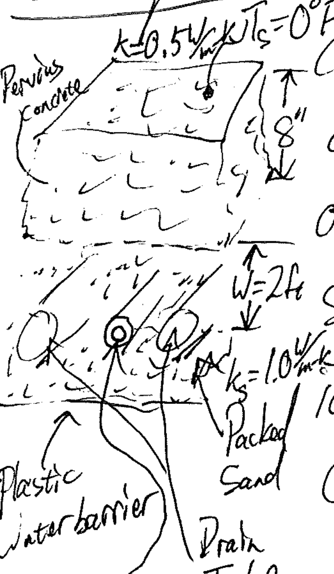
# Analytical 2-D Conduction

ME576F19 8.1

Thus far we've dealt with 1-D conduction problems & developed the extended surface approximation to handle some 2-D problems as 1-D. We could spend more time on fins to cover moving fins & variable area fins. The approach remains the same, the math just gets harder. Luckily many libraries are available that tabulate most known solutions for convenient use. (Show EES: Options → Function Info → HT & Fluid Flow → Fins) At some point though, we will no longer be able to use 1-D approximation.

Example: WSU's Civil Engineering Department helped to develop pervius concrete pavement solutions such as the rotunda in front of the Allen School of Global Animal Health or the sidewalk in front of Sloan. These systems collect ground water within a packed sand & gravel layer to reduce runoff. One day during conversations the idea of using one of these systems as a ground coupled heat pump came up. The water stored in the "basin" is basically a giant thermal mass at relatively constant temp. But the concern is that heat transfer to the surface on a cold day will freeze fluid in the observation/research 'well' that we would use as the heat exchanger. At what depth 'W' should the well be placed in the sand bed?



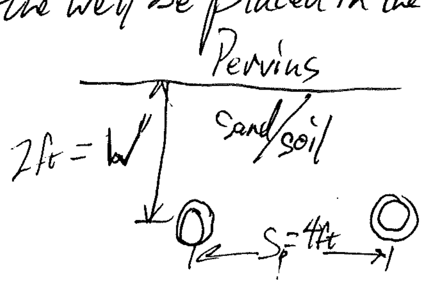

Pipe "Well" research

$T_w = 35°F$

$h_w = 200\,W/m^2K$

$k_{pipe} = 1.5\,W/mK$

$D_i = 1.5"$ $D_o = 2"$




Step 0: Quick calcs to figure out what problem we are trying to solve.

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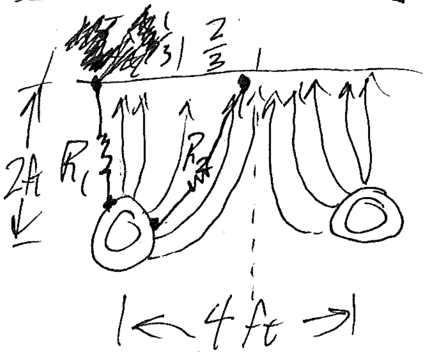


Heat transfer is definitely 2D in this problem, but how much so? $B_i = \dfrac{\text{resistance you want to neglect}}{\text{resistance you want to consider}}$

$$B_i = \frac{R_2}{R_1} = \frac{\sqrt{2}\,w/kA}{w/kA} \Rightarrow B_i = \underline{\underline{\sqrt{2} = 1.414}} \text{ not } \lll 1$$

It would be very convenient if there was an "integrated average 2-D" resistance for this problem. Many common 2-D & 3-D surface problems have been solved & are known as "Shape Factors". The Shape Factor is: $S = \frac{1}{kR}$ where $k$ is the material thermal conductivity & $R$ is the thermal resistance between the surfaces. Recall for a plane wall that $R = \frac{th}{kA_c}$

$\Rightarrow S \approx \frac{A_c}{th}$ this means that Shape Factors should always be of the same order as the ratio of the cross sectional area to the average length that heat transfer is occurring ← great check.

☆ Many Heat transfer libraries like EES often include Shape factors (Show EES: Options → Function Info → Conduction Shape Factors)

Now show EES code & plot → I could answer many of the questions to determine the feasibility of this concept with quick Shape Factor estimates!

But what if we can't find the Shape Factor we need?

# Separation of Variables

→ Most common technique for solving Partial Differential Equations used in a number of engineering fields

→ ~~Split your~~ solution involves splitting your PDE into 2 ODEs that can be solved using techniques we have developed for 1-D problems

## Requirements for Separation of Variables (SOV)

1) PDE must be linear; i.e. cannot contain any products of temperature or its derivative

Example non-linear: $$T\frac{\partial^2 T}{\partial x^2}+\frac{\partial T}{\partial x}\frac{\partial^2 T}{\partial y^2}=0$$

linear: $$\frac{\partial^2 T}{\partial x^2}+\frac{\partial T}{\partial x}+\frac{\partial^2 T}{\partial y^2}+T=0$$

2) PDE must be homogeneous: i.e. if $T$ is a solution, then $CT$ must also be a solution (where $C$ is any constant)

non-homogeneous: $$\frac{\partial^2 T}{\partial x^2}+\frac{\partial T}{\partial x}+\frac{\partial^2 T}{\partial y^2}+T=T_\infty$$

$CT$ doesn't always $= T_\infty$

homogeneous: $$\frac{\partial^2 T}{\partial x^2}+\frac{\partial T}{\partial x}+\frac{\partial^2 T}{\partial y^2}+T=0$$

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3) All BCs must be linear

→ cannot contain any products of temperature or its derivative

nonlinear: $-k\frac{\partial T}{\partial x}\Big|_{x=W} = \sigma\varepsilon T^4_{x=W}$

linear: $-k\frac{\partial T}{\partial x}\Big|_{x=W} = h(T_{x=W} - T_\infty)$

4) Both BC's ~~must be~~ in ~~one directi~~ the homogeneous direction must be homogeneous

→ if T satisfies BC's, then CT must also satisfy BCs

→ BCs in other direction do not have to be homogeous

non-homogeneous: $-k\frac{\partial T}{\partial x}\Big|_{x=W} = h(T_{x=W} - T_\infty)$

homogeneous: $-k\frac{\partial T}{\partial x}\Big|_{x=W} = h\,T_{x=W}$

5) Computational domain must be "simple", i.e. boundaries must lie along lines of constant coordinates (e.g. $x=W$)

Now we are ready to ~~attempt a problem~~ work through the solution steps.