When we started 2-D conduction we considered Shape Factors known as analytical solutions that provide effective thermal resistances based on the geometries of the problem. These were fast & got us close. We can use resistance networks in more ways as quick hacks to help us understand & simplify 2D conduction problems.

Isothermal & Adiabatic Limits

Example: Superconducting Heat "Switch". Heat switches are challenging & important in complex systems when you need to turn the heat transfer on or off quickly & reliably.

When a material goes superconducting all of the electrons are mobile for electrical conduction & are not available for heat conduction. By varying the magnetic field on the superconductor you can drive it between superconducting & normal states. This effectively forms a "thermal switch". What's the difference in Q between on/off?

Step 0: Simple resistance calcs! But how? Before we used Boltzmann to compare single resistors. We could compare $R_{sc}$ to $R_p$, but we still have 2D thermal gradients. It would be nice & convenient if we used resistances to estimate upper & lower limits on the overall HT through the switch in both the on & off cases.
Isothermal limit: Assumes perfect (no resistance) to HT in one direction. In this case we'll assume no ohmic drop for perfect & thin polymer layers:

\[ R_{iso} = \frac{\Delta x}{k_A} = \frac{56}{k_A W^2 a} \]
across SC

\[ R_{iso} = \frac{4a}{k_A W^2 a} \]

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Adiabatic limit: Assumes no (total resistance) to HT in one direction. In this case no \( \dot{q} \). All HTs go.

along polymer + \[
R_{al} = \frac{\Delta x}{k_A} = \frac{(4a + 56)}{k_p W^2 a} \]

across SC

\[ R_{al} = \frac{4a}{k_p W^2 a} \]

\[ \frac{R_d}{R_{iso}} = 11.2 \Rightarrow \text{the upper & lower limits are within } \pm 12\% \]

Show EES code of limits & solution, if possible. Note that \( R_d > R_{iso} \) generally.

This is a handy way to bound a 2-D analysis, & sometimes determine that one is not needed!
Example: Heat transfer through a Cap screw. Length constant but the area changes.

$$R_{rad} \rightarrow \text{no radial HT} \rightarrow R_{rad} = \frac{2L}{kA_{c1}}$$

$$R_{iso} \rightarrow \text{perfect radial HT} \rightarrow R_{iso} = \frac{L + L}{kA_{c1} + kA_{c2}}$$

The bounds between $R_{rad}$ and $R_{iso}$ may be used.

An alternative: Average Area Resistance Limit: assumes an effective area for HT through the problem.

$$R_{AA} = \frac{4L}{k(A_{c1} + A_{c2})} \text{ The average area method will underpredict } R \text{ even more than the isothermal limit. As } A \to 0, \text{ this remains finite but isothermal will diverge.}$$

$$R_{AA} = \frac{2}{kA_{c}} \text{ The average length model overestimates the resistance to a greater extent than the adiabatic limit. When } L \to 0 \text{ causes } R \to 0 \text{ however } R_{AA} \text{ will remain finite.}$$

$$\frac{R_{AA}}{R_{iso}} \leq R_{actual} \leq R_{rad} \leq R_{iso}$$
Example: Block of C-10 composite, silica fibers at 45° relative to block. Epoxy layers bond the silica strands strongly.

\[ K_{C10} = 0.2798 \text{ W/mK} \quad K_{\text{Epoxy}} = 0.3864 \text{ W/mK} \]

Which resistance approach should we use?

None seem to be appropriate.

\[ \dot{Q}_{\text{Total}} = \dot{Q}_{\text{Normal}} + \dot{Q}_{\text{Warp}} = \frac{\dot{Q}_{\text{A}}}{L} \left( T_h - T_c \right) \]

We can simplify this via \( \frac{\dot{Q}_{\text{A}}}{L} = k_{\text{Eff}} \) \( \Rightarrow \)

\[ \dot{Q}_{\text{Total}} = \frac{K_{\text{Eff}} \cdot A}{L} \left( T_h - T_c \right) \]

This is known as the effective thermal conductivity method. It assumes the material is homogenous with an effective thermal conductivity that would lead to the same overall HT rate.

The challenge with all of these methods is being able to quickly & confidently use them to bound or simplify a problem. You should make a diagram or process map for applying these to simplify any 2D problem as much as possible to determine if a full analytical or numerical analysis is needed.