Advanced Extended Surfaces

- Moving Extended Surfaces
- Example
- Numerical Extended Surfaces

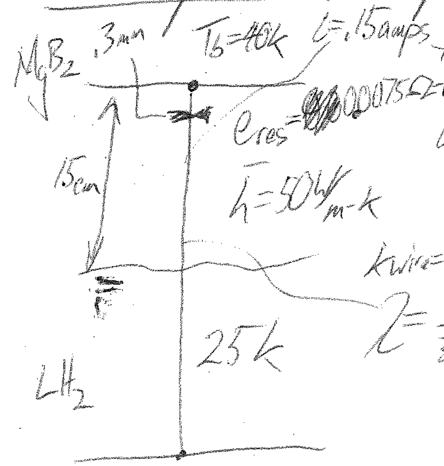
→ Angel forum for research & project interests

→ Last time we introduced the concept of an the extended surface approximation & showed the deriation of analytical solutions for simple geometries.

→ The methodology of the extended surface approximation can be used to analyze complex problems including: volumetric generation, heat flux, moving surfaces, multiple computational domains, ... etc.

→ The solution steps we have used thus far still apply

Example: Superconducting MgB_2 fuel level gauge for Liquid hydrogen fuel tank. We'll work this problem with both analytical & numerical techniques.



$e_{res} = R\frac{A}{L}$

$k_{wire} = 15\,W/m\text{-}k$

$k = \frac{15\,(W/m\text{-}k)}{39\,k}\,T(k)$

a) Is extended surface approximation applicable?

→ Calculate Biot number

$$B_i = \frac{R_{thermal\ to\ neglect}}{R_{thermal\ you\ need}} = \frac{R_{cond,r}}{R_{conv}} \Rightarrow$$

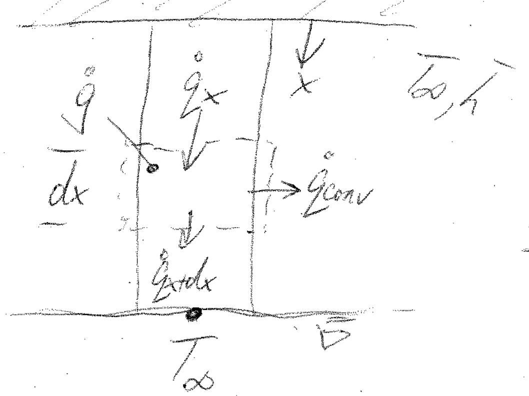
$$\Rightarrow \frac{R_{cond,r}}{R_{conv}} = \frac{\frac{\ln\left(\frac{R_{out}}{r_{in}}\right) \to 1}{2\pi L k}}{\frac{1}{hA_s}} \Rightarrow \frac{h\pi D L}{2\pi k L} \Rightarrow B_i = \frac{hD}{2k} = \frac{50(0.0003m)}{2(15)} \Rightarrow B_i = 0.0004875$$

$B_i << 1$

So fin rules apply

b) Develop analytical solution to problem

5.2 ME516 Sp2013



Step 1: Define CV

Step 2: Define energy terms

Step 3 & 4: Energy Balance: IN = OUT + STORED

$$\cancel{\dot{q}_x} + \dot{g} = \dot{q}_{conv} + \cancel{\dot{q}_x} + \frac{d\dot{q}}{dx}dx$$

Step 5: Apply Rate Equations $\dot{g} = I^2 \frac{e_{res}\,dx}{A_c}$ (R), $\dot{q}_{conv} = h\,per\,dx(T - T_\infty)$

$$\dot{q} = -kA_c \frac{dT}{dx}$$

$$\frac{e_{res}\cancel{dx}}{A_c} I^2 = h\,per\cancel{dx}(T - T_\infty) + \frac{d}{dx}\left[-kA_c \frac{dT}{\cancel{dx}}\right]\cancel{dx}$$

$$-kA_c$$

→ 2nd order → nonhomogeneous → linear → ODE

$$\Rightarrow \frac{d^2T}{dx^2} - \frac{h\,per}{kA_c}T = \frac{-h\,per}{kA_c}T_\infty - \frac{e_{res}}{kA_c}I^2$$

Step 6: Solve the ODE: Split into homogeneous & particular ODEs: $T = T_h + T_p$

$$\frac{d^2(T_h+T_p)}{dx^2} - \frac{h\,per}{kA_c}(T_h+T_p) = \frac{-h\,per}{kA_c}T_\infty - \frac{e_{res}}{kA_c^2}I^2$$

$$\frac{d^2T_h}{dx^2} - \frac{h\,per}{kA_c}T_h = 0 \quad \& \quad \frac{d^2T_p}{dx^2} - \frac{h\,per}{kA_c}T_p = \frac{-h\,per}{kA_c}T_\infty - \frac{e_{res}}{kA_c^2}I^2$$

recognize solution

$$T_h = C_1\sinh(mx) + C_2\cosh(mx)$$

Guess simple solution:

If T_p = constant then $\frac{-h\,per}{kA_c}\text{constant} = \frac{-h\,per}{kA_c}T_\infty - \frac{e_{res}}{kA_c^2}I^2$

$$\text{Constant} = T_\infty + \frac{e_{res}I^2}{h\,per\,A_c} \qquad \frac{-h\,per}{kA_c}$$

$$T = T_h + T_p \Rightarrow T = C_1\sinh(mx) + C_2\cosh(mx) + T_\infty + \frac{e_{res}}{h\,per\,A_c}I^2$$ ← General Solution

Step 7: Apply BC's: $T_{x=0} = 40K = T_b$, $T_{x=L} = 25K = T_\infty$

BC1: $$T_{x=0} = T_b = C_1 \underbrace{\sinh(m(0))}_{=0} + C_2 \underbrace{\cosh(m(0))}_{=1} + T_\infty + \frac{\rho_{res}}{h\,per\,A_c} I^2$$

$$C_2 = T_b - T_\infty - \frac{\rho_{res}}{h\,per\,A_c} I^2$$

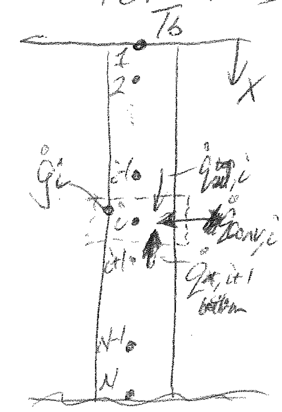
BC2: $$T_{x=L} = T_\infty = C_1 \sinh(m(L)) + C_2 \cosh(m(L)) + T_\infty + \frac{\rho_{res}}{h\,per\,A_c} I^2$$

Step 8: Analyze solution in EES

→ Go to EES, make a plot of T vs. i @ end & T vs. x

→ However there are 2 problems with our analytical solution, the 1st is that the thermal conductivity is a strong function of temperature $k \cong \frac{15\ \frac{W}{mK}}{39\,(K)} T$ & the electrical resistivity $\rho_{res} \to 0$ if $T < T_c \approx 35K$.

→ We need a numerical model to account for these changes! The steps for numerical fin analysis are very similar to the steps for 1-D numerical problems



Step 1: Distribute Nodes: $x_i = \frac{L(i-1)}{(N-1)}$ for $i = 1 \ldots N$

$$\Delta x = \frac{L}{(N-1)}$$

Step 2: Energy Balance on Internal Node: IN = OUT + STORED

$$\cancel{\dot q_{x,i} + \dot g_i = \dot q_{x,i+1} + \dot q_{conv,i}} \quad \text{for } i = 2 \ldots (N-1)$$

$$\dot q_{top,i} + \dot q_{bottom,i} + \dot q_{conv,i} + \dot g_i = 0$$

5.4 ME516 Sp2013

Step 3: Rate equations for internal node

$$\dot{q}_{top,i} = \frac{k_{top}\pi D^2}{4\Delta x}(T_{i-1} - T_i), \quad \dot{q}_{bottom,i} = \frac{k_B \pi D^2}{4\Delta x}(T_{i+1} - T_i), \quad \dot{q}_{conv,i} = \bar{h}\pi D \Delta x (T_\infty - T_i)$$

$$\dot{g}_i = \frac{4 e_{res,i} I^2}{\pi D^2}\Delta x$$

So

$$\frac{k_{top}\pi D^2}{4\Delta x}(T_{i-1} - T_i) + \frac{k_B \pi D^2}{4\Delta x}(T_{i+1} - T_i) + \bar{h}\pi D\Delta x(T_\infty - T_i) + \frac{4 e_{res,i} I^2}{\pi D^2}\Delta x = 0$$

Boundary Nodes:

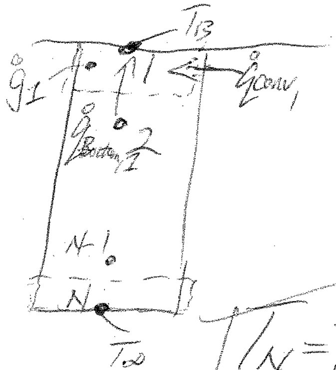
$$\boxed{T_1 = T_B} \leftarrow \text{~~yes its that easy~~}$$

$$\dot{q}_{Bottom,1} + \dot{q}_{conv,1} + \dot{g}_1 = 0$$

$$\dot{q}_{Bottom,1} = \frac{k_B \pi D^2}{4\Delta x}(T_2 - T_1)$$

$$\dot{q}_{conv,1} = \bar{h}\pi D\frac{\Delta x}{2}(T_\infty - T_1)$$

$$\dot{g}_i = \frac{4 e_{res} I^2}{\pi D^2}\frac{\Delta x}{2}$$



~~$T_N = T_\infty$~~

$$0 = \frac{k_B \pi D^2}{4\Delta x}(T_2 - T_B) + \bar{h}\pi D\frac{\Delta x}{2}(T_\infty - T_B) + \frac{4 e_{res,1} I^2}{\pi D^2}\frac{\Delta x}{2}$$

$$\boxed{T_N = T_\infty}$$

$$\cancel{0 = \frac{k_{Top,N}\pi D^2}{4\Delta x}(T_{N-1} - T_\infty) + \bar{h}\pi D\frac{\Delta x}{2}(T_\infty - T_\infty) + \frac{4 e_{res,N} I^2}{\pi D^2}\frac{\Delta x}{2}}$$

Step 4: Implement model in EES

Step 6: Check mesh convergence via important quantity