Closed System Energy Balances:

Last week we covered properties of fluids, the week before we discussed systems & balances.

Today we start putting everything together: Balances, Energy, Fluid properties.

Example: Water is compressed in a series of steps:

1) Predicted volume of liquid water is 0.05 m³.
2) First it is compressed @ constant temperature until liquid water droplets begin to form. This process is maintained @ constant temperature by heat transfer to the surroundings.

2→3) The piston is pushed in further until $V = 0.03 m^3$ again @ constant temperature.

Find: the heat transfer from 2→3 to maintain the constant temperature.

How do we begin? Let's establish some general steps for problem solving:

Step 1: Draw & define the system.

Step 2: List assumptions about the problem:
1) Pure water (real fluid model), 2) Sealed piston/closed system, 3) Rigid cylinder, 4) No PE, KE, or NPE, 5) Process is incremental.

Step 3: Apply balances & simplify (if needed):

Mass Balance: $M_{in} + M_{produced} = M_{out} + M_{stored} + \Delta M_{stored} \Rightarrow M_{out} = M_{in} - M_{stored}$
Energy Balance: \[ E_{\text{in}} + E_{\text{produced}} = E_{\text{out}} + E_{\text{destroyed}} + E_{\text{stored}} \]

\[ \Rightarrow Q_{\text{in}} + W_{\text{in}} = Q_{\text{out}} + W_{\text{out}} + \Delta U + \Delta (PE + KE + ME) \]

\[ \Rightarrow W_{\text{in}} = Q_{\text{out}} + \Delta U \Rightarrow m(u_2 - u_1) \]

PT work \( = P \Delta V \)

\[ \Rightarrow P \Delta V = Q_{\text{out}} + m(u_2 - u_1) \]

This is about as simplified as we can make it, this sign is needed.

Step 4: Apply fluid model, Draw diagram, Fix states

State 1: \( P = 15 \text{ bar}, T = 150^\circ C, \text{v} = 1 \text{ m}^3 \)

Vapor Table A-5: \( T_s = 113.3^\circ C \) so we must be in the saturated vapor region

Go to Table A-6 to find \( v \) & \( u \)

@ 100 kPa & 150°C \( u = 1936.7 \text{ m}^3 / \text{kg} \)

@ 200 kPa & 150°C \( u = 2582.9 \text{ m}^3 / \text{kg} \)

We need to interpolate!

\[ \frac{P_{\text{High}} - P_{\text{Low}}}{P_{\text{High}} - P_{\text{Low}}} = \frac{150 - 100}{200 - 100} \]

\[ \Rightarrow u = 1936.7 \left( \frac{150}{100} \right) = 2582.9 \text{ m}^3 / \text{kg} \]

\[ \frac{P_{\text{High}} - P_{\text{Low}}}{P_{\text{High}} - P_{\text{Low}}} = \frac{150 - 100}{200 - 100} \]

\[ \Rightarrow u = 2582.9 \left( \frac{150}{100} \right) = 2582.9 \text{ m}^3 / \text{kg} \]
We can find the mass of water in the tank now:

\[ M = \frac{T_i}{V_i} = \frac{1\text{ m}^3}{1.448\text{ m}^3} = 0.690\text{ kg} \]

Now find state 2; compressed at constant \( T \) until it drops to form \( x_2 = 10 \)

\[ T_2 = 150^\circ C \]

Which table? Saturated water A-4 @ \( T_2 = 150^\circ C \)

We see that \( P_2 = 4.76\text{ bar} \)

\[ V_2 = 0.39248\text{ m}^3 \]

\[ \rho_2 = 2539.11\text{ kg/m}^3 \]

\[ T_2 = ? \]

\[ \rho_2 = 0.690\text{ kg} \left( 0.39248\text{ m}^3 \right) \Rightarrow T_2 = 0.271\text{ m}^3 \]

Now find state 3; compressed at constant \( T \) until \( T_3 = 0.05\text{ m}^3, P_3 = 150^\circ C \)

What can we find? We know that \( V_2 = 0.00109\text{ m}^3 \)

\[ V_3 = \frac{T_3}{M} = 0.05\text{ m}^3 \Rightarrow V_3 = 0.0742\text{ m}^3 \]

\[ \rho_3 = 0.690\text{ kg} \]

\[ x_3 = \frac{V_3 - V_i}{V_i - V_f} \Rightarrow x_3 = \frac{0.0742 - 0.00109}{0.00109 - 0.000612} \approx 0.187 \]

But how to find \( x_2 \)? Find the quality:

\[ x_3 = \frac{V_3 - V_f}{V_f - V_i} \Rightarrow V_3 = V_f + x (V_f - V_i) \Rightarrow V_3 = 0.3166 + 0.187(0.3927) \Rightarrow V_3 = 0.4078\text{ m}^3 \]

\[ \frac{V_3 - V_f}{V_f - V_i} = \frac{0.4078 - 0.000612}{0.000612} \approx 0.187 \]

Volume of just the liquid?

\[ x = 1 - \frac{M_f}{M} \Rightarrow M_f = (1-x)M = (1-0.187)0.690\text{ kg} \]

\[ M_f = 0.561\text{ kg} \]

\[ V_f = M_f \rho_f = 0.561\text{ kg} \times 0.00109\text{ m}^3 \Rightarrow V_f = 0.000612\text{ m}^3 \]
Step 5: Now solve balance: 

\[ P \left( \frac{V_2^3}{V_3} \right) = Q_{\text{out}} + m_l (u_3 - u_2) \]

Since \( 2 \rightarrow 3 \) process occurs at constant pressure:

\[ P(T_2 - T_3) = Q_{\text{out}} + m_l (u_3 - u_2) \Rightarrow Q_{\text{out}} = P(T_2 - T_3) + m_l (u_3 - u_2) \]

\[ Q_{\text{out}} = 472,000 \text{ Pa}(0.271 \text{ m}^3 - 0.05 \text{ m}^3) + 0.80 \text{ kg}(2,539,100 \frac{\text{J}}{\text{kg}} - 992,780 \frac{\text{J}}{\text{kg}}) \]

\[ Q_{\text{out}} = 1,186,000 \text{ J} \Rightarrow Q_{\text{out}} = 1.186 \text{ MJ} \]

Heat lost the system as the sign is still positive, makes sense with engineering judgement.