Supplement to:

CAN PRIVATIZATION OF U.S. HIGHWAYS IMPROVE MOTORISTS' WELFARE?

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This supplement describes additional findings and methodological procedures indicated in the text of our paper.

1. Demand Model Coefficients and Value of Time Estimates

The coefficients of the utility function based on motorists' choices among six alternative considerations of route (free or tolled) and vehicle occupancy (solo, HOV2, or HOV3) are presented in table A1. Estimates of the value of time and reliability and the heterogeneity of those values are presented in table A2.

2. Wardrop Equilibrium

We show that a unique Wardrop equilibrium for the third stage of the overall game exists for a price vector $p \geq 0$. Define *allocation* $g : N \rightarrow \Omega$, which maps potential travelers to the choice set, to represent travelers' choices. Traveler $i's$ choice is denoted $g(i)$. The market share of alternative j under allocation g is denoted by S_j^g, and the corresponding traffic volume is $V_j^g \equiv \frac{N \cdot S_j^g}{O_j}$, where O_j is vehicle occupancy. The transportation network is a simple two-route network and traffic volumes on the two routes $(r1, r2)$ under allocation g are $V_{r1}^g = \sum_{j \in \Omega_{r1}} V_j^g$ and $V_{r2}^g = \sum_{j \in \Omega_{r2}} V_j^g$.

The utility of choosing not to travel in equation (1) of the text is not a function of traffic volume. Given a price, the utility function of a traveler choosing a travel alternative that is associated with a route in our choice set is a function of the traffic volume on the route because both travel time and travel time unreliability are increasing functions of the route's traffic volume. Formally, traveler $i's$ utility for choosing a travel alternative under allocation g can be expressed as

$$U_{ig(i)} = U_{ig(i)}\left(V_r^g\right) \text{ for } g(i) \neq 0, \tag{A.1}$$

where the choice $g(i)$ is associated with route r and subscript i indicates that the utility function is individual specific. The utility function has two properties. The first is

$$U_{ig(i)}\left(V_r^g\right) < U_{ig(i)}\left(V_r^{g'}\right) \Leftrightarrow V_r^g > V_r^{g'}, \qquad \text{(A.2)}$$

which says that a traveler's utility of choosing a travel alternative decreases as the volume on the chosen route r increases. The second property is for $j, k \in \Omega_r$

$$U_{ik}\left(V_r^g\right) < U_{ij}\left(V_r^g\right) \Leftrightarrow U_{ik}\left(V_r^{g'}\right) < U_{ij}\left(V_r^{g'}\right). \qquad \text{(A.3)}$$

Since j and k are two travel alternatives on the same route, they must have different car occupancies. The property in (A.3) says that a traveler's preference for two alternatives on the same route, but with different car occupancies, is invariant to the traffic volume on the route.[1]

Allocation g is a Wardrop equilibrium if and only if under the allocation each traveler maximizes her utility given the traffic volumes and price; that is, $U_{ig(i)} = \max\left\{U_{i0}, U_{i1}, \ldots, U_{iJ} \middle| V_{r1}^g, V_{r2}^g, p\right\}$ for all i. Konishi (2004) proves the uniqueness of the Wardrop equilibrium in transportation networks with heterogeneous commuters for a model with only route choice. We consider travelers' route and vehicle occupancy choices, but the proof of the uniqueness of the Wardrop equilibrium follows Konishi's idea.

We first show the existence of the Wardrop equilibrium. The third stage game of the paper is an example of the atomless game considered by Schmeidler (1973). The game is anonymous in the sense that travelers care only about the number of travelers for each alternative but do not care about who they are. Under anonymity, Schmeidler's result is that the Wardrop equilibrium exists.

The uniqueness of the Wardrop equilibrium depends on whether traffic volumes of the alternatives are the same for any equilibria g and g'. If the traffic volumes are the same, then the pricing decisions at the first stage and the welfare effects of the overall game are also the same for any Wardrop equilibria g and g'. To prove uniqueness, we first show that traffic volumes on the two routes are the same for any two equilibria; that is, if g and g' are two Wardrop equilibria, we have $V_{r1}^g = V_{r1}^{g'}$ and $V_{r2}^g = V_{r2}^{g'}$.

[1] For example, if a traveler prefers driving alone to using a carpool on a route when the route is not congested, the traveler still prefers driving alone to using a carpool on the route when the route is congested.

We prove this by contradiction. Suppose allocations g and g' are both Wardrop equilibria and traffic volumes on the two routes are different for g and g'. Without loss of generality, we can assume that $V_{r1}^{g} < V_{r1}^{g'}$. This implies that there exists a traveler i and an alternative $j \in \Omega_{r1}$ such that $g'(i) = j$ and $g(i) \neq j$. We can divide this possibility into the following cases:

Case 1: $g(i) = 0$. This case indicates that traveler i chooses alternative j under allocation g' but switches to the no-travel option under allocation g. We are able to construct the following inequalities, $U_{i0} \geq U_{ij}\left(V_{r1}^{g}\right)$, $U_{i0} \leq U_{ij}\left(V_{r1}^{g'}\right)$, and $U_{ij}\left(V_{r1}^{g'}\right) \geq U_{ij}\left(V_{r1}^{g}\right)$; from property (A.2), the last inequality means that $V_{r1}^{g} \geq V_{r1}^{g'}$. Thus, we obtain a contradiction.

Case 2: $g(i) \in \Omega_{r1}$. This case indicates that traveler i stays on the same route but switches to another alternative with larger vehicle occupancy. This case contradicts property (A.3) because it implies that $U_{ig(i)}\left(V_{r1}^{g}\right) \geq U_{ij}\left(V_{r1}^{g}\right)$ and $U_{ij}\left(V_{r1}^{g'}\right) \geq U_{ig(i)}\left(V_{r1}^{g'}\right)$.

Case 3: $g(i) \in \Omega_{r2}$. This case indicates that traveler i selects alternative j under allocation g' but switches to an alternative that is associated with route *r2* under allocation g. By (A.2), we have $U_{ij}\left(V_{r1}^{g}\right) > U_{ij}\left(V_{r1}^{g'}\right)$. Since g and g' are both equilibrium allocations, we can have the other two inequalities, $U_{ig(i)}\left(V_{r2}^{g}\right) \geq U_{ij}\left(V_{r1}^{g}\right)$ and $U_{ij}\left(V_{r1}^{g'}\right) \geq U_{ig(i)}\left(V_{r2}^{g'}\right)$. Combining the three inequalities, we have $U_{ig(i)}\left(V_{r2}^{g}\right) > U_{ig(i)}\left(V_{r2}^{g'}\right)$ which implies $V_{r2}^{g} < V_{r2}^{g'}$. Given the total number of travelers on the routes is fixed (from case 1), both routes can be less congested under the allocation g only when some travelers on the routes switch to alternatives with larger vehicle occupancy. Case 2 indicates that it is impossible for travelers to make such changes on the same route, so we can conclude that: (a) there exists one traveler (denoted by n) who switches from an alternative on route $r1$ to an alternative (denoted by m_2) with larger vehicle occupancy on route $r2$; (b) there exists one traveler (denoted by h) who switches from an alternative on route $r2$ to an alternative (denoted by k_1) with larger vehicle occupancy on route $r1$. From

(a) we have $U_{nm_2}\left(V_{r2}^{g}\right) > U_{nm_1}\left(V_{r1}^{g}\right)$ with m_1 denoting the alternative on route $r1$ with the same vehicle occupancy as m_2; from (b) we have $U_{hk1}\left(V_{r1}^{g}\right) > U_{hk_2}\left(V_{r2}^{g}\right)$ with k_2 denoting the alternative on route $r2$ with the same vehicle occupancy as k_1. The first inequality requires $V_{r1}^{g} > V_{r2}^{g}$ and the second inequality requires $V_{r1}^{g} < V_{r2}^{g}$, which again results in a contradiction. Summarizing the three cases, we can conclude that if g and g' are two equilibrium allocations, $V_{r1}^{g} = V_{r1}^{g'}$ and $V_{r2}^{g} = V_{r2}^{g'}$. But although traffic volumes are the same, the composition of vehicles with different occupancies can be different for g and g'. If this is true, pricing decisions at the second stage and the welfare effects of the overall game can be different for the two equilibria.[2] It is also the case that we can find an alternative j such that $V_{j}^{g} \neq V_{j}^{g'}$; that is, there exists one traveler with different choices for these two equilibria. However, since $V_{r1}^{g} = V_{r1}^{g'}$ and $V_{r2}^{g} = V_{r2}^{g'}$, a traveler obtains the same utility under the two equilibria from choosing an alternative; accordingly, her ranking of the alternatives should be the same for g and g'. Thus, given prices, we obtain a unique Wardrop equilibrium.

3. Computational Procedures

The simulations require us to compute users' (Wardrop) equilibrium and equilibrium tolls under different competitive and regulatory scenarios. Users' equilibrium is determined by market demand represented by travelers' choices (equations (4)-(7) in the text) and market supply represented by travel time costs (equations (10)-(11) in the text). Demand is simulated by sample enumeration, using the random sample collected by the Brookings Institution in the year 2000 on the California SR91 corridor. The enumeration sample is assumed to represent a fixed population of size *N* potential travelers. Given the tolls and travel time costs, travelers are assumed to choose an alternative to maximize their utility. We then compute the choice probability of each traveler in the enumeration sample choosing each alternative,

[2] For example, operators charge different prices for carpoolers and solo drivers under a policy of high-occupancy-tolls (HOT).

multiplying the choice probability by the population size N and aggregating across the sampled travelers to obtain total demand for the alternative. Travel time costs are then updated by using the simulated traffic volumes. We iterate between demand and supply until we obtain convergence, which occurs when the absolute change in demand from two consecutive iterations is less than a very small number (0.001).

Equilibrium tolls under government regulation or private monopoly provision are solutions to a bi-level programming problem (or a mathematical program with equilibrium constraints), in which the government's or the monopolist's toll decision at the upper level accounts for the users' equilibrium constraint at the lower level. Embedding the procedure for computing the users' equilibrium as a constraint, we solve the optimal tolls by employing MATLAB's constrained nonlinear optimization package.

Computing equilibrium tolls under duopoly or public-private competition is more complicated. We obtain equilibrium tolls by first solving the best-response functions of the operators. The optimal toll on a route, given the toll on the other route, is the solution to a bi-level programming problem. We express the toll on the other route in discrete units of 100 equally spaced points in the range from \$0 to \$30, and solve for the optimal toll on the route under consideration for each of those points. The operator's best-response function is then approximated by linear interpolation based on the 100 points. The equilibrium tolls are determined by the intersection(s) of the best-response functions of the two operators.

4. Solutions to Duopoly Equilibrium

We present graphical solutions to private and public-private Bertrand duopoly competition. Figure A1 pertains to private duopoly competition without bargaining between operators and travelers. Two equilibria exist; one can be obtained by switching the tolls of the other. Tolls are strategic substitutes when a rival's toll is low and strategic complements when a rival's toll is high. The main explanation for the V-shape toll reaction functions is motorists' preference heterogeneity. Users with low VOT and VOR take the route with the lower toll. So, if the toll on route r2 is low, operator r1 prefers to cater to high VOT users (analogous to the long side of the market in a Hotelling line model) and sets a high value for the toll on route r1. As the toll on route r2 rises, the mix of travelers on route r1 shifts toward a lower VOT and operator 1 finds it profitable to cut the toll on route r1 even though usage increases. This explains why the reaction function of operator r1 is downward-sloping for low values of the toll on route r2.

At some point, the toll on route r2 is close to $10 according to the figure; operator r1 finds it profitable to switch from a high-toll high-quality strategy to a lower toll and lower quality. The figure shows that the reaction function takes a small downward jump. As the toll on route r2 rises further, the mix of travelers on route r1 shifts toward a higher VOT and operator 1 responds by raising the toll on route r1; the response function of operator r1 is therefore upward-sloping for higher values of the toll on route 2.

Figure A2 presents duopoly bargaining equilibria for the travelers' solution given that the optimal sale price of the highway is the government's reservation price (the highway's construction cost). Again, two equilibria exist as one can be obtained by switching the tolls of the other and one of the two operators breaks even. Tolls are strategic complements when a rival's toll is low and they are strategic substitutes when a rival's toll exceeds a threshold. When a rival's toll is very low, an operator can improve consumer surplus by increasing its toll to maintain toll differences that benefit high VOT users. When a rival's toll exceeds a certain level, about $2.70 according to Figure A2, the operator cuts its toll to the break-even level to benefit low VOT users. Because of the break-even requirement, tolls are quite unresponsive to a rival's toll when they are strategic substitutes. At the break-even level, a higher sale price of the highway would shift up operator r1's response curve and shift operator r2's response curve to the right such that tolls under the equilibria would be higher and reduce consumer surplus.

Figure A3 presents duopoly bargaining equilibria under the operators' solution. The key difference between this competitive environment and duopoly competition without bargaining is that tolls are also strategic substitutes when a rival's toll is high; thus, an operator's response curve overlaps with its rival's response curve leading to multiple equilibria. Tolls are strategic substitutes when a rival's toll is high because in response to an increase in the rival's toll, an operator has to reduce its own toll to satisfy the requirement that the change in consumer surplus will be nonnegative. Tolls cannot be too low given the assumption that the purchase price of the highway cannot be lower than the government's reservation price (construction cost).

Figures A4 and A5 present the travelers' solution and the operators' solution under Bertrand competition with unequal capacity allocation. Changing the capacity allocation does not alter the basic findings from duopoly bargaining with equal capacity. However, the two equilibria for the travelers' solution presented in Figure A4 have different welfare effects; the one with the higher toll on route r1 (the route with smaller capacity) generates a larger increase in consumer surplus. The government's optimal sale price is at the lowest possible level, the reservation price; a higher sale price would lead to higher tolls under the equilibria and eventually reduce consumer surplus.

Finally, figure A6 presents duopoly equilibria under public-private competition. There are two equilibria for Bertrand public-private competition but the two equilibria have different welfare effects; equilibrium A dominates equilibrium B in terms of both consumer surplus change and social welfare. The profits to the private operator under equilibrium A are also higher than the profits under equilibrium B. Under the SPE, the government can restrict the equilibrium of the Bertrand public-private competition to the better outcome by setting the sale price of the highway as the present value of private firm's excess operating profits under equilibrium A.

The reaction function for the private operator in Figure A6 is qualitatively similar to the reaction functions in Figure A1 except for a much bigger discontinuity at the transition point. In contrast, the public operator's reaction curve is nearly flat. This may be because the public operator takes into account not only the VOT for the marginal user but also the VOT for the inframarginal users. The public operator is therefore reluctant to cut the toll because this would exacerbate congestion for users with the highest VOT.

5. Traffic Growth

By the time a privatization policy is implemented, the growth in traffic will result in delays that are greater than they are today. Under such conditions, tolls become even more essential to allocate scarce highway capacity. Table A3 indicates that privatization's benefits to motorists and social benefits increase if we assume the population of potential travelers grows 20 percent, which generates additional congestion. It is interesting that the travelers' solution yields a much more differentiated toll in response to greater congestion and produces a social welfare gain that now exceeds the gain produced under the operator's solution. The toll of $8.77 is clearly within motorists' actual willingness to pay to improve the speed and reliability of their trips because tolls on SR 91 during January 2010 were as high as $9.90 during certain peak travel periods.

6. Negotiating Outcomes

We indicated in the text that a wide range of negotiating outcomes could enable both the private operator and motorists to gain from privatization thereby enhancing the feasibility of the policy. Figure A7 shows the tradeoffs under different allocations of highway capacity and finds that the potentially largest "win-win" outcomes occur when highway service is differentiated by a price-service package of one route consisting of 5 lanes and another route consisting of one lane. Given this allocation of capacity, figure A8 shows the feasible tradeoffs of tolls and consumer surplus on the two routes, and figure A9 shows the feasible tradeoffs of consumer surplus and the sale price of the highway.

7. Capacity Expansion

The median cost to build a lane-mile on an existing rural highway in the United States is roughly $2 million (Washington State Department of Transportation (2002)). Given this figure and the other empirical parameters, the private operator will add another lane if the present value of operating profits from doing so minus the construction costs exceeds the present value of operating profits before the capacity was expanded. We find that duopoly operators would not add a lane to the highway because it would give rise to unequal capacity among competitors and they both cannot gain from the additional lane if excess operating profits are constrained to zero as in the travelers' solution. (We could not find a unique equilibrium for the operators' solution under capacity expansion.) A monopoly would also not expand capacity if its excess profits were constrained to zero in negotiations, but if that constraint were relaxed then it would find it profitable to add a lane that would benefit motorists. Based on the parameters of our model and assuming the construction cost figure of $2 million per mile, Figure A10 indicates that the potentially largest “win-win” bargaining outcomes occur when the additional lane is used to expand the number of (higher speed) lanes in route 2, meaning that route 1 would consist of 1 lane and route 2 would now consist of 6 lanes. Figure A11 shows for this allocation of capacity that given an initial bargaining solution of either the travelers’, motorists’, or an intermediate solution, feasible “win-win” outcomes exist where an additional lane would increase marginal operating profits and consumer surplus. Because the construction costs are likely to be higher for adding a lane-mile on an urban highway; the likelihood of expanding capacity on such highways may be reduced because the private operator is unable to recover the initial investment.

8. Homogeneous Preferences

Table A4 shows that motorists’ welfare does not improve under privatization if we ignore preference heterogeneity and assume that road users have homogeneous preferences. Generally, monopoly tolls are lower than they are for heterogeneous users because when users are heterogeneous, the monopolist sets high tolls to serve users with the highest value of time and reliability. With homogenous users, the travelers’ bargaining solution results in a corner solution (i.e., the monopoly operator sets the lowest tolls that enable it to break even), and different market structures—monopoly, Bertrand duopoly, and Bertrand public-private competition—are less feasible politically because consumer surplus is always negative.

References

Konishi, H., 2004. Uniqueness of user equilibrium in transportation networks with heterogeneous commuters. Transportation Science 38, 315-330.

Schmeidler, D., 1973. Equilibrium points of nonatomic games. Journal of Statistical Physics 7, 295-300.

Small, K., Winston, C., Yan, J., 2006. Differentiated road pricing, express lanes, and carpools: exploiting heterogeneous preferences in policy design. Brookings-Wharton Papers on Urban Affairs, 53- 96.

Small, K., Winston, C., Yan, J., 2005. Uncovering the distribution of motorists' preferences for travel time and reliability. Econometrica 73, 1367−82.

Washington State Department of Transportation, 2002. Highway Construction Cost Comparison Survey: Final Report, Washington, April.

Table A1. Estimated Coefficients for the Route-Vehicle Occupancy Choice Model

Variable	Coefficient[a]
Toll ($)	-1.4580
Toll × dummy for high household annual income (> $60K)	0.8411
Median travel time (minutes) × trip distance (units of 10 miles)	-0.3489
Median travel time × trip distance squared	0.0684
Median travel time × trip distance cubed	-0.0030
Travel-time uncertainty (80th percentile minus the median) (minutes)	-0.4541
HOV2 dummy	-6.9854
HOV3 dummy	-12.580
Female × age 30−50 × household size × carpool dummy[b]	0.8735
Random components of coefficients	
Standard deviation of travel-time coefficient	0.3866
Standard deviation of travel-time uncertainty coefficient	0.6009
Common standard deviation of HOV2 and HOV3 dummies	6.2597

Source: Small, Winston, and Yan (2006).

[a] All coefficients are statistically significant at the five percent level.
[b] The carpool dummy is set to one if the route-vehicle occupancy choice includes HOV2 or HOV3.

Table A2. Value and Heterogeneity of Travel Time and Reliability [a]

Item	Median estimate
Value of Median Travel Time	
Dollars per hour	19.63
As a percent of the hourly wage [b]	85
Value of Reliability	
Dollars per hour	20.76
As a percent of the hourly wage [b]	90
Heterogeneity [c]	
Median travel time	19.02
Reliability	35.51

[a] Source: Small, Winston, and Yan (2006).
[b] The wage rate, estimated in Small, Winston, and Yan (2005), is about $23 per hour.
[c] Heterogeneity is expressed here as the interquartile range of the quantity in question across individuals.

Table A3. Welfare Effects under Monopoly Provision and Traffic Growth (with gas tax rebate)

	Base case: with traffic growth [d]	Monopoly with traffic growth [d]	Monopoly bargaining with traffic growth: travelers' solution [d]	Monopoly bargaining with traffic growth: operator's solution [d]
Capacity (vehicles/hour) [a]				
Route r1	6000	6000	2000	2000
Route r2	6000	6000	10000	10000
Sale price ($ million/mile)	N. A.	75.8	43.0	53.9
Toll ($)				
Route r1	0.00	22.77	0.00	1.74
Route r2	0.00	22.77	8.77	12.95
Travel times (min.):				
Route r1	24.40	9.69	66.76	67.53
Route r2	24.40	9.69	13.76	11.13
Aggregated choice shares (%):				
No travel on the corridor	16	32	10	12
Travel on the corridor	84	68	90	88
For those who travel on the corridor				
Solo driving	80	8	47	33
HOV2	17	50	38	44
HOV3	3	42	15	23
Operating profits: one period ($/person) [b,c]	0.00	4.67	2.67	3.33
Operating profits: present value ($ million/mile) [e]	0.00	75.8	43.0	53.9
Change in government budget: one period ($/person) [c,f]	0.00	4.44	2.44	3.10
Change in consumer surplus: one period ($/person) [c]	0.00	-4.91	1.56	0.00
Change in welfare: one period ($/person) [c, g]	0.00	-0.47	4.00	3.10

[a] Capacity is allocated optimally for each scenario.
[b] Operating profits are determined as 65% of the toll revenues.
[c] The change in consumer surplus, government budget, and social welfare change are measured relative to the no-toll scenario. These items and operating profits are divided by the total number of potential users *N*.
[d] Population size (*N*) is increased 20% in the scenarios with traffic growth.
[e] We assume a 4.5% discount rate.
[f] The change in the government's budget is calculated by subtracting the government's gas tax revenues and maintenance expenditures under the no-toll scenario from the highway sale revenue under privatization. Gas tax revenues are calculated assuming average gas mileage of 16 miles per gallon and a gasoline tax rate of $0.49 per gallon.
[g] The welfare change is the sum of the change in the government budget and consumer surplus because the government's sale price extracts excess operating profits.

Table A4. Simulation Results with Homogeneous Road Users (with the gas tax rebate)[a]

	Base: current situation	Monopoly	Monopoly bargaining: travelers' solution	Bertrand Duopoly Competition	Free Public Route without the gas tax rebate
Capacity (vehicles/hour)					
Route r1	6000	6000	6000	6000	2000
Route r2	6000	6000	6000	6000	10000
Sale price ($ million/mile)					
Route r1	N. A.	13.8	6.0	13.6	2.4
Route r2	N. A.	13.8	6.0	13.6	N. A.
Toll ($)					
Route r1	0.00	5.71	1.73	5.51	4.56
Route r2	0.00	5.71	1.73	5.51	0.00
Travel times (min.):					
Route r1	20.00	11.39	18.19	11.68	11.38
Route r2	20.00	11.39	18.19	11.68	20.65
Aggregated choice shares (%):					
No travel on the corridor	28	52	31	50	32
Travel on the corridor	72	48	69	50	68
For those who travel on the corridor					
Solo driving	100	100	100	100	100
HOV2	0	0	0	0	0
HOV3	0	0	0	0	0
Operating profits: one period ($/person)					
Route r1	0.00	0.89	0.38	0.88	0.24
Route r2	0.00	0.89	0.38	0.88	0.00
Change in government budget: one period ($/person)	0.00	1.56	0.54	1.54	0.02
Change in consumer surplus: one period ($/person)	0.00	-0.48	-0.09	-0.45	-0.10
Operating profits: present value ($ million/mile)					
Route r1	0.00	13.8	6.0	13.6	2.4
Route r2	0.00	13.8	6.0	13,6	0.00
Change in social welfare: one period ($/person)	0.00	1.08	0.45	1.09	-0.08

[a] We modify the demand model in Table 1 in the following ways to eliminate the heterogeneity in motorists' preferences: 1. set the random components of the coefficients to zero; 2. evaluate the toll, time, and carpool dummy coefficients at the means of the motorists' profiles. We recalibrate the parameters $\left(N,\lambda,\bar{\delta},\hat{\delta}\right)$ after those modifications following the same procedures described in the text.

Figure A1. Solutions to Private Duopoly Competition

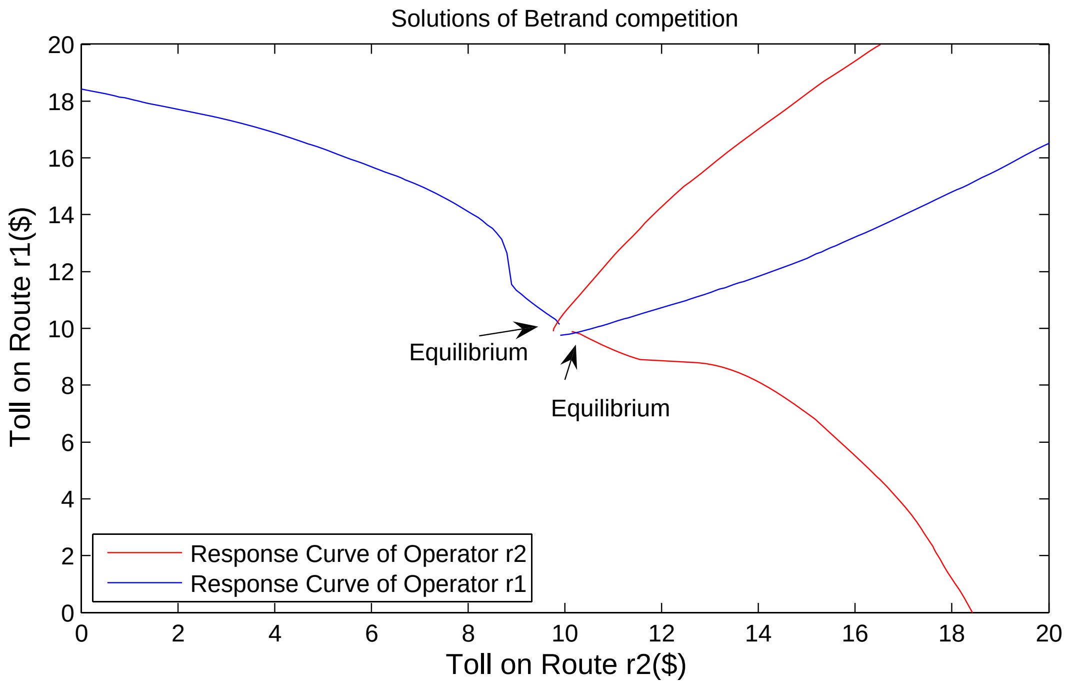

Figure A2. Solutions to Private Duopoly Competition with Bargaining: Travelers' Solution

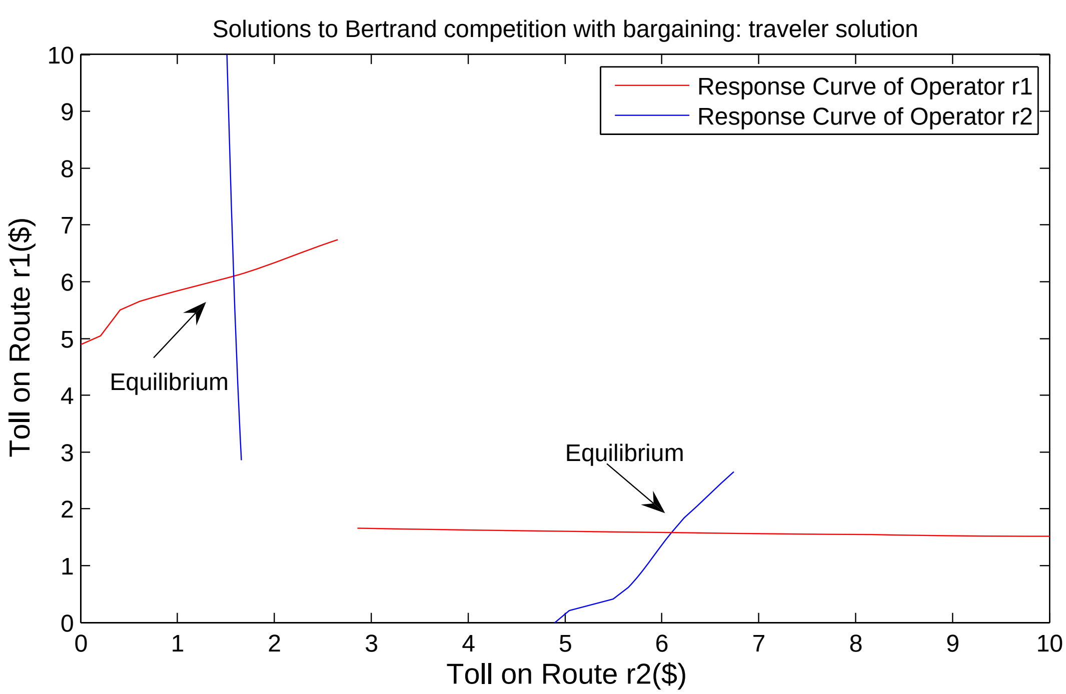

Figure A3. Multiple Equilibria under Duopoly Competition with Bargaining: Operator's Solution

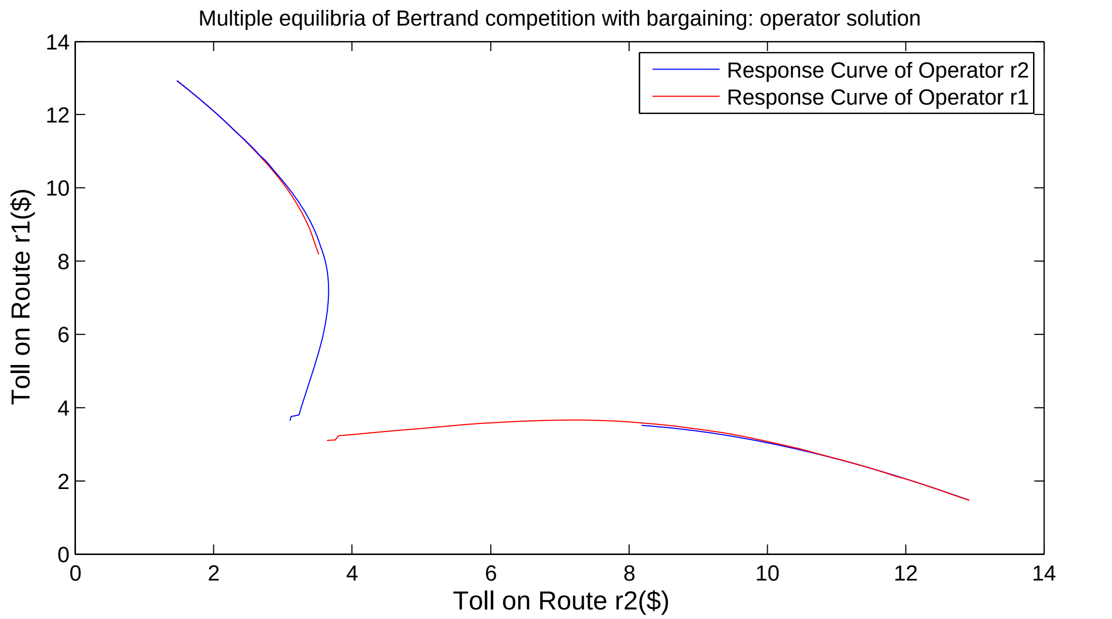

Figure A4. Solution to Duopoly Competition with Bargaining and Unequal Capacity: Travelers' Solution

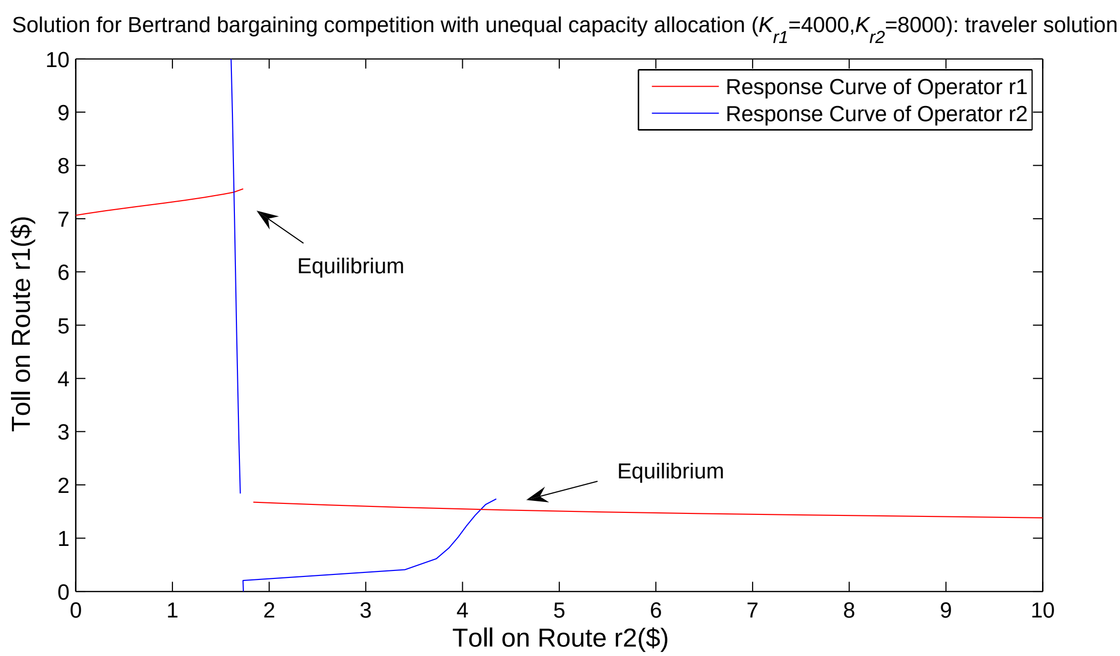

Figure A5. Multiple Equilibria under Duopoly Competition with Bargaining and Unequal Capacity: Operators' Solution

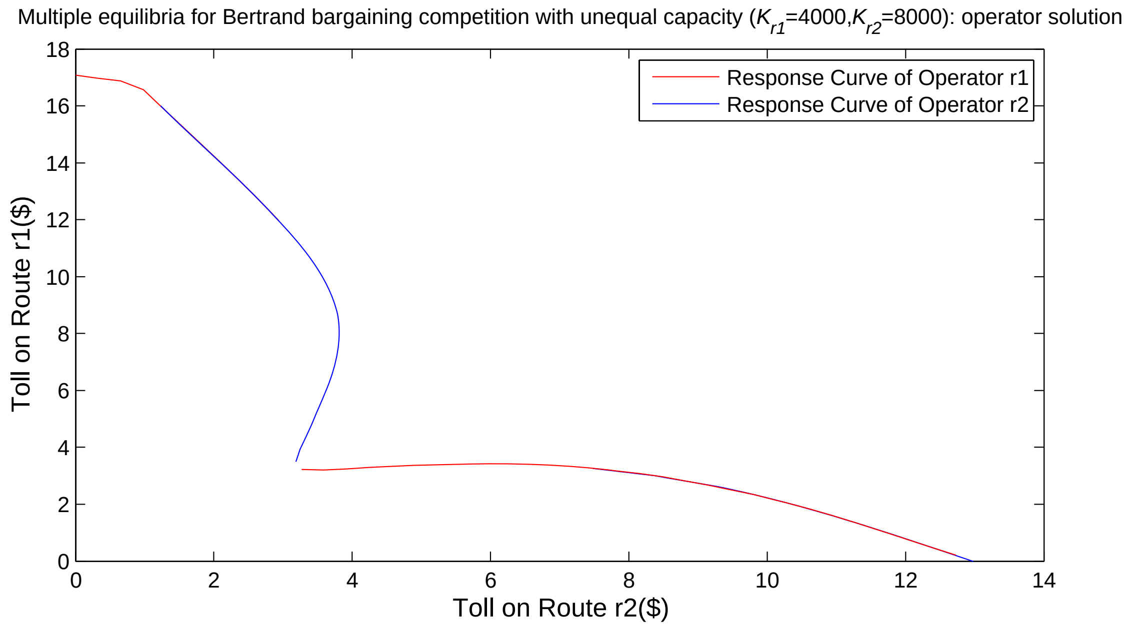

Figure A6. Solutions to Public-Private Duopoly Competition

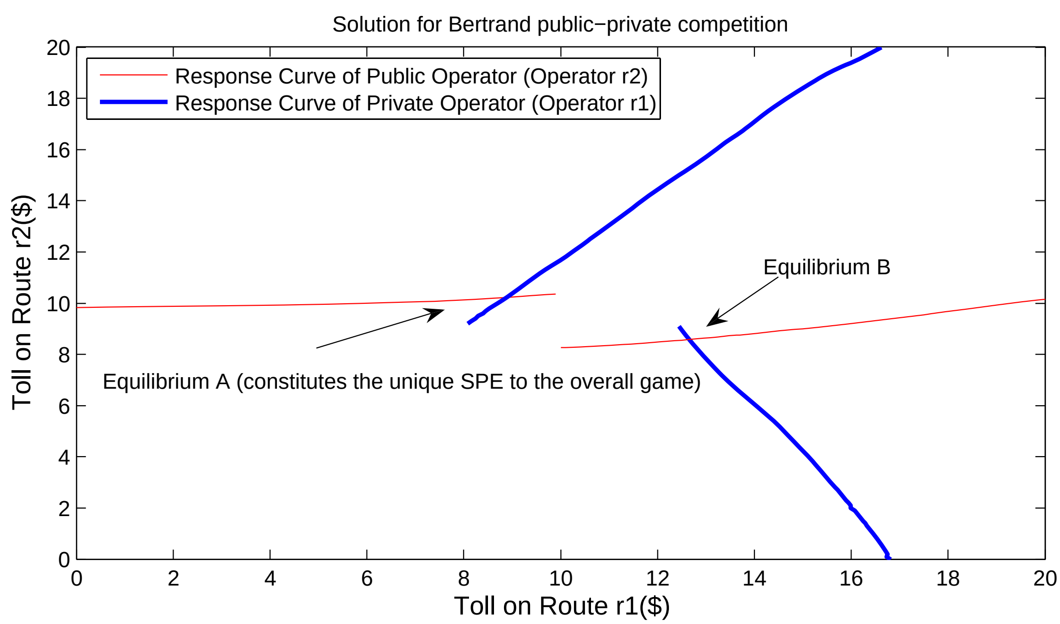

Figure A7. Trade-off between monopoly operating profits and the change in consumer surplus under bargaining solutions for different allocations of capacity.

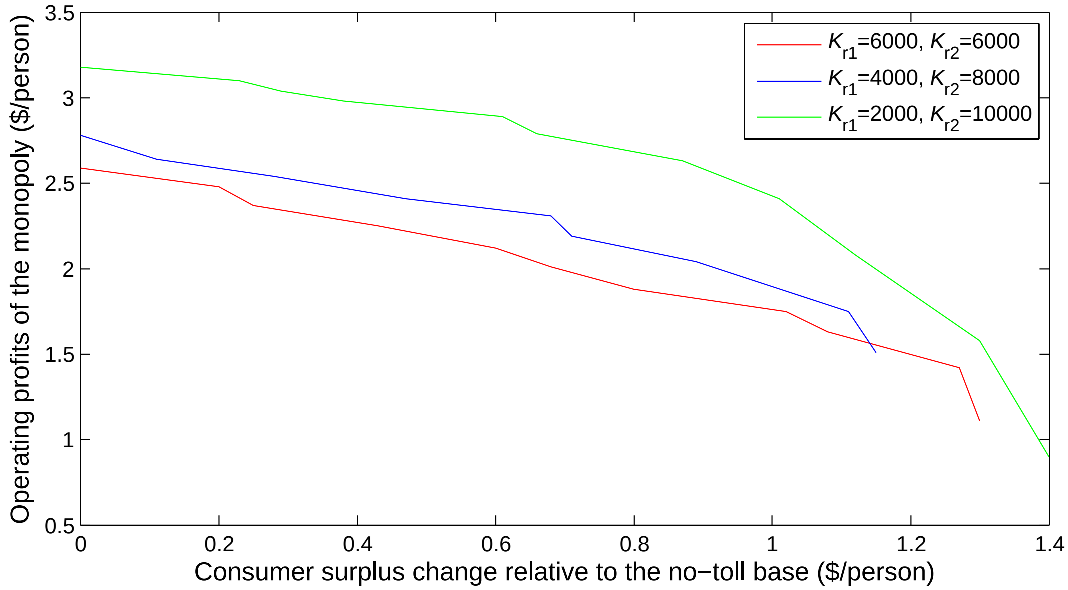

Figure A8. Optimal tolls and consumer surplus under bargaining solutions

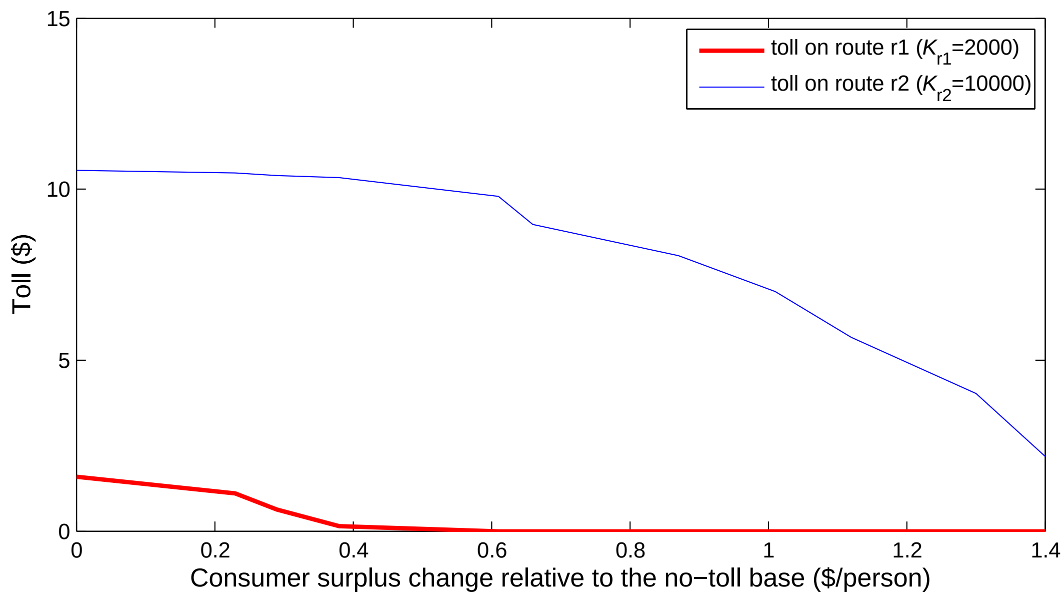

Figure A9. Government's optimal sale price and consumer surplus under bargaining solutions

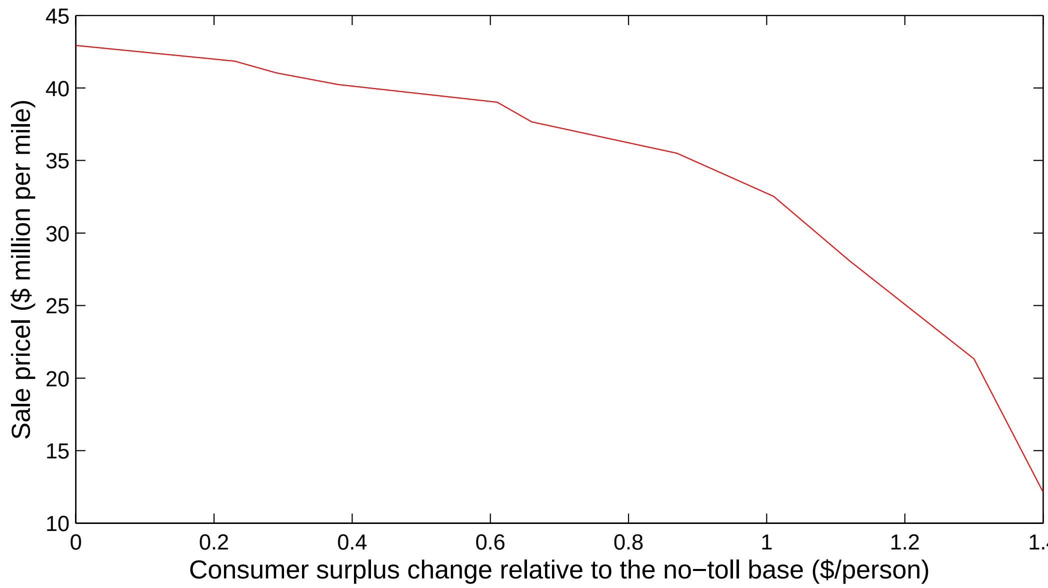

Figure A10. Trade-off between monopoly operating profits and the change in consumer surplus under bargaining solutions after adding one more lane for different allocations of capacity

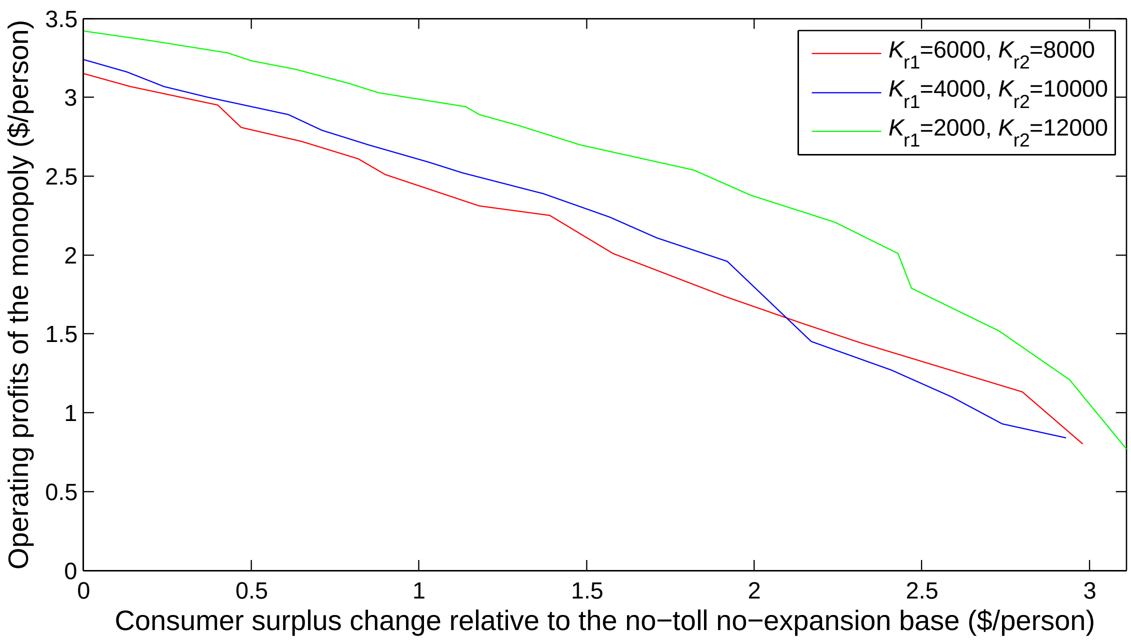

Figure A11. Feasible win-win outcomes for travelers and the operator from expanding capacity