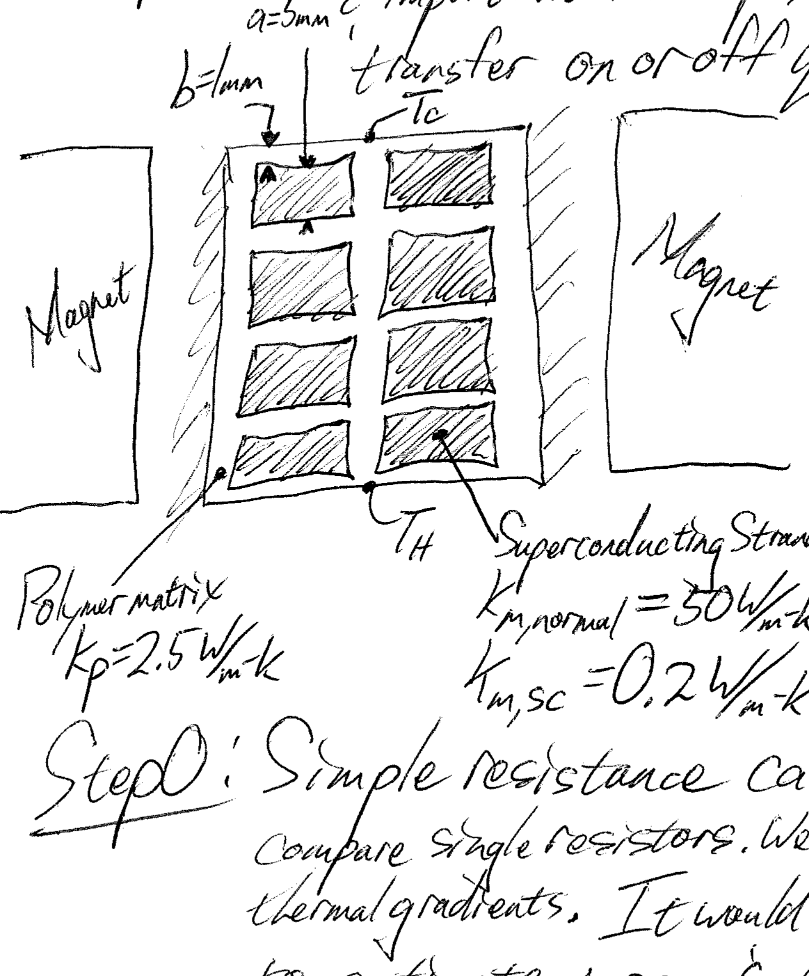
2D Resistance Hacks

When we started 2-D conduction we considered Shape Factors ⇒ known analytical solutions that provide effective thermal resistances based on the geometries of the problem. These were fast & got us close. We can use resistance networks in more ways as quick hacks to help us understand & simplify 2D conduction problems.

Isothermal & Adiabatic Limits

Example: Superconducting Heat "Switch". Heat switches are challenging & important in complex systems when you need to turn the heat transfer on or off quickly & reliably.



When a material goes superconducting all of the electrons are mobile for electrical conduction & are not available for heat conduction. By varying the magnetic field on the superconductor you can drive it between superconducting & normal states. This effectively forms a "thermal switch". What's the difference in Q between "on" or "off"?

Step 0: Simple resistance calcs! But how? Before we used Biot # to compare single resistors. We could compare R_{SC} to R_p, but we still have 2-D thermal gradients. It would be nice & convenient if we used resistances to estimate upper & lower limits on the overall HT through the switch in both the on & off cases.

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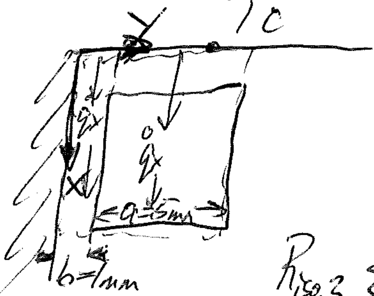
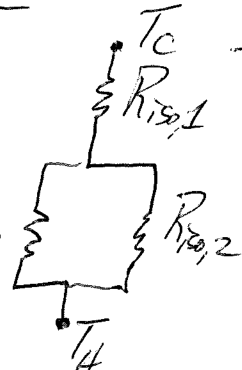
Isothermal limit: Assumes perfect (no resistance) to HT in one direction.

In this case we'll assume no $\frac{dT}{dy}$ or perfect $\dot{q}_y$ in





just polymer layers

$$R_{iso,1}=\frac{\Delta x}{kA_c}=\frac{5b}{k_pW(2a+3b)}$$

polymer between SC

$$R_{iso,2}=\frac{4a}{k_pW3b}$$

across SC

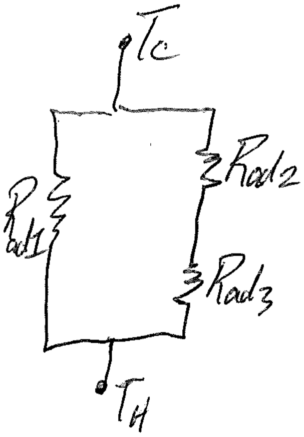
$$R_{iso,3}=\frac{4a}{k_{SC}W2a}$$

☆ Isothermal limit is a lower bound on the resistance (perfect y conduction)

$$R_{iso}=R_{iso1}+\left[\frac{1}{R_{iso,2}+R_{iso3}}\right]^{-1} \Rightarrow \underline{R_{iso}=225.9\,\frac{K}{W}}$$

Adiabatic Limit: Assumes no (total resistance) to HT in one direction. (adiabatic)

In this case no $\dot{q}_y$. All HT is $\dot{q}_x$.



along polymer in x

$$R_{ad1}=\frac{\Delta x}{kA_c}=\frac{(4a+5b)}{k_pW3b}$$

Polymer between SC

$$R_{ad,2}=\frac{5b}{k_pW2a}$$

across SC

$$R_{ad3}=\frac{4a}{k_{SC}W2a}$$

$$R_{ad}=\left[\frac{1}{R_{ad1}}+\frac{1}{(R_{ad_2}+R_{ad_3})}\right]^{-1}=\underline{R_{ad}=251.2\,\frac{K}{W}}$$

☆ Adiabatic limit is an upper bound on the resistance (no y conduction)

$$B_i=\frac{R_{ad}}{R_{iso}}=1.112 \rightarrow$$ the upper & lower limits are within $\pm 12\%$!

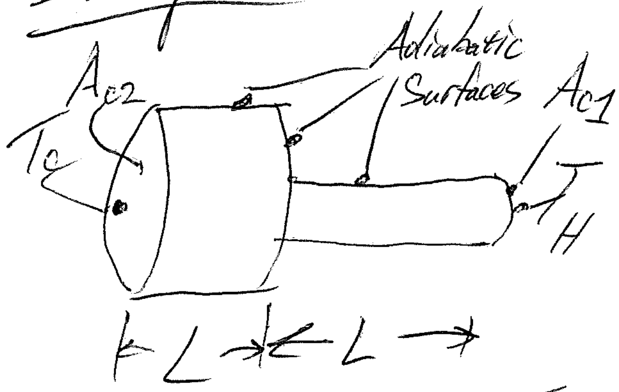
Note that $R_{ad}>R_{iso}$ regardless of k_{SC}.

Show EES code of limits & solution.

When $k_{SC}\approx k_p$ $R_{ad}\approx R_{iso}$

☆ This is a handy way to bound a 2-D analysis, & sometimes determine that one is not needed!

Example: Heat transfer through a Socket Head Cap screw. Length constant but the area changes.



$R_{ad} \to$ no radial HT $\to R_{ad} = \dfrac{2L}{kA_{c1}}$

$R_{iso} \to$ perfect radial HT $\to R_{iso} = \dfrac{L}{kA_{c,1}} + \dfrac{L}{kA_{c,2}}$

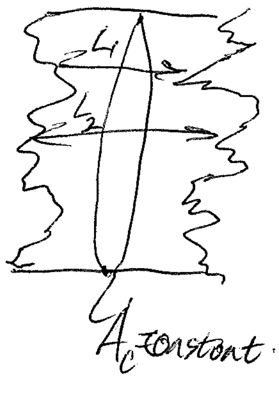
The bounds between R_{ad} & R_{iso} may not be as useful.

An alternative: <u>Average Area Resistance limit</u>: assumes an effective area for HT through the problem.

$$R_{AA} = \frac{4L}{k(A_{c1}+A_{c2})} \qquad \text{Average Area} = \frac{A_{c1}+A_{c2}}{2}$$

☆ The average area method will underpredict R even more than the isothermal limit. As $A_1 \to 0$, this remains finite but isothermal will diverge.

Example: Heat transfer through a rough concrete block. Area is ~constant but the length changes.



$$R_{AL} = \frac{\bar{L}}{kA_c}$$ Integrated average length

☆ The average length model overestimates the resistance to a greater extent than the adiabatic limit. When $L \to 0$ causes $R \to 0$ however R_{AL} will remain finite.

$$R_{AA} \le R_{iso} \le R_{actual} \le R_{ad} \le R_{AL}$$

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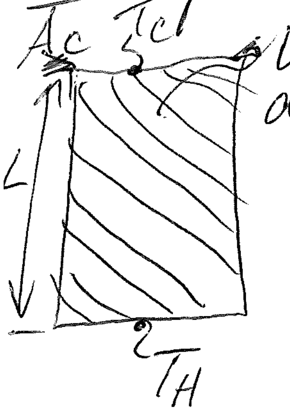
Example: Block of G-10 composite, silica fibers @ 45° relative to block. Epoxy layers bond the silica strands together.



From EES @ 77K

$$k_{G10_{Normal}} = 0.2798 \frac{W}{mK} \qquad k_{G10_{warp}} = 0.3856 \frac{W}{mK}$$

Which resistance approach should we use?

None seem to be appropriate.

$$\dot{q}_{Total} = \dot{q}_{Normal} + \dot{q}_{Warp} = \frac{k_N A_N}{L}(T_H - T_c) + \frac{k_W A_W}{L}(T_H - T_c)$$

We can simplify this via $\left(\frac{k_N + k_W}{2}\right) = k_{eff} \Rightarrow \dot{q}_{total} = \frac{k_{eff} A_c}{L}(T_H - T_c)$

This is known as the effective thermal conductivity method. It assumes the material is homogeneous with an effective thermal conductivity that would lead to the same overall HT rate.

☆ The challenge with all or any of these methods is being able to quickly & confidently using them to bound or simplify a problem. You should make a diagram or process map for applying these to simplify any 2D problem as much as possible to determine if a full analytical or numerical analysis is needed.