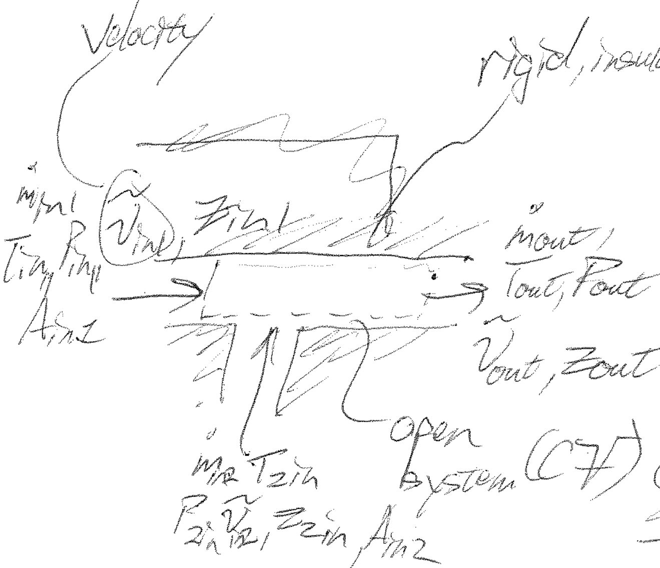
ME 301 S2011 8.1

Energy Balances on Open Systems

Up until now we have only had 2 ways to transfer energy across a system boundary: 1) Work, 2) Heat

is there another way? Yes! → 3) Energy carried by a flow of mass

only apples to open systems



Step 1: Define System

Step 2: System Assumptions

1) Open System 2) Rigid, Insulated T, no shafts 3) Steady

Step 3: Apply Balances

Mass Balance (Incremental): $m_{in} = m_{out} + \Delta m_{stored} \Rightarrow \dot{m}_{in1}t + \dot{m}_{in2}t = \dot{m}_{out}t + m_2 - m_1$

It's sometimes easier to write an incremental balance, especially if this could run on indefinitely.

Mass Balance (Rate): $\dot{m}_{in} = \dot{m}_{out} + \frac{dm}{dt} \Rightarrow \dot{m}_{in,1} + \dot{m}_{in,2} = \dot{m}_{out}$ Steady State

→ what happens if this is not zero,

how do we find $\dot{m}$ in terms of velocity?

$\dot{V}$ = volumetric flow rate $\frac{\dot{V}}{v_{in}} = \dot{m}_{in}$

specific volume

$\dot{V} = \tilde{v}_{in} A_{in}$ cross sectional area of inlet

$= \frac{\pi}{4}D^2$ ← for circle

$\tilde{v}_1 = \frac{4\dot{V}}{\pi D_1^2}$

Sometimes we'll be given volumetric flow rate, others velocity & area.

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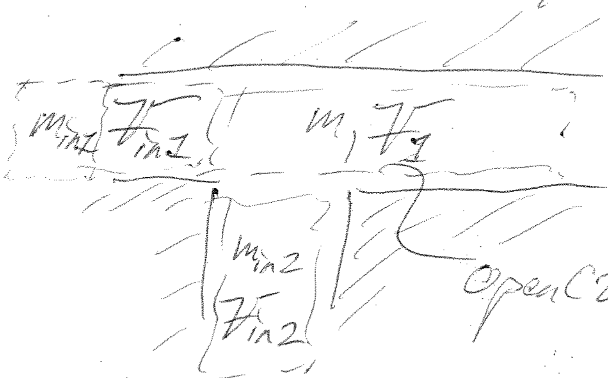
$\Delta U + \Delta KE + \Delta PE$

Energy Balance (Incremental): $E_{in} = E_{out} + \Delta E_{stored}$

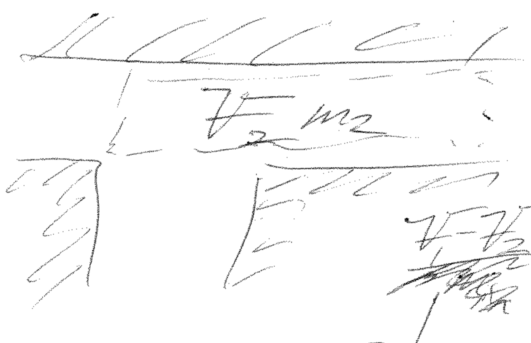
$E_{in} = ?$ Our flow coming in has thermal energy, kinetic energy, & potential energy relative to our CV/open system.

Is it just: $E_{in} = Q_{in} + W_{in} + m_{in1}\left(u_1 + \frac{V_1^2}{2} + g z_1\right) + m_{in2}\left(u_2 + \frac{V_2^2}{2} + g z_2\right)$

No! It's not that easy! There is an additional energy carried by the flow required to push the mass into the system. Why? Not obvious



To push this mass into the original volume, we have to change the system volume



As we change the system Volume $W_{flow} = \int P dV$

$$W_{flow} = -\int P_{in1} dV_{in1} - \int P_{in2} dV_{in2}$$

$$W_{flow} = P_{in1}\left(\overset{m_{in1}v_{in1}}{V_{in1,1}} - \overset{0}{V_{in1,2}}\right) + P_{in2}\left(\overset{m_{in2}v_{in2}}{V_{in2,1}} - \overset{0}{V_{in2,2}}\right)$$

$$W_{flow} = P_{in1} m_{in1} v_{in1} + P_{in2} m_{in2} v_{in2}$$

This appears in every open system & we have to account for it

$$E_{in} = Q_{in} + W_{in} + \sum_{i=1}^{N_0} m_{in,i}\left(u_{in,i} + P_{in,i} v_{in,i} + \frac{V_{in,i}^2}{2} + g z_{in,i}\right)$$

Shaft work

since u, P, & v are all intensive properties, this is a property combination

$h = u + Pv$ is defined as the property enthalpy

Our full energy balance on a rate basis is then: $\dot{E}_{in} = \dot{E}_{out} + \frac{dE_{stored}}{dt}$

$$\dot{Q}_{in} + \dot{W}_{in} + \sum_{i=1}^{N} \dot{m}_{in,i}\left(h_{in,i} + \frac{\tilde{v}_{in,i}^2}{2} + gz_{in,i}\right) = \dot{Q}_{out} + \dot{W}_{out} + \sum_{i=1}^{M} \dot{m}_{out,i}\left(h_{out,i} + \frac{\tilde{v}_{out,i}^2}{2} + gz_{out,i}\right)$$

$$+ \frac{dU}{dt} + \frac{dKE}{dt} + \frac{dPE}{dt}$$ Applied to our system

$$\dot{m}_{in,1}\left(h_{in,1} + \frac{\tilde{v}_{in,1}^2}{2} + gz_{in,1}\right) + \dot{m}_{in,2}\left(h_{in,2} + \frac{\tilde{v}_{in,2}^2}{2} + gz_{in,2}\right) = \dot{m}_{out}\left(h_{out} + \frac{\tilde{v}_{out}^2}{2} + gz_{out}\right)$$

→ Where do we get values for the enthalpy? 3 different models

1) Real fluid: enthalpy is listed next to internal energy in the tables in your book. $x = \frac{h - h_f}{h_g - h_f}$, in EES: h=enthalpy(Fluid, Prop1=Value1, Prop2=Value2)

2) Ideal gas: What do we know about ideal gas?

1) $Pv = RT$, 2) $u = f(T)$ What does this say about enthalpy $h = u + Pv$ ← $f(T)$
It's only a function of temp!

$C_v = \left(\frac{\partial u}{\partial T}\right)_v$ & $C_p = \left(\frac{\partial h}{\partial T}\right)_p$ h is only $f(T)$, not P → $dh = C_p dT \Rightarrow \int_{h_1}^{h_2} dh = \int_{T_1}^{T_2} C_p dT$

$\Rightarrow h_2 - h_1 = \int_{T_1}^{T_2} C_p dT$ if C_p is constant $\Rightarrow h_2 - h_1 = C_p(T_2 - T_1)$

If C_p is not constant, ideal gas tables in book, or in EES h_1=enthalpy(Ideal Fluid, T=T_1)

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3) Incompressible substance? 1) v = constant, 2) $u = f(T)$

$h = u + Pv \Rightarrow h_2 - h_1 = (u_2 + P_2 v_2) - (u_1 + P_1 v_1) \Rightarrow$

$\Rightarrow h_2 - h_1 = (u_2 - u_1) + (P_2 v_2 - P_1 v_1)$ if v is constant then

$\Rightarrow h_2 - h_1 = (u_2 - u_1) + v(P_2 - P_1)$ where $u_2 - u_1$ for incompressible substance is

$\Rightarrow h_2 - h_1 = \int_{T_1}^{T_2} C\,dT + v(P_2 - P_1)$ if C is constant

$$\underline{\underline{h_2 - h_1 = C(T_2 - T_1) + v(P_2 - P_1)}}$$