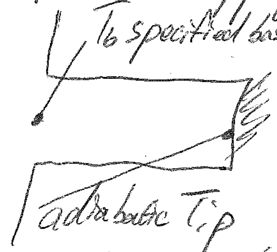
Step 7: Apply BC's (Problem Specific)



T_b is easy, no interface balance required $T_{x=0} = T_b$

$T = C_1 \exp(mx) + C_2 \exp(-mx) + T_\infty \Rightarrow \underline{T_b = C_1 + C_2 + T_\infty}$

Interface balance @ tip: $\dot{q}_{in} = \dot{q}_{out} \nearrow 0 \Rightarrow \dfrac{-kA_c \left(\frac{dT}{dx}\right)_{x=L} = 0}{-kA_c} \Rightarrow \left(\dfrac{dT}{dx}\right)_{x=L} = 0$

Take derivative of general solution:

$$\frac{dT}{dx} = C_1 m \exp(mx) - C_2 m \exp(-mx) \Rightarrow \left(\frac{dT}{dx}\right)_{x=L} = \underline{C_1 m \exp(mL) - C_2 m \exp(-mL) = 0}$$

2 eqs & 2 unknowns: $C_1 = \dfrac{(T_b - T_\infty)\exp(-mL)}{\exp(-mL) + \exp(mL)}$ & $C_2 = \dfrac{(T_b - T_\infty)\exp(mL)}{\exp(-mL) + \exp(mL)}$

Substituting in we arrive @ the well known solution:

$$\underline{\frac{T - T_\infty}{T_b - T_\infty} = \frac{\exp\left(-mL\left(1 - \frac{x}{L}\right)\right) + \exp\left(mL\left(1 - \frac{x}{L}\right)\right)}{\exp(-mL) + \exp(mL)}} = f\left(mL, \frac{x}{L}\right)$$

remember that $m = \sqrt{\dfrac{per\, h}{kA_c}}$ & $R_{cond,x} = \dfrac{L}{kA_c}$, $R_{conv} = \dfrac{1}{h\, per\, L} \Rightarrow m = \sqrt{\dfrac{R_{cond,x}}{R_{conv}}}$

* Show mL plot

→ There is actually a more compact way to write fin equations.

→ hyperbolic cosine (cosh) & hyperbolic sine (sinh) are defined as

$$\cosh(A) = \frac{1}{2}\big[\underbrace{\exp(A) + \exp(-A)}_{\text{look familiar?}}\big] \quad \& \quad \sinh(A) = \frac{1}{2}\left[\exp(A) - \exp(-A)\right]$$

Cosh & Sinh behave a lot like cosine & sine

$\cos^2(A) + \sin^2(A) = 1$	$\cosh^2(A) - \sinh^2(A) = 1$
$\dfrac{d}{dx}\left[\cos(A)\right] = -\sin(A)\dfrac{dA}{dx}$	$\dfrac{d}{dx}\left[\cosh(A)\right] = \sinh(A)\dfrac{dA}{dx}$
$\dfrac{d}{dx}\left[\sin(A)\right] = \cos(A)\dfrac{dA}{dx}$	$\dfrac{d}{dx}\left[\sinh(A)\right] = \cosh(A)\dfrac{dA}{dx}$

4.6 ME516 Sp2013

We could've solved $\frac{d^2\bar{T}_h}{dx^2} - m^2\bar{T}_h = 0$ by either $\bar{T}_h = C_1\exp(mx) + C_2\exp(-mx)$ or $\bar{T}_h = C_3\sinh(mx) + C_4\cosh(mx)$

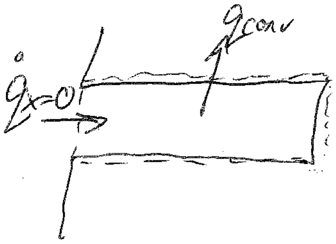
$\Rightarrow$ this would've lead to

$$\frac{T - T_\infty}{T_b - T_\infty} = \frac{\cosh\left[mL\left(1-\frac{x}{L}\right)\right]}{\cosh(mL)}$$

$\leftarrow$ much more compact & easier to write

Table 1-4 pg 104

$\rightarrow$ What about total HT from fin?



$$\dot{q}_{conv} = \int_0^L h\,per\,(T - T_\infty)\,dx$$

$$\dot{q}_{x=0} = -kA_c\left(\frac{dT}{dx}\right)_{x=0}$$

Should give same answer

This one likely easier as T is constant.

$$\dot{q}_{x=0} = -kA_c\frac{d}{dx}\left[T_\infty + (T_b - T_\infty)\frac{\cosh\left[mL\left(1-\frac{x}{L}\right)\right]}{\cosh(mL)}\right]_{x=0} \Rightarrow$$

$$\Rightarrow \dot{q}_{x=0} = \frac{-kA_c(T_b - T_\infty)}{\cosh(mL)}\frac{d}{dx}\left(\cosh\left[mL\left(1-\frac{x}{L}\right)\right]\right)_{x=0} \Rightarrow$$

$$\Rightarrow \dot{q}_{x=0} = -\frac{kA_c(T_b - T_\infty)}{\cosh(mL)}\left(-m\sinh\left[mL\left(1-\frac{x}{L}\right)\right]\right)_{x=0} \Rightarrow$$

$$\Rightarrow \dot{q}_{x=0} = kA_c(T_b - T_\infty)\sqrt{\frac{h\,per}{kA_c}}\tanh(mL)$$

$\leftarrow$ same as table 1-4

Also in table 1-4 is the fin efficiency:

$$\text{Fin efficiency (for any geometry)} = \eta_{fin} = \frac{\text{fin heat transfer}}{\text{heat transfer to a "perfect" fin } (k \to \infty)}$$

$R_{cond,x} \to 0$ so the fin is isothermal at the base temp

$$\eta_{fin} = \frac{\dot{q}_{fin}}{\bar{h} A_{s,fin} (T_b - T_\infty)}$$

for our rectangular fin: $\dot{q}_{fin} = \sqrt{\bar{h}\, per\, k A_c}\,(T_b - T_\infty)\tanh(mL)$

$$\eta_{fin} = \frac{\sqrt{\bar{h}\, per\, k A_c}\,(T_b - T_\infty)\tanh(mL)}{\bar{h}\, per\, L\,(T_b - T_\infty)}$$

$$\eta_{fin} = \frac{\tanh(mL)}{\sqrt{\frac{\bar{h}\, per}{k A_c}}\, L} \Rightarrow \boxed{\eta_{fin} = \frac{\tanh(mL)}{mL}}$$ fin efficiency

~~* Show fin library in EES.~~

Fin Resistance is a convenient way of accounting for fins

$$\eta_{fin} = \frac{\dot{q}_{fin}}{\bar{h} A_{s,fin} (T_b - T_\infty)} \Rightarrow \dot{q}_{fin} = \eta_{fin} \bar{h} A_{s,fin} (T_b - T_\infty)$$

$$\underbrace{\eta_{fin} \bar{h} A_{s,fin}}_{R_{fin}} \rightarrow R_{fin} = \frac{1}{\eta_{fin} \bar{h} A_{s,fin}}$$

accounts for both convection & conduction

* Show fin library in EES