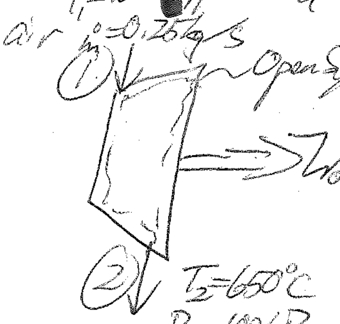
Last time we ~~defined the~~ balanced entropy on open systems. Today we continue our 2nd Law analysis of open systems → this will allow us to define meaningful efficiencies for them.

Turbine: Find Turbine power & its efficiency



Mass Balance: $\mathring{m}_{in}=\mathring{m}_{out}+\cancel{\frac{dm_{stored}}{dt}}^{\,0}$

Energy Balance: $\mathring{E}_{in}=\mathring{E}_{out}+\cancel{\frac{dE_{stored}}{dt}}^{\,0(ss)}$

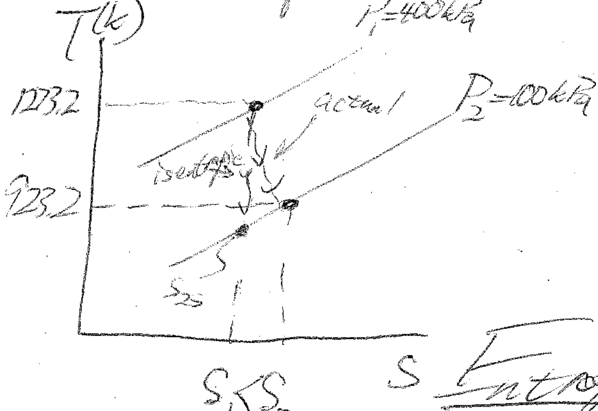
$$(\underbrace{\mathring{Q}_{in}}_{0\,(adiabatic)}+\underbrace{\mathring{W}_{in}}_{0}+\mathring{m}_{in}(h_{in}+\frac{V_{in}^2}{2}+gz_{in}))=\underbrace{\mathring{Q}_{out}}_{0\,(adiabatic)}+\mathring{W}_{out}+\mathring{m}_{out}(h_{out}+\frac{V_{out}^2}{2}+gz_{out})$$

0 (ΔPE, ΔKE negligible)

$$\mathring{m}_{in}h_{in}=\mathring{W}_{out}+\mathring{m}_{out}h_{out}\Rightarrow\mathring{W}_{out}=\mathring{m}(h_{in}-h_{out})$$

assume ideal gas constant C_p

$$\mathring{W}_{out}=\mathring{m}C_p(T_{in}-T_{out})\Rightarrow\frac{0.25kg}{s}\left|\frac{1005J}{kg\text{-}k}\right|(1273.2-923.2)K\Rightarrow\mathring{W}_{out}=87,938W$$



$s_1 < s_2$

T-h & T-s diagrams are more useful in industry. Why?

Entropy is a description of efficiency & Enthalpy a measure of work possible.

Entropy Balance: $\mathring{S}_{in}+\mathring{S}_{Gen}=\mathring{S}_{out}+\cancel{\frac{dS_{stored}}{dt}}^{\,0}$

$$\mathring{m}s_{in}+\mathring{S}_{Gen}=\mathring{m}s_{out}\Rightarrow\mathring{S}_{Gen}=\mathring{m}(s_{out}-s_{in})\Rightarrow\mathring{S}_{Gen}=\mathring{m}\left[C_p\ln\left(\frac{T_{out}}{T_{in}}\right)-R\ln\left(\frac{P_{out}}{P_{in}}\right)\right]$$

$$\mathring{S}_{Gen}=\frac{0.25kg}{s}\left[\frac{1005J}{kg\text{-}k}\ln\left(\frac{923.2}{1273.2}\right)-287.1\frac{J}{kg\text{-}k}\ln\left(\frac{100}{400}\right)\right]\Rightarrow\mathring{S}_{Gen}=18.74\frac{W}{k}$$

$s_2 > s_1$ just like figure

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But how good is the turbine? How do we define it's efficiency?

What's the best we could do? ⇒ Reversible turbine $s_2 = s_1$

Compare this to actual ΔS

⇒ 2 ways to define efficiency

1) System → $\dfrac{\text{What you want}}{\text{What you pay}}$ ⇐ approach used for heat engines, refrigerators, heat pumps. Sometimes called a 1st Law efficiency.

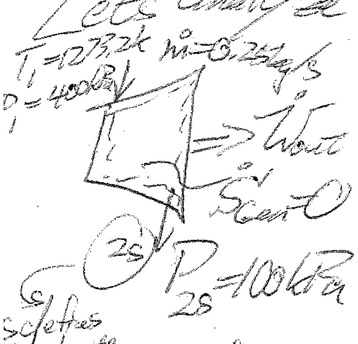
2) Component → $\dfrac{\text{What you got} \Rightarrow \text{Actual}}{\text{What you could've} \Rightarrow \text{Ideal/Reversible}}$ ⇐ approach used for turbines, compressors, etc... any component. Sometimes called 2nd Law efficiency

Turbine Efficiency: $\eta_T = \dfrac{\dot{W}_{actual}}{\dot{W}_{ideal}}$ ⇐ actual turbine produced 87,938 W; ⇐ power produced by a reversible turbine that does the same job.

⇒ In limit of ΔPE & ΔKE, adiabatic: $\eta_T = \dfrac{h_1 - h_2}{h_1 - h_{2s}}$

What do I mean by the same job? 1) recieves same fluid @ same inlet conditions
2) exhausts to the same exit pressure

Let's analyze the reversible turbine that does the same job.



Whenever you analyze a reversible device it's a good idea to start with an entropy balance / 2nd law.

Entropy Balance: $\dot{S}_{in} + \dot{S}_{gen} = \dot{S}_{out} + \cancel{\dfrac{dS_{sys}}{dt}}^{0}$ (reversible $\dot{S}_{gen}=0$) $\Rightarrow \dot{S}_{in} = \dot{S}_{out} \Rightarrow \dot{m}_{in} s_{in} = \dot{m}_{out} s_{2s}$

$s_1 = s_{2s}$ & P_{2s} fixes state

recall from Monday: $\dfrac{T_{2s}}{T_1} = \left(\dfrac{P_{2s}}{P_1}\right)^{R/c_p} \Rightarrow T_{2s} = 1273.2\text{K}\left(\dfrac{100}{400}\right)^{\frac{287.1}{1005}} \Rightarrow T_{2s} = 856.8\text{K}$

Energy Balance: $\dot{E}_{in} = \dot{E}_{out} \Rightarrow \dot{W}_{out} = \dot{m}c_p(T_1 - T_{2s}) \Rightarrow \dfrac{0.25\text{kg}}{\text{s}} \Big| \dfrac{1005\text{J}}{\text{kg}\cdot\text{K}} \Big| (1273.2 - 856.8\text{K})$ $\dot{W}_{out} = 104,600\text{W}$

$\eta_T = \dfrac{87,938\text{W}}{104,600\text{W}} \Rightarrow \eta_{turbine} = 0.841 \Rightarrow 84.1\%$ efficient

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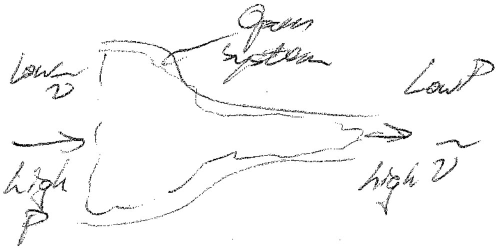
Nozzle

: What is the purpose of a nozzle? ⇒ create exit stream with high $\tilde{v}$.



So what's the logical definition of a nozzle?

actual relative to best you can do

$$\eta_{noz} = \frac{\frac{1}{2}\tilde{v}_2^2}{\frac{1}{2}\tilde{v}_{2s}^2}$$

$\frac{1}{2}\tilde{v}_2^2$ → actual kinetic energy of exit stream

$\frac{1}{2}\tilde{v}_{2s}^2$ → kinetic energy of exit stream produced by a reversible nozzle that does the same job

same job: 1) Same inlet state conditions 2) exhausts to same exit pressure

Energy Balance

: $\dot{E}_{in} = \dot{E}_{out} + \frac{\Delta E_{stored}}{dt}$ (0 css)

$$\dot{Q}_{in} + \dot{W}_{in} + \dot{m}_{in}\left(h_{in} + \frac{\tilde{v}_{in}^2}{2} + g z_{in}\right) = \dot{Q}_{out} + \dot{W}_{out} + \dot{m}_{out}\left(h_{out} + \frac{\tilde{v}_{out}^2}{2} + g z_{out}\right)$$

0 (adiabatic) 0 (no work)

$\dot{m}_{in} = \dot{m}_{out}$

$$h_{in} + \frac{\tilde{v}_{in}^2}{2} = h_{out} + \frac{\tilde{v}_{out}^2}{2}$$

$$h_1 - h_2 \quad h_{in} - h_{out} = \frac{\tilde{v}_{out}^2}{2}$$

$$h_1 - h_{2s} = \frac{\tilde{v}_{2s}^2}{2} \quad \text{so} \quad \eta_{noz} = \frac{h_1 - h_2}{h_1 - h_{2s}}$$

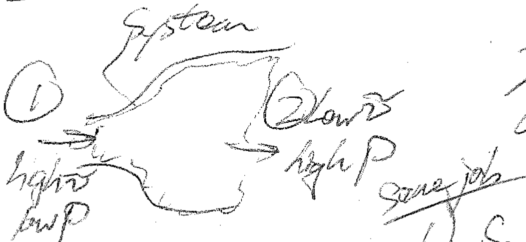
actual

← for adiabatic nozzle with no ΔPE

This means if you have the nozzle efficiency, inlet conditions, & exit pressure, you can solve for the final state! Example 7-16

Diffuser

; Purpose: Increase in Pressure $P_2 > P_1$ ← Could have also done Velocity



$$\eta_{diff} = \frac{P_2 - P_1}{P_{2s} - P_1}$$

$P_2 - P_1$ ← actual ΔP

$P_{2s} - P_1$ ← ideal ΔP doing same job

same job: 1) Same inlet conditions, 2) Same exit velocity