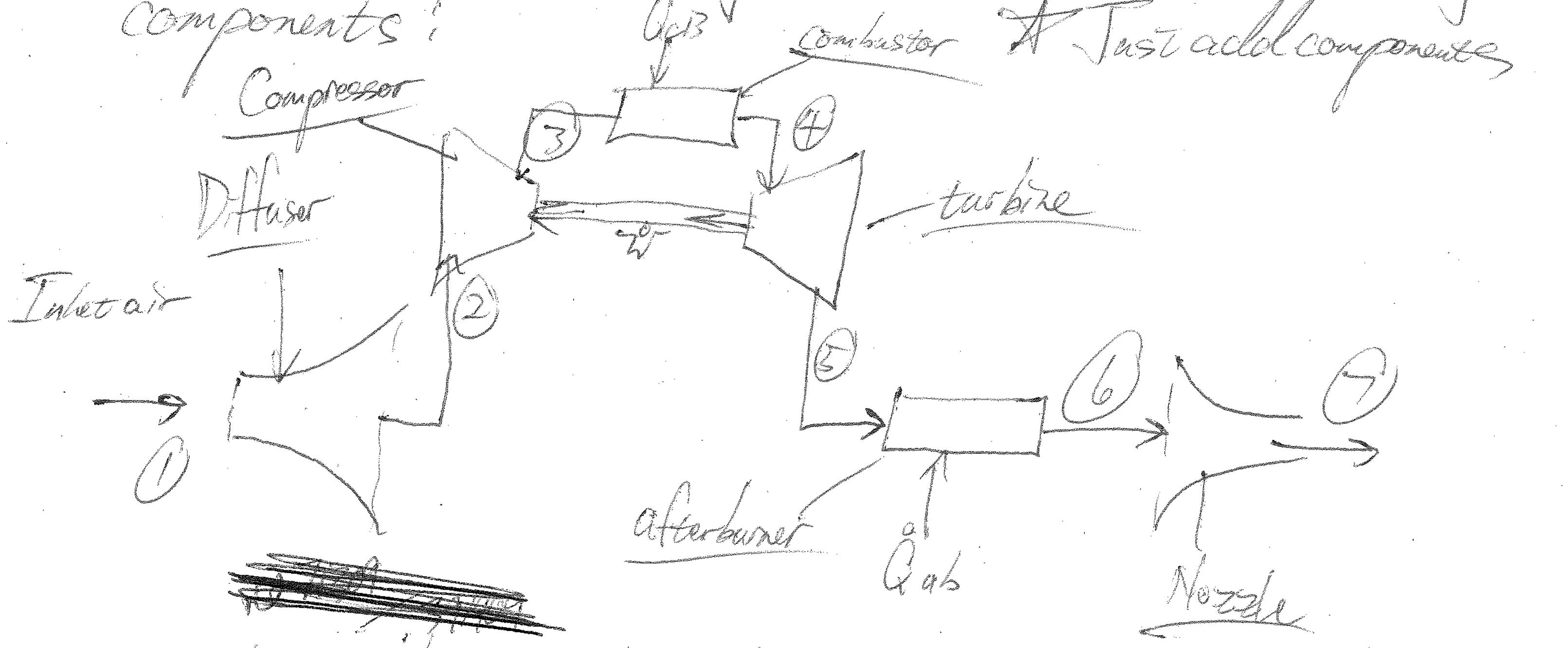
Thermodynamic Analysis of Engineering Components

- ~~The plan in the syllabus was to cover pipe flow, nozzles, diffusers then move to comp~~

- Now that we can apply energy balances to open systems we can begin to look @ engineering devices:
  We'll spend the next 2 days looking @ the Goblin-II turbojet engine used on F80 military jet airplanes
  - Show picture

- Most ~~will~~ gas turbine engines consist of the following components: ★ Just add components



We'll use what we've learned to analyze this system piece by piece...

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Diffuser: The purpose of a diffuser is to convert a high velocity low pressure flow into a low velocity, high pressure flow.

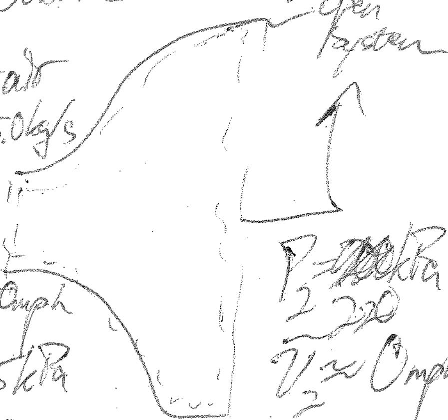
For Goblin-II

Inlet air $\dot{m} = 25.0\,kg/s$

$\tilde{v}_1 = 350\,mph$

$P_1 = 75\,kPa$

$T_1 = 5°C$



$P_2 = 200\,kPa$

$\tilde{v}_2 \approx 0\,mph$

Assuming air is ideal with constant heat capacity, find a) Inlet diameter of Diffuser

~~b) Temperature of air exiting diffuser~~

Mass balance (Rate): $\dot{m}_{in} = \dot{m}_{out} + \frac{dM}{dt}$ → 0 (steady state)

$\dot{m}_{in} = \dot{m}_{out} = 25.0\,kg/s$ how do we relate to A_{in}?

$\dot{m}\left(\frac{kg}{s}\right) = \rho_1 A_1 \tilde{v}_1 \qquad \rho_1 = \frac{1}{v_1} \quad \text{where } v_1 = \frac{RT_1}{P_1}$

$R_{air} = \frac{R_{ideal}}{mw_{air}} = \frac{8314\,\frac{J}{kmol\cdot K}}{28.97\,\frac{kg}{kmol}} \Rightarrow R_{air} = 287.1\,\frac{J}{kg\,K} \qquad v_1 = \frac{287.1\frac{J}{kg\,K}\,278.15\,K}{75000\,Pa}$

$v_1 = 1.065\,m^3/kg$

$\dot{m} = \frac{\frac{\pi}{4} D_{in}^2 \left(156.6\,m/s\right)}{1.065\,m^3/kg} = 25\,kg/s \Rightarrow \underline{D_{in} = 0.4654\,m}$

B) Temperature of air leaving the diffuser? → **Assumptions:** 1) Steady state 2) ΔPE negligible 3) No $\dot{Q}$ or shaft work 4) Adiabatic 5) ideal gas

Energy Balance: $\dot{E}_{in} = \dot{E}_{out} + \frac{dE}{dt}$ → 0 (steady state)

$\dot{Q}_{in} + \dot{W}_{in} + \dot{m}_1\left(h_1 + \frac{\tilde{v}_1^2}{2} + g z_1\right) = \dot{Q}_{out} + \dot{W}_{out} + \dot{m}_2\left(h_2 + \frac{\tilde{v}_2^2}{2} + g z_2\right) \Rightarrow \dot{m}_1\left(h_1 + \frac{\tilde{v}_1^2}{2}\right) = \dot{m}_2(h_2)$

algebra $\dot{m}\frac{\tilde{v}_1^2}{2} = \dot{m}(h_2 - h_1)$ — for ideal gas constant C $\Delta h = C_p(T_2 - T_1)$

$\Rightarrow \dot{m}\frac{\tilde{v}_1^2}{2} = \dot{m}C_p(T_2 - T_1) \Rightarrow$ ~~$\frac{\dot{m}\tilde{v}^2 + \dot{m}C_pT_1}{\dot{m}C_p}$~~ $= T_2$

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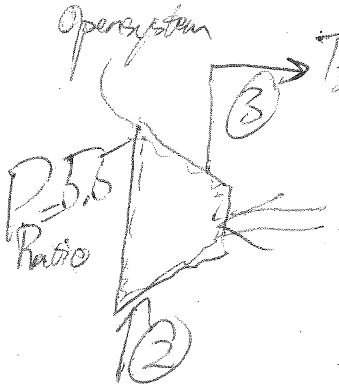
~~$\frac{25\frac{kg}{s}(1565\frac{m}{s})^2}{2} + 25\frac{kg}{s}(1005\frac{J}{kg\,k})(278.15k) \;/\; 25\frac{kg}{s}(1005\frac{J}{kg\,k})$~~ $\Rightarrow$ $T_2 = 290.3k$ — Flow heated? Makes sense?

$$T_2 = \frac{1}{C_p}\frac{\tilde{v}_1^2}{2} + T_1$$

($\tilde{v}_1 = 156.5$, $C_p = 1005\frac{J}{kg\,k}$)

Compressor: The purpose of a compressor is to use work to increase the pressure of a fluid.



$T_2 = 290.3k$
$P_2 = 220kPa$
$\dot{M} = 25kg/s$
$\tilde{v}_2 = 0$

c) Determine the volumetric flow rate leaving the compressor.

$$\dot{V}_3 = \frac{m^3}{s} = \frac{kg}{s}\frac{m^3}{kg} = \dot{m}v_3$$

Mass Balance

$$\dot{M}_{in} = \dot{M}_{out}, \quad \dot{M}_2 = \dot{M}_3 = 25kg/s$$

$$v_3 = \frac{RT_3}{P_3} \quad \frac{P_3}{P_2} = P_{ratio} \Rightarrow P_3 = PR_c P_2 \Rightarrow v_3 = \frac{RT_3}{PR_c P_2}$$

$$v_3 = \frac{287.1\frac{J}{kg\,k}(500.2k)}{5.5(220{,}000Pa)} \Rightarrow v_3 = 0.1187\,m^3/kg$$

$$\dot{V}_3 = 25\frac{kg}{s}(0.1187\,m^3/kg) \Rightarrow \dot{V}_3 = 2.967\,m^3/s$$

D) Find the power required by the compressor. → Energy Balance

$$\dot{E}_{in} = \dot{E}_{out} + \frac{dE}{dt}^{\,0} \Rightarrow \dot{Q}_{in} + \dot{W}_{in} + \dot{M}_2\left(h_2 + \frac{\tilde{v}_2^2}{2} + gz_2\right) = \dot{Q}_{out} + \dot{W}_{out} + \dot{M}_3\left(h_3 + \frac{\tilde{v}_3^2}{2} + gz_3\right)$$

$$\dot{W}_{in} = \dot{M}(h_3 - h_2) \quad (h_{out} - h_{in})$$

⟸ general equation for a compressor

Assumptions: 1) SS 2) ΔKE, ΔPE negligible 3) adiabatic

$$\dot{W}_{in} = \dot{M}C_p(T_3 - T_2) \Rightarrow 25\frac{kg}{s}\,1005\frac{J}{kg\,k}(500.2k - 290.3k) \Rightarrow \dot{W}_{in} = 5{,}272{,}000\,W$$

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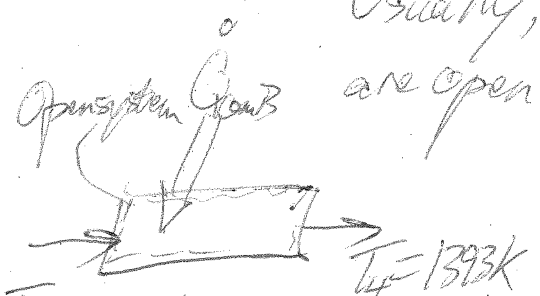
Finished ↑↑

Combustor: The purpose of a combustor is to add heat to a flow of fluid. Sometimes just the heat is specified. Sometime rate of fuel & heating value of fuel is specified. Usually, they are large combustion chambers that are open at either end. This means they are isobaric $P_3 = P_4$.



c.) Find the heat transfer that must be added to the combustor.

$T_3 = 300.2K$

$\dot{M}_3 = 25 kg/s$

$P_3 = 1,210,000 Pa$

Mass Balance: $\dot{M}_{in} = \dot{M}_{out}$ $\quad \dot{M}_3 = \dot{M}_4$

Assumptions: 1) Steady State 2) ΔPE, ΔKE negligible 3) No shaft or PV work 4) ideal constant C_p

Energy Balance: $\dot{E}_{in} = \dot{E}_{out} + \frac{dE}{dt}$

$$\Rightarrow \dot{Q}_{in} + \dot{W}_{in} + \dot{M}_3\left(h_3 + \frac{V_3^2}{2} + gz_3\right) = \dot{Q}_{out} + \dot{W}_{out} + \dot{M}_4\left(h_4 + \frac{V_4^2}{2} + gz_4\right)$$

$\dot{Q}_{in} = \dot{M}_4(h_4 - h_3)$ ← general equation for combuster with above assumptions

for ideal gas (constant C_p) $\Rightarrow \dot{Q}_{in} = \dot{M}C_p(T_4 - T_3) \Rightarrow 25\frac{kg}{s}\left(1005\frac{J}{kg\cdot k}\right)(1393k - 300.2k)$

$25 kg/s(1515.42 - 503.02) = 25,310,000 W$

$$\dot{Q}_{comb} = 22,440,000 W$$

Noticing a pattern?

Problem solving steps: Included

1) Make a drawing & note the known property/state information.
2) Choose the system to solve for important specifications with feasible assumptions
3) Apply mass balance with assumptions to simplify
4) Apply energy balance with assumptions to simplify
5) (Apply fluid model: real (EES or tables), ideal (Constant or not C)
6) Solve resulting set of equations