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The Value of ''Value Pricing'' of Roads: Second-Best Pricing and Product Differentiation[1]

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Some road-pricing demonstrations use ''value pricing,'' in which travelers can choose between a free but congested roadway and a priced roadway. Recent research has uncovered a potentially serious problem for such demonstrations: second-best tolls may be far lower than those typically charged, and from a welfare perspective, the latter may be worse than not pricing at all. That research, however, assumes that all travelers are identical and it therefore neglects the benefits of product differentiation. Using a model with two user groups, we find that accounting for heterogeneity in value of time is important in evaluating constrained policies, and improves the relative performance of policies that offer differential prices. © 2000 Academic Press

Key Words: value pricing; congestion pricing; value of time; road pricing; high occupancy/toll lanes.

1. INTRODUCTION

Road-pricing concepts have moved to center stage in many transportation planning and policy-making venues around the world. Small and Gómez-Ibanez [22] describe 13 significant applications under consideration in nine countries, 7 of them implemented as of mid-1997. More projects have been undertaken subsequently, including an innovative no-cash system using combined electronic and video collection technology on a new expressway near Toronto, Ontario, which opened in October 1997. Meanwhile, hardly an issue of the monthly ''Toll Roads Newsletter'' goes by without accounts of new pricing proposals by government agencies.

Yet in only one case (Singapore) has congestion pricing been adopted in something like a first-best form: significant time-of-day variations applying to an entire road network. All other applications are limited, such as toll rings with fixed or nearly fixed tolls (Norway), behavioral experiments (Stuttgart), or

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pricing on a single facility (France, Ontario, California, Texas, Florida). Increasingly, the favored approach is to adopt small-scale ‘‘demonstration projects’’ intended to test and publicize pricing concepts and their associated technologies. This approach is specifically funded in U.S. legislation passed in 1991 and reauthorized in 1998.

Three of the demonstrations currently operating—in Orange County (California), San Diego, and Houston—let travelers choose between two adjacent roadways: one free but congested, the other priced but free-flowing. This scheme is sometimes called ‘‘value pricing’’ because people are given the option to pay for a more highly valued service, much as train or air travelers can purchase a first-class ticket. In these particular examples, the express lanes also serve carpools at zero or at reduced rates, and so are known as ‘‘High Occupancy/Toll’’ (HOT) lanes. (In Houston, furthermore, the value-pricing option is available only to people in two-person carpools.)

Many criteria might be used to design a value-pricing program. One is to apply the ‘‘second-best’’ toll to the express roadway, chosen to maximize social welfare subject to the zero-toll constraint on the other roadway. Another is to apply a ‘‘profit-maximizing’’ toll which maximizes revenue, subject to the same constraint. A third is to set the toll just high enough to keep it flowing at a minimum specified speed.

By comparing these first two alternate criteria, recent research has uncovered a potential problem with current implementations of value pricing as a demonstration of road pricing (Braid [3], Verhoef *et al*. [26], Liu and McDonald [14]). This research focuses on the profit-maximizing version of value pricing, comparing its outcomes to those of the second-best version. An application of these methods by Liu and McDonald [13] is designed to approximate conditions for California’s State Route 91 (SR91), the site of the Orange County value-pricing demonstration; their results suggest that in a second-best optimum, the express lanes would have a far lower toll and considerably more congestion than under the profit-maximizing regime, which presumably is what actually exists. Furthermore, Liu and McDonald find that pricing the express lanes lowers welfare compared to leaving them free.

However, the Liu−McDonald analysis, like the other papers mentioned above, makes the simplifying assumption that all travelers are identical. This assumption obscures the benefits of offering a differentiated product in order to allow people to indulge their varying preferences. To analyze the situation fully, we need a model that includes heterogeneity in the values that users place on the service quality offered by the express lanes.

This paper uses such a model to explore the importance of heterogeneity in value of time for value-pricing demonstrations. We extend the Liu−McDonald model to two user groups differing by value of time (after first simplifying their model by considering just one time period). Value of time here proxies for value of reliability as well, since travel time and reliability are closely correlated in such corridors.

We find that heterogeneity can make a significant difference in evaluating profit-maximizing and second-best policies. Still, under many conditions the profit-maximizing policy produces welfare losses compared to making all the lanes free; this is especially likely when heterogeneity is low and the proportion of high-value-of-time (VOT) users is larger than the proportion of capacity that can be priced. Profit maximization performs relatively better when we allow for an exogenous number of carpools who use the express lanes for free and who have a high VOT per vehicle. We also examine a policy, adopted in the San Diego demonstration, of setting the toll just high enough to maintain a specified level of service on the express lanes; in most cases, this policy performs little better, and often worse, than the profit-maximizing policy.

Like the studies cited earlier, ours does not constitute a comprehensive assessment of the SR91 experiment or of any other actual demonstration project, because such projects are often designed with additional objectives or constraints in mind. In particular, we do not account for a desire to encourage the use of high-occupancy vehicles (HOVs);[4] nor do we consider capacity costs or the financial viability of private road provision.[5] A pricing demonstration might legitimately be considered successful, even if welfare would be improved by eliminating pricing, if the no-toll baseline is not relevant to the policy context. Nevertheless, we still would regard such a success as a fragile one, given the political appeal of free and unrestricted roads—an appeal recently manifested, in fact, on SR91.[6]

We also do not fully account for the benefits of time-varying prices because we consider only a single uniform peak period. De Palma and Lindsey [8, Table 1] illustrate how the benefits of either second-best or profit-maximizing tolls are substantially increased when these tolls can vary smoothly over the peak period so as to just eliminate queueing while maintaining full use of the capacity of the priced roadway.

Only a few other papers have addressed user heterogeneity in a two-route problem. Schmanske [18, 19] and Arnott *et al*. [1] show that with heterogeneous users, differential tolls on separate roadways may be superior to a single toll. Bradford [2] shows that in a queue system with multiple servers, a revenue-maximizing system administrator would charge higher tolls, hence offer lower congestion, than is socially optimal. Verhoef and Small [27]

[4] HOV lanes are treated, for example, by Mohring [16], Small [20], Dahlgren [7], and Yang and Huang [30].

[5] See Viton [28] on this topic.

[6] An abortive plan to sell the privately built SR91 Express Lanes, combined with a controversial suit by the private owner to prevent parallel capacity expansions in violation of its franchise, have led political opponents of toll roads to reopen the question of whether the Express Lanes should be priced at all. See Garvey and James [10] and James and Garvey [11].

consider heterogeneity using a continuous VOT distribution, calibrated from Dutch stated-preference data, and also account for the possibility that users of the two roadways interact on a congested serial link elsewhere as part of their trips; their conclusions are broadly consistent with those of this paper.[7]

2. THE MODEL

We consider two roadways, A and B, connecting the same origin and destination. Both have the same length L and the same free-flow travel-time $T_f L$. A user of type i $(i = 1, 2)$ traveling on road r $(r = A, B)$ incurs travel cost c_{ir} which consists of operating cost β plus a time cost $\alpha_i T_r$ per unit distance. The parameter α_i is the value of time, and it is this parameter for which we introduce heterogeneity, by assuming that $\alpha_1 > \alpha_2$. Unit travel time T_r (the inverse of speed) is represented by flow congestion of a standard type, depending on volume-capacity ratio N_r/K_r so that:

$$c_{ir}(N_r) = \beta L + \alpha_i T_f L\left[1 + \gamma(N_r/K_r)^k\right] \qquad i = 1, 2;\ r = A, B \quad (1)$$

where γ and k are parameters. The congestion-dependent part of cost, $d_{ir} \equiv \alpha_i T_f L \gamma(N_r/K_r)^k$, is what we call *delay cost*. This particular functional form has the property that the marginal external cost is k times the average delay cost: $MEC_r \equiv \sum_i N_{ir} \partial c_{ir}/\partial N_r = k \cdot (\sum_i N_{ir} d_{ir})/N_r$, where N_{ir} is the number of type-i users on road r. We use values $\gamma = 0.15$ and $k = 4$, following common practice.[8]

Demand by each group has the linear form

$$N_i(P_i) = a_i - b_i P_i \quad (2)$$

where a_i and b_i are positive parameters, and P_i is the ''inclusive price'' or ''full price,'' defined as the minimum combination of travel cost plus toll (τ) for this user group:

$$P_i = \operatorname*{Min}_r\left[c_{ir} + \tau_r\right]. \quad (3)$$

The inverse demand function corresponding to (2) is denoted $P_i(N_i)$.

[7] Our results are also consistent with the literature on monopoly, which suggests that a monopolist might divide its market, for purposes of price discrimination, by letting queues ration some of its output (Donaldson and Eaton [9]). We do not, however, consider distributional issues and therefore do not follow up on the demonstration by Bucovetsky [4] that under certain constraints a second-best distributional policy can include rationing by queueing.

[8] See Small [21, pp. 69–72], for a discussion of empirical evidence for this functional form. These particular parameters are known as the Bureau of Public Roads formula.

The social welfare function is defined as the area under the inverse demand curve, less total cost:

$$W = \sum_{i=1}^{2} \int_0^{N_i} P_i(t)\, dt - \sum_{i=1}^{2} \sum_{r=A}^{B} N_{ir} c_{ir} \tag{4}$$

This function is strictly concave in the four variables N_{ir}.

2.1. *Types of Solution*

The equilibrium conditions are those of Wardrop [29], stating (i) that users of a given type choose the road or roads that minimize inclusive price, and (ii) that inclusive price be equalized across the two roads for any user group that uses both roads. We assume that if the roads are differentiated, it is road A that offers faster travel, so that $N_{1A} > 0$ and $N_{2B} > 0$. (This is a substantive assumption if the roads are of unequal capacity.) Wardrop's conditions can then be written:

$$c_{1A}(N_A) + \tau_A \le c_{1B}(N_B) + \tau_B \tag{5a}$$

$$c_{2A}(N_A) + \tau_A \ge c_{2B}(N_B) + \tau_B \tag{5b}$$

$$N_{1B} \cdot (c_{1A} + \tau_A - c_{1B} - \tau_B) = 0 \tag{5c}$$

$$N_{2A} \cdot (c_{2A} + \tau_A - c_{2B} - \tau_B) = 0 \tag{5d}$$

$$N_{1B}, N_{2A} \ge 0 \tag{5e}$$

It is useful to distinguish four possible cases, depending on whether each of (5a) and (5b) is an inequality or an equality.

Case SE (fully separated equilibrium). Both (5a) and (5b) are inequalities, i.e., each group strictly prefers a different roadway. Because we assumed $\alpha_1 > \alpha_2$, these conditions require that road A be more expensive but less congested than road B,[9] i.e., $\tau_A > \tau_B$ and $(N_A/K_A) < (N_B/K_B)$.

*Case SE*1 (partially separated equilibrium with group 1 separated). Group 1 strictly prefers road A, but group 2 is indifferent: that is, (5a) is an inequality, but (5b) an equality. Like the fully separated equilibrium, SE1 requires that road A have higher toll but lower travel time. Note it is not impossible that $N_{2A} = 0$, if this condition happens to yield indifference for group 2; but we would expect this only by coincidence.

[9] Subtracting (5b) from (5a) and applying (1) yields $(\alpha_1 - \alpha_2)(N_A/K_A)^k < (\alpha_1 - \alpha_2)(N_B/K_B)^k$, which (given $\alpha_1 > \alpha_2$ and $k > 0$) implies $N_A/K_A < N_B/K_B$. This in turn implies $c_{2A} < c_{2B}$, so (5b) requires $\tau_A > \tau_B$.

*Case SE*2 (partially separated equilibrium with group 2 separated). Group 2 strictly prefers road B, but group 1 is indifferent: (5a) is an equality, (5b) an inequality. Again, road A must have a higher toll but is faster. The boundary solution $N_{1B} = 0$ can occur, but again only by chance.

Case IE (fully integrated equilibrium). Both groups are indifferent between the two roads; (5a–b) both hold with equalities. Since the two groups have different values of time, this can occur only if the roads have equal tolls and equal speeds.

2.2. *Pricing Regimes*

We consider five alternative pricing regimes, also called policies.

First-best regime (FB): a public operator charges tolls on both roads that maximize welfare (4). It can be shown that this policy yields conventional marginal-cost pricing on each road.

Second-best regime (SB): the same objective is pursued but subject to the constraint $\tau_B = 0$.

Third-best regime (TB): like SB but with an additional constraint designed to guarantee a minimum level of service on the priced roadway, namely[10]

$$\frac{N_A}{K_A} \leq 0.887. \tag{6}$$

Profit-maximizing regime (PM): τ_A is chosen to maximize revenues on road A subject to the constraint $\tau_B = 0$. (By calling this "profit-maximizing," we implicitly assume there are no variable costs to the road owner of serving traffic.)

No-toll regime (NT): $\tau_A = \tau_B = 0$.

The no-toll regime is determined by solving (1)–(3) and (5) with equalities in (5a) and (5b); the solution is assumed to be of the integrated equilibrium (IE) type, since there is nothing to distinguish the two roadways from each other. (This is in fact the only regime where IE can occur, due to our assumption of strictly unequal values of time.) Each of the other regimes calls for maximizing either welfare, as given by (4), or revenues $R = \sum_r \tau_r N_r$, while imposing constraints (5) and, in the TB regime, constraint (6).

[10] Legislation authorizing the San Diego HOT lane specifies that the express lanes must operate with volumes that permit level of service C or better. At the time of our simulations, the authority operating the lanes was attempting to loosen this constraint to level of service D, which corresponds to a maximum volume-capacity ratio of 0.887 (Transportation Research Board [25, Table 3-1]), so we used this value.

Our solution strategy[11] is first to choose an equilibrium case (SE1, SE2, or SE) to test. We form the relevant Lagrangian, simplifying by taking advantage of the requirement, by (5c–d), that one or both of N_{1B} and N_{2A} be zero, depending on the regime. (Specifically, $N_{1B} = 0$ in regime SE1, $N_{2A} = 0$ in SE2, and both are zero in SE.) We then solve the first-order conditions numerically for N_{ir} and τ_r. Next, we check the non-negativity constraints (5e); if either of them is not satisfied, we impose it as an equality and again solve the first-order conditions. In the case of TB, we also check the level-of-service constraint and, if it is violated, we impose it as an equality and start over. Finally, we check the appropriate inequality (5a or 5b or both) defining the equilibrium type under consideration; if it is violated, we conclude that this equilibrium type cannot exist for this set of parameters. In this manner, we generate up to three candidate solutions, one for each equilibrium type, and we choose the one for which the maximized objective function is largest.

An example is instructive. Consider the SE1 equilibrium for the TB policy regime. For this case, $\tau_B = 0$, (5a) holds as an inequality (consequently $N_{1B} = 0$), and (5b) holds as an equality. Therefore Eqs. (3) and (5a–d) simplify to:

$$\tau_A = P_1 - c_{1A} \tag{7a}$$

$$P_1 - c_{1A} = P_2 - c_{2A} \tag{7b}$$

$$P_2 - c_{2B} = 0 \tag{7c}$$

$$P_1 - c_{1B} < 0 \tag{7d}$$

where it is to be remembered that P_i is a function of $(N_{iA} + N_{iB})$ through (2) and c_{ir} is a function of $(N_{1r} + N_{2r})$ through (1). We solve the problem by first using ordinary Lagrangian methods to find the values of N_{1A}, N_{2A}, and N_{2B} that maximize (4) subject to equality constraints (7b) and (7c); then τ_A is calculated from (7a). The non-negativity constraint for N_{2A} is then checked, and is imposed as an equality if needed. Similarly, the level-of-service constraint (6) is checked and imposed as an equality if needed. Finally, the inequality (7d) is checked to see if the trial solution is a valid SE1 equilibrium.

3. SIMULATION RESULTS

In this section, we design several scenarios to explore the effects of user heterogeneity on the efficiency of various pricing policies. We begin with a base scenario that resembles SR91, the demonstration site in Orange County, California. We then consider alternate demand parameters, first changing price elasticities, then total demands, then the relative sizes of groups 1 and 2. Next,

[11] In the Appendix, we enumerate the full set of possible solutions. For most cases, they are not of closed form, so require numerical maximization procedures to find them.

TABLE 1
Parameter Values Used in Simulations[a, b]

Parameter	Base scenario	High-elasticity scenario	High-congestion scenario	Proportional-demand scenario	Reversed-capacity scenario	Carpool scenario[c]
$\underline{\alpha}$ (cents/mi.)[d]	34.38	34.38	34.38	34.38	34.38	23.00
K_A (veh./hour)	2000	2000	2000	2000	4000	2000
K_B (veh./hour)	4000	4000	4000	4000	2000	4000
α_1	5700	7150	6780	3800	5700	5580
α_2	5700	7150	6780	7600	5700	5580

[a] The following parameters are the same in all scenarios: $L = 10$ miles, $\gamma = 0.15$, $k = 4$, $T_f = 0.9231$.

[b] The slopes of the demand functions are varied, as $(\alpha_1 - \alpha_2)$ is varied, in order to maintain demand elasticities of both groups at -0.33 in most scenarios and at -0.60 in the high-elasticity scenario.

[c] See text, Section 3.4, for differences in the meaning of these parameters compared to other scenarios.

[d] Average value of time is defined as $\underline{\alpha} = (N_1^{NT}\alpha_1 + N_2^{NT}\alpha_2)/(N_1^{NT} + N_2^{NT})$, where N_i^{NT} is the number of type-i users in no-toll regime.

we reverse the relative capacities of the two roadways, making road A the larger one.

The choice of parameters for these scenarios is explained in the following subsections and is summarized in Table 1. Where possible, we maintain comparability with the Liu–McDonald paper: specifically we maintain their choices for road length ($L = 10$ miles), vehicle operating cost ($\beta = 6.8$ cent per vehicle-mile), and free-flow speed ($60/T_f = 65$ miles per hour). In all but one scenario, we also use their assumed capacities ($K_A = 2000$, $K_B = 4000$ vehicles per hour). Actual capacities on SR91, figured at 2000 veh./hour per lane, are twice this; had we used the actual values, we would simply have doubled the demand parameters and thereby obtained identical results.

3.1. *Base Scenario*

In this scenario, we choose the demand parameters so that in the NT regime the price elasticity of demand is -0.33 as in Liu and McDonald, and so that our PM policy produces a toll of about \$2.75 and a travel time differential between routes of about 8 minutes, thereby replicating actual conditions on SR91 in June 1997.[12] This is achieved with an average VOT of 34.38 cents/minute (\$20.63/hour), which is much higher than the value of \$6.36 per

[12] The time difference of 8 minutes is computed from Sullivan [23, Figure 2-13, p. 28], averaging the time differences shown over the 4-hour peak period (3:00–7:00 p.m.) to which the peak toll of \$2.75 applied during June 1997. The 1-hour peak time difference is 12 minutes.

TABLE 2

Results for Base Scenario under Near Homogeneity

Pricing regime[a]	FB	SB	TB	PM	NT
Type of equilibrium[b]	SE2	SE2	SE2	SE2	IE
Toll[c] $-$ A	389	73	267	276	0
Toll $-$ B	389	0	0	0	0
Speed[d] $-$ A	49.6	44.8	59.4	60	40
Speed $-$ B	49.6	38.7	33.5	33.3	40
Delay Cost:[c]					
1 A	97	144	29	26	198
1 B	97	217	297	302	198
2 A	—	—	—	—	198
2 B	97	217	297	302	198
Rel. Use[e] $-$ 1	0.84	0.99	0.94	0.94	1.00
Rel. Use $-$ 2	0.84	0.99	0.94	0.94	1.00
Elast.[f] $-$ 1	-0.59	-0.34	-0.41	-0.41	-0.33
Elast. $-$ 2	-0.59	-0.34	-0.41	-0.41	-0.33
Welfare gain per vehicle[g]	61	4	-40	-45	0

[a] Pricing regimes: FB = first best; SB = second best; TB = third best; PM = profit maximization; NT = no toll (see Section 2.1).

[b] Types of equilibrium: SE2 = partially separated eq., group 2 separated; IE = integrated eq. (see Section 2.1).

[c] All costs (toll, delay cost, welfare gain) are in cents per vehicle. Delay cost is defined as $\alpha_i T_f L\gamma(N_r/K_r)^k$.

[d] Speed is in miles per hour.

[e] Relative use of group is relative to the no-toll regime, i.e., N_i/N_i^{NT}.

[f] Demand elasticity at usage level in the solution.

[g] Welfare gain divided by usage in the NT regime, i.e. $(W - W^{NT})/N^{NT}$.

hour in Liu and McDonald's paper and equal to about 88% of the average wage rate of peak users of the corridor.[13] As we shall see in Section 3.4, when carpools are taken into account, the calibration produces a more moderate average VOT.

The simulation results for nearly homogeneous users are shown in Table 2. (Because the algorithm requires strict inequality of VOT, we set $\alpha_1 - \alpha_2$ equal to 0.02 instead of zero.) The pattern of results is the same as in Liu and McDonald [13]. The welfare gain from second-best pricing (SB) is small, and that from one-route PM policy is negative; their relative efficiencies (relative to the FB policy)[14] are 6 and -74%, which compare to 9 and -50% in Liu and McDonald. In addition, the SB toll is much lower than the FB toll; thus it has

[13] The average self-reported wage rate of peak corridor users was \$23.40 in June 1997 (Parkany [17, p. 45]).

[14] Relative efficiency is defined as $(W^{SB} - W^{NT})/(W^{FB} - W^{NT})$, where W is defined in Eq. (3) and the superscripts indicate policy regimes.

little effect on total traffic, reducing it by only 1%. The FB toll is about 40% higher than the PM toll, and it reduces total traffic by nearly three times as much. With no toll (NT), speed would be 40 miles per hour.

Now we turn to the effects of product differentiation, by examining how the simulation results change when the two groups are assigned different values of travel time. We let α_1 and α_2 diverge by a given amount $\Delta\alpha$. At the same time, we alter the slopes of demand functions so that in the NT regime, the demand elasticities of both user types remain unchanged, as does the average value of travel time. We allow $\Delta\alpha$ to cover nearly the full range of possible values (from zero to twice the average α) in order to portray the properties of the model.[15]

Selected results are shown in Fig. 1. At the far left of each panel, users are homogeneous. At the far right, the two groups' values of time are 2.4 and 66.4 cents/minute, almost the largest difference possible. In the middle, $\Delta\alpha = (\alpha_1 + \alpha_2)/2 = 34.4$ cents/minute, the value we believe most realistic. The partially separated equilibrium SE2 remains optimal for all pricing policies; that is, group 1 users use both roads, which is not surprising because group 1 contains half the population of potential users but the express road contains only a third of the total capacity.

Figure 1a shows the tolls as the function of heterogeneity. In each of the three constrained pricing policies, the toll rises sharply with the difference in value of time. At the middle of the diagram, the SB toll is more than double what it was with identical VOT, although it is still less than half the PM toll. The TB toll is nearly identical to that of PM.

The FB toll is indeed differentiated, but there is a surprise here: the toll differential gets larger at first but then gets smaller again when heterogeneity is extreme. The reason is that when heterogeneity is large, the marginal benefit of a trip by a type 1 user (whose equilibrium value depends strongly on α_1) is much bigger than that of a type 2 user. The FB policy therefore accommodates many more type 1 users than type 2 users, even on route B. As a result, the difference between average values of travel time on the two routes diminishes. Furthermore, route B carries more vehicles than route A, which increases the marginal external cost of a vehicle there, and this effect is more pronounced the more heterogeneity there is.

[15] No doubt both extremes are unrealistic. Still, it is worth noting that variation in VOT occurs for more reasons than income—in fact, VOT appears to vary from day to day even for the same individual, based on observed usage patterns of the SR91 express lanes (Sullivan [23]; Parkany [17]). While the variation in VOT with observed characteristics has been studied, to the best of our knowledge only one study, by Erik Verhoef, has attempted to measure empirically the net effects of both systematic and random variation (see Verhoef and Small [27]). This was a stated-preference study of peak-hour road users in the Dutch Randstad area. The resulting distribution has an interquartile range comparable in magnitude to its mean. Therefore we consider $\Delta\alpha = (\alpha_1 + \alpha_2)/2$, half-way between the extremes of $\Delta\alpha$, to be the most realistic value.

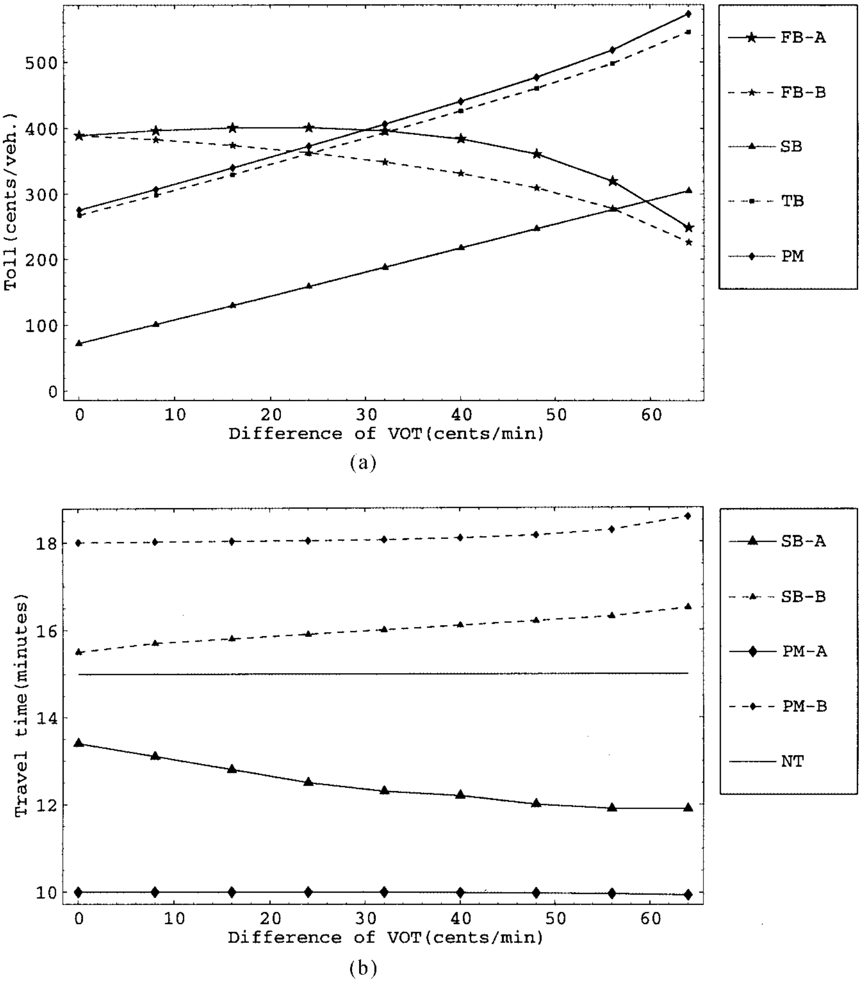
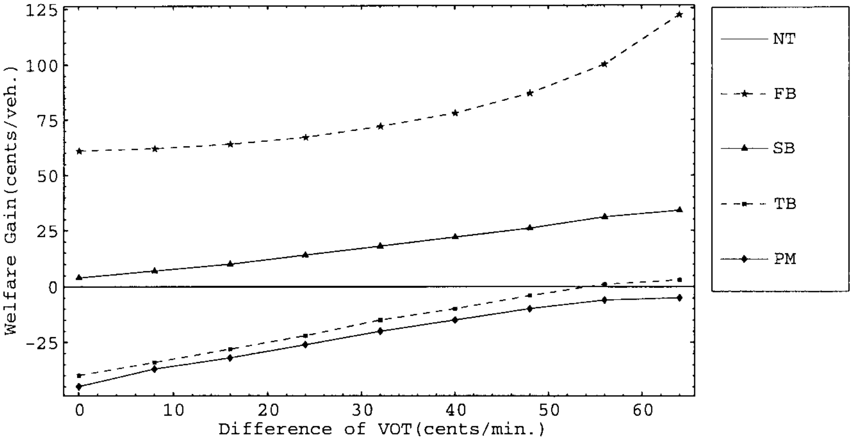


FIG. 1. Base scenario. (a) toll, (b) travel time, (c) welfare gain.

Figure 1b shows the travel time on both routes under the SB and PM policies, as well as under the NT regime. Profit maximization creates a much greater quality differential between the two roads than does SB, an indication of exercise of monopoly power on the priced roadway. The TB regime (not shown) is almost identical to PM.

Figure 1c shows the welfare changes, all relative to no toll. The welfare gains from all the differential-pricing policies are much greater when there is more heterogeneity. The efficiencies of the three constrained regimes also improve



(c)

FIG. 1—*Continued*

when measured as fractions of possible FB welfare gains: for example, the SB welfare gain increases from 6 to 28% of FB. Even so, the PM policy always produces a welfare loss (compared to no toll), and TB pricing almost always does; both perform much worse than SB.

To check the sensitivity of our results to average VOT, we recalculated the base scenario using half the previous value, i.e., \$10.32 per hour, while adjusting intercepts and slopes to maintain the same price elasticity and the same PM time differential. This lowers the tolls charged, but otherwise does not change the qualitative results. We also recalculated the base scenario changing exponent k in the cost function to 2.5, based on evidence in Small [21, pp. 70−71] that a likely range for k is between 2.5 and 5. The results change hardly at all.

3.2. *High-Elasticity and High-Congestion Scenarios*

The next three subsections describe scenarios, each of which deviates from the base scenario in just one respect. Often this requires changing more than one parameter, as is described. In each case, the slopes of the demand functions under homogeneity are set so that under the NT regime, the price elasticities of demand of both groups are the same as in the base scenario (-0.33) or, in the ‘‘high-elasticity’’ scenario, are equal to a stated amount. In each case, furthermore, the slopes are adjusted as heterogeneity is introduced so as to maintain constant elasticities under the NT regime. Average VOT is kept the same as in the base case, except for the last scenario considered (Section 3.4).

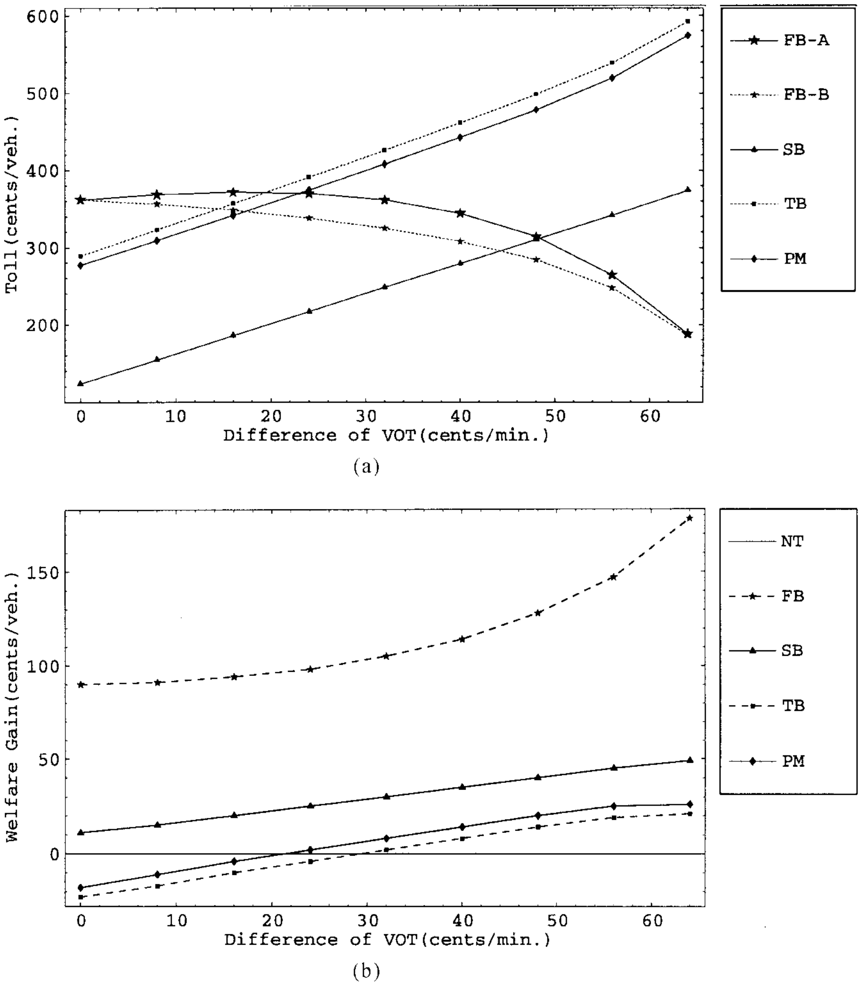


FIG. 2. High-elasticity scenario. (a) toll, (b) welfare gain.

In this subsection, we consider two such scenarios: first one with a higher demand elasticity (-0.60), then one with greater total demand and hence greater congestion. Results for the high-elasticity scenario are shown in Fig. 2. The SB toll is much higher, and the FB toll is lower, than in the base scenario. This result is well known from previous studies, e.g., Verhoef *et al.* [26, Fig. 3]; with more elastic demand, welfare-maximizing policies shift their aim from distributing demand across the two roads to moderating total demand. Furthermore, the efficiencies of the PM and TB policies are improved significantly, resulting in positive welfare gains when the VOT difference is close to

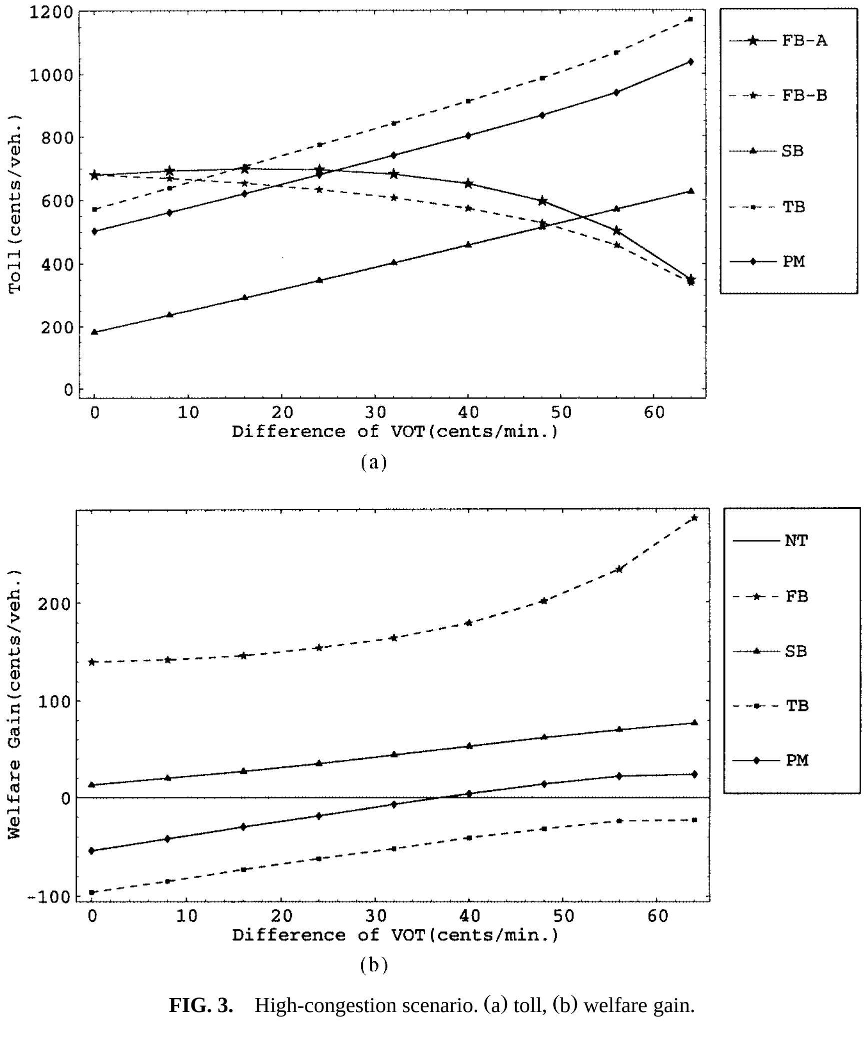


FIG. 3. High-congestion scenario. (a) toll, (b) welfare gain.

or greater than the average value. The SB policy is only slightly improved, however, so the gap between it and the other two constrained policies diminishes; this narrowing of the gap between SB and PM is also observed by Verhoef *et al*. [26].

Next, we consider a scenario with higher congestion, namely a travel-time differential of 15 minutes under PM. We accomplish this by increasing the intercepts of the demand functions. The results, shown in Fig. 3, are mostly similar to the base scenario, but with two exceptions. First, the PM policy now allows substantial congestion on the toll lanes. Second, the TB policy, because it cannot allow such congestion, produces a markedly higher toll than PM, with consequent welfare losses.

3.3. *Proportional-Demand and Reversed-Capacity Scenarios*

In order to examine cases where product differentiation might be more important, we next consider two scenarios in which the numbers of users in the two groups are approximately proportional to the capacities of the two roadways.

First, we set the intercepts of the demand functions proportionally to the relative capacities, i.e., $\alpha_1/\alpha_2 = K_A/K_B = 1/2$, while keeping the total demand under no toll unchanged. The slopes are also adjusted to keep elasticities unchanged. We introduce heterogeneity by increasing α_1 twice as fast as we decrease α_2; thus the distribution of values of time becomes not only dispersed but also skewed. Results are shown in Fig. 4. At the far right of each panel, the value of time of type 1 users is 2.37 cents/minute, while that of type 2 users is 98.40 cents/minute.

The pattern of tolls is similar to that in the base scenario. All the constrained policies (SB, PM, and TB) have considerably improved welfare effects, with PM and TB generating positive welfare gains and with SB reaching almost half the efficiency of FB pricing even under moderate amounts of heterogeneity. The reason for these results is that the differentiated products are better matched to the different user types in this scenario; fewer users are forced into the wrong quality.

Next, we try an even more drastic change by interchanging the two roadway capacities: 4000 veh/hour for the express lanes and 2000 for the free lanes. Results are shown in Fig. 5. The three constrained policies have higher tolls in this scenario because the constraints have been substantially relaxed by making them apply to a smaller roadway. Furthermore, even with homogeneous users, both the welfare gains from SB and the welfare losses from PM and TB are more than doubled, consistent with simulations by Liu and McDonald [14] and theoretical analysis by Braid [3]. What is different here is that increasing heterogeneity has, in this scenario, a much bigger positive effect on all three constrained policies. This appears to be due to better matching of group sizes to capacity. With enough heterogeneity, the welfare gain from SB becomes almost as much as from FB, and the relative efficiency even of TB reaches 77%. Profit-maximization, however, performs quite poorly (on welfare grounds, not on profits) relative to other policies, due to its setting an excessively high toll.

We get different types of equilibria in this scenario. As heterogeneity is increased, user differences become too great to be worth accommodating on a shared roadway, so the optimal equilibria tend to become fully separated (SE). The exception is PM, where equilibrium remains partially separated (SE2) due to the very high toll charged. These equilibria are shown in Fig. 5b as differently sized symbols (larger for SE2, smaller for SE). In one regime (SB),

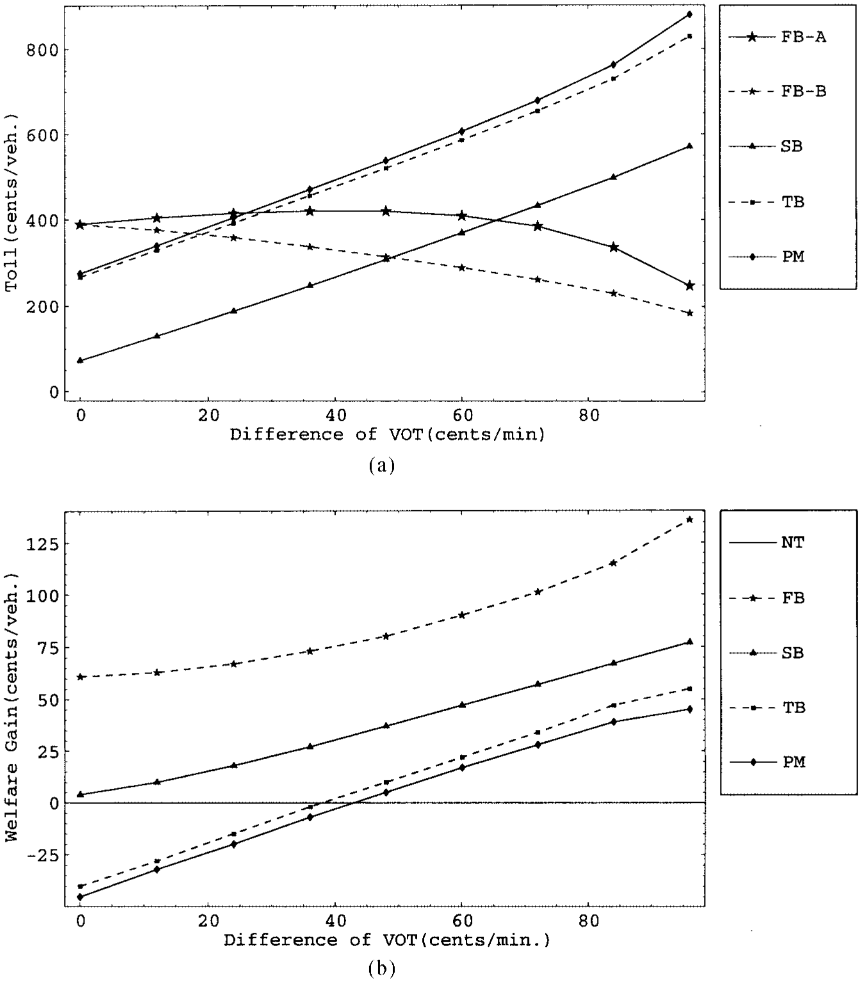


FIG. 4. Proportional-demand scenario. (a) toll, (b) welfare gain.

a change in equilibrium type is accompanied by a rather sudden change in toll, as seen in Fig. 5a.

3.4. *Carpool Scenario*

As noted in the introduction, our model does not allow us to assess the importance of maintaining low congestion in the express lanes for purposes of

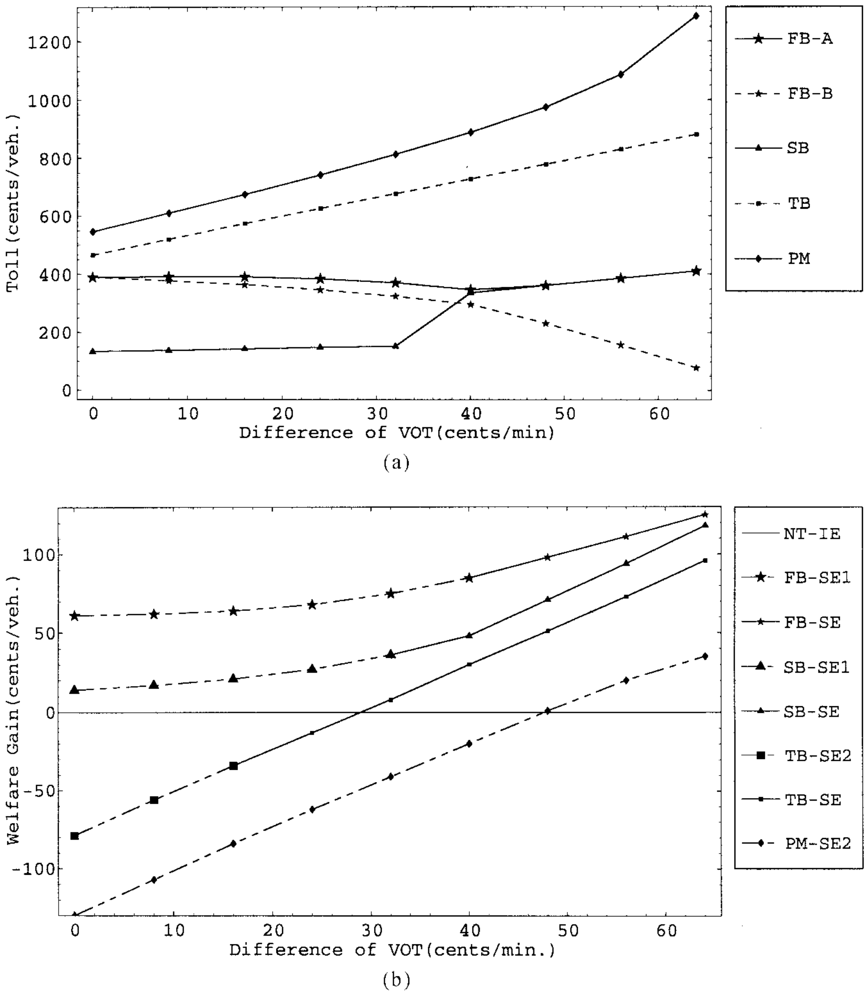


FIG. 5. Reversed-capacity scenario. (a) toll, (b) welfare gain.

encouraging carpools.[16] However, another goal of carpool lanes is to improve efficiency by speeding up HOVs at the expense of lower-occupancy vehicles. This can be viewed either as increasing the throughput of people or, on the assumption that higher-occupancy vehicles have a higher value of time per vehicle, as reducing total time costs.

[16] The presence of HOV lanes does appear to encourage carpooling, based on evidence from several user surveys cited by Long [15] and on econometric evidence provided by Brownstone and Golob [5].

Here we adopt the latter interpretation and define a scenario in which carpools are exogenous and each has three times the average VOT of other vehicles. We also assume that carpools use the express lanes without charge. These assumptions are an attempt to capture a feature of the HOT-lane demonstrations thus far ignored, which is that vehicles with three or more people (HOV3+) travel free in the express lanes.[17] In the case of SR91 (as well as the Katy Expressway in Houston), two-person carpools have to pay the announced toll; we can therefore account for them roughly in our model by assuming that they are part of user group 1.

Our objective is to get an idea of how much difference the existence of free carpools in the express lanes would make to our main findings. A secondary objective is to calibrate the model more carefully by assuming that conditions observed on SR91 resulted from the PM regime being applied in the presence of non-paying HOV3+ vehicles and a moderate heterogeneity in value of time. The number of HOV3+ vehicles is assumed constant in all pricing regimes at an amount equal to 4.4% of the total vehicle flow in the PM regime; this figure is based on observed peak flows on SR91 in June 1997 (Sullivan [23, p. 35]). Moderate heterogeneity is taken to mean that the VOT difference, $\alpha_1 - \alpha_2$, is equal to the average value to time. Again, we calibrate to achieve a toll of $2.75 and travel-time difference of 8 minutes during the peak period under a PM policy. The resulting parameter set has average value of time (not counting carpools) of 23 cents/minute or $13.80/hour; this is 59% of the average wage rate, much closer to the central tendency of most empirical measurements of value of time than the value we used in our baseline calibration.

The results for the carpool scenario are presented in Fig. 6.[18] The presence of carpools, with their high VOT, substantially increases the benefits from making the express lanes faster, and this effect changes the nature of the results considerably from our base scenario.

First, although FB tolls are slightly lower overall (due to the lower average value of time), the FB toll differentials are considerably greater than before. This is because the presence of high-VOT carpools increases the marginal external cost on road A, but not on road B.

Second, the benefits of all three constrained policies are larger than in the base scenario. The reason is the same as the reason for the higher FB toll differential: the presence of carpools increases the importance of product differentiation, which is a strength of the constrained policies.

Third, the gap between the SB and the other two constrained policies is less than in the base scenario. At a moderate VOT difference of 23 cents/minute, equal to the average VOT for non-carpools, the relative efficiencies of these

[17] On SR91, this was the case until Jan. 1998, when HOV3+ vehicles began paying half the regular price.

[18] In order to ensure that carpools voluntarily take the express lanes, even in FB where they have to pay, we limit the range of $\Delta\alpha$ to that for which their VOT exceeds α_1.

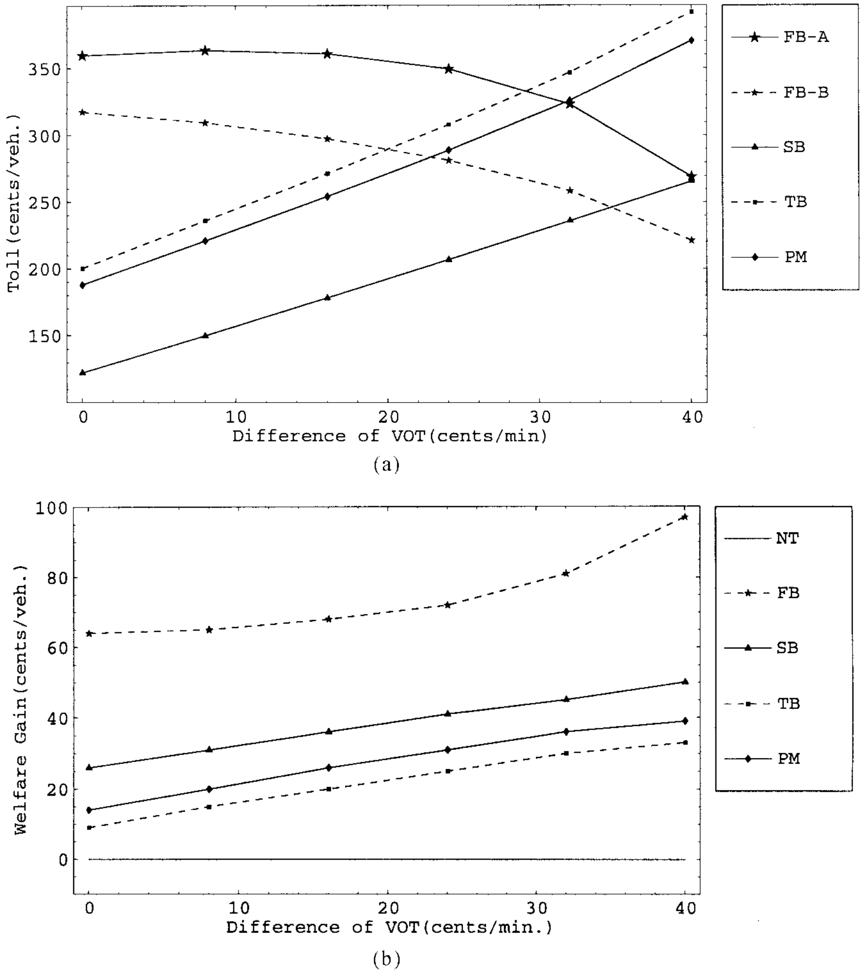


FIG. 6. Carpool scenario. (a) toll, (b) welfare gain.

policies (relative to first best) are 57% for SB, 43% for PM, and 34% for TB. No longer do PM and TB produce welfare losses, even with no heterogeneity. However, TB still performs worse than all other policies.

4. CONCLUSION

Our results demonstrate the importance of heterogeneity in VOT for evaluating congestion policies that offer pricing as an option. Generally, the existence of heterogeneity favors such policies because product differentiation then offers a greater advantage: those with high VOT reap more benefits from the high-priced option, while those with low VOT find it all the more important not to be subjected to policies aimed at the average user.

Nevertheless, in a ''pure'' setting without carpools, insisting that one of the products be free imposes quite a large penalty except when heterogeneity is extreme. In our base scenario for moderate amounts of heterogeneity, a SB one-route pricing policy achieves only one-sixth to one-third the possible welfare gains of FB pricing, and uses a much smaller toll. Even more alarming, a revenue-maximizing policy sets the price far higher, and achieves benefits far lower, than SB pricing. This is true no matter what the heterogeneity, and it also applies to a policy that maintains nearly congestion-free travel in the priced roadway. In the majority of cases, the overall benefits from pricing are negative for these policies. We recognize that such policies may sometimes be the only way the express lanes can be built at all, or the only way they can be opened to general traffic, but their potential inferiority to a NT policy is still troubling.

If, however, carpools travel for free and carry a higher VOT than other vehicles, the benefits of the constrained policies greatly increase, and the gap between SB and the other two decreases. No welfare losses are then encountered in our simulations, and the benefits from profit maximization are about three-quarters of the benefits from SB pricing at moderate amounts of heterogeneity. This finding treats carpools as exogenous, so does not take into account any social value from inducing more people to carpool. It suggests that policymakers should be mindful of a secondary purpose of policies favoring carpools, namely, to lower total transportation costs by allowing vehicles with high VOT to bypass those with lower VOT. This observation is relevant to current controversies over decommissioning carpool lanes, although this is not a policy we have examined here.

From these observations, we draw three conclusions about partial-pricing policies under highly congested conditions. The first two are in accord with studies based on homogeneous users. First, when politics or other considerations dictate that one roadway be free, aggregate costs can be reduced by letting the priced roadway become at least moderately congested; carpooling mandates or privatization goals may prevent this, but they do so at a cost. Second, under many conditions, partial-pricing policies are inadequate substitutes for more thorough-going pricing policies. Third, accounting for heterogeneity does improve the performance of partial-pricing policies by creating significant value for product differentiation, especially when the price elasticities for total demand is high and congestion in the absence of tolls is extreme.

APPENDIX

A.1. *General Form of the Nonlinear Programming Problem and Its Possible Solutions*

We assume that at least some type 1 users use road A and at least some type 2 users use road B. We consider a congested traffic condition, so the toll charged under a policy regime is strictly greater than zero. The general form of the FB problem is therefore:

$$\max W = \int_0^{N_{1A}+N_{1B}} P_1(t)\, dt + \int_0^{N_{2A}+N_{2B}} P_2(t)\, dt - \sum_i \sum_r N_{ir} c_{ir}$$

$$s.t.\ h_1 \equiv P_1(N_{1A} + N_{1B}) - c_{1A}(N_{1A} + N_{2A}) - \tau_A = 0 \tag{A.1a}$$

$$h_2 \equiv P_2(N_{2A} + N_{2B}) - c_{2B}(N_{1B} + N_{2B}) - \tau_B = 0 \tag{A.1b}$$

$$h_3 \equiv N_{1B} \cdot (P_1 - c_{1B} - \tau_B) = 0 \tag{A.1c}$$

$$h_4 \equiv N_{2A} \cdot (P_2 - c_{2A} - \tau_A) = 0 \tag{A.1d}$$

$$g_1 \equiv P_1(N_{1A} + N_{1B}) - c_{1B}(N_{1B} + N_{2B}) - \tau_B \le 0 \tag{A.1e}$$

$$g_2 \equiv P_2(N + N_{2B}) - c_{2A}(N_{1A} + N_{2A}) - \tau_A \le 0 \tag{A.1f}$$

$$g_3 \equiv -N_{1B} \le 0 \tag{A.1g}$$

$$g_4 \equiv -N_{2A} \le 0 \tag{A.1h}$$

where $P(\cdot)$ and $c(\cdot)$ are the functions defined by (2) and (1). Certain constraints are added for the SB, TB, and PM policy, and the objective function is replaced by toll revenues in PM policy. Because we assume $N_{1A}, N_{2B} > 0$, (A.1a, b) are the same as (3) of the paper; (A.1c, d) are equivalent to (5c, d); (A.1e, f) to (5a, b); and (A.1g, h) to (5e).

Suppose $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ are the Lagrangian multipliers for the four equality constraints, and $\gamma_1, \gamma_2, \gamma_3, \gamma_4$ are those for the inequality constraints. According to the Kuhn−Tucker theorem, the necessary condition for the optimal solution $N^* = (N_{1A}^*, N_{1B}^*, N_{2A}^*, N_{2B}^*)$, $\lambda^* = (\lambda_1^*, \lambda_2^*, \lambda_3^*, \lambda_4^*)$, $\gamma^* = (\gamma_1^*, \gamma_2^*, \gamma_3^*, \gamma_4^*)$ are:

$$\nabla W(N^*) - \sum_{i=1}^{4} \lambda_i^* \nabla h_i(N^*) - \sum_{j=1}^{4} \gamma_j^* \nabla g(N^*) = 0 \tag{A.2a}$$

$$\gamma_j^* g_j(N^*) = 0, \quad j = 1, 2, 3, 4 \tag{A.2b}$$

$$\gamma_j^* \ge 0, \quad j = 1, 2, 3, 4 \tag{A.2c}$$

$$g_j \le 0, \quad j = 1, 2, 3, 4 \tag{A.2d}$$

If constraints (A.1e) and (A.1f) are binding at the same time, the tolls on both routes must be equal, as shown in Section 2. This is impossible for SB, TB, and PM policy, and our numerical results also show that this case is never optimal for FB policy. As a result, the possible solution cases for the programming problem are only three:

1. $\gamma_1^* = 0,\ \gamma_2^* > 0$ (SE1);

In this case, (A.2c) $\Rightarrow g_2 = 0$, i.e., (A.1f) must be binding. This means type 2 users are indifferent for two routes. Then (A.1e) cannot be binding, i.e., type 1 users strictly prefer road A and, from (A.1c), $N_{1B}^* = 0$.

2. $\gamma_1^* > 0$, $\gamma_2^* = 0$ (SE2)

In this case, constraint (A.1e) is binding and constraint (A.1f) is not binding, and $N_{2A}^* = 0$.

3. $\gamma_1^* = 0$ and $\gamma_2^* = 0$;

In this case, we can only say (from the argument above) that (A.1e) or (A.1f) or both must be non-binding, therefore N_{1B}^* or N_{2A}^* or both must be zero. Thus there are three solution cases:

3a. (A.1f) is binding and (A.1e) is not; $N_{1B}^* = 0$ (SE1).
3b. (A.1e) is binding and (A.1f) is not; $N_{2A}^* = 0$ (SE2).
3c. both (A.1e) and (A.1f) are non-binding; $N_{1B}^* = N_{2A}^* = 0$ (SE).

In the paper, we divide the programming problem into different cases (SE, SE1, SE2) and solve each case under each policy. The above classification shows that the solutions from these cases include all of the possible solutions for the whole problem.

A.2. *Derivation of Optimal Tolls*

In this section, we show how the general problem simplifies in each policy and equilibrium type (here described as ‘‘case’’). As noted in the paper, we first ignore the non-negative constraints (A.1g, h), then check each of them separately and impose it as an equality if required.

A.2.1. *FB Policy*

Case SE. Substituting $N_{1B} = 0$ and $N_{2A} = 0$ into the welfare function, the welfare maximizing problem can be written as:

$$\max W = \int_0^{N_{1A}} P_1(t)\,dt + \int_0^{N_{2B}} P_2(t)\,dt - N_{1A}\cdot c_{1A}(N_{1A}) - N_{2B}\cdot c_{2B}(N_{2B})$$

The objective function is strictly concave because it equals the sum of four strictly concave functions. Therefore, the solution to the first-order conditions must be unique. The optimal traffic (N_{1A}^*, N_{2B}^*) in this case can be solved out from those first-order conditions. The corresponding tolls on the two routes, determined by (A.1a, b), are:

$$\tau_A = P_1 - c_{1A} = N_{1A}\cdot c'_{1A}(N_{1A}) \equiv MEC_{1A}$$
$$\tau_B = P_2 - c_{2B} = N_{2B}\cdot c'_{2B}(N_{2B}) \equiv MEC_{2B}$$

The optimal toll on each road is equal to the difference between social and private marginal cost on that road, known as ‘‘marginal external cost’’ *MEC*, just as in a single-route model.

*Case SE*1. Substituting $N_{1B} = 0$ into the welfare function, we get:

$$\max W = \int_0^{N_{1A}} P_1(t)\, dt + \int_0^{N_{2A}+N_{2B}} P_2(t)\, dt$$
$$- N_{1A} \cdot c_{1A}(N_{1A} + N_{2A}) - N_{2A} \cdot c_{2A}(N_{1A} + N_{2A}) - N_{2B}c_{2B}(N_{2B})$$

This objective function is also strictly concave because it equals the sum of five strictly concave functions. The corresponding tolls are:

$$\tau_A = P_1(N_{1A}) - c_{1A} = N_{1A}c'_{1A}(N_{1A} + N_{2A}) + N_{2A}c'_{2A}(N_{1A} + N_{2A})$$
$$\equiv MEC_A = P_2 - c_{2A}$$
$$\tau_B = P_2(N_{2A} + N_{2B}) - c_{2B}(N_{2B}) = N_{2B}c'_{2B}(N_{2B}) \equiv MEC_{2B}$$

The tolls are again the differences between social and private marginal costs on each route. The social cost on route A includes the users of both groups; the social cost on route B includes just the users of group 2. We also check the corner solution of $N_{2A} = 0$ in the simulation study.

*Case SE*2. This case is symmetric to SE1.

A.2.2. *SB and TB Policies*

Case SE. The welfare-maximizing problem under SB pricing policy for the fully separated equilibrium case can be written as:

$$\max W = \int_0^{N_{1A}} P_1(t)\, dt + \int_0^{N_{2B}} P_2(t)\, dt - N_{1A}c_{1A}(N_{1A}) - N_{2B}c_{2B}(N_{2B})$$
$$s.t.\ P_2(N_{2B}) = c_{2B}(N_{2B})$$

N_{2B} is determined solely by the constraint, and numerical results in the paper show that there is only one positive real solution for N_{2B}. The objective function is a strictly concave function of N_{1A}, so if this case can occur, the solution is unique. The corresponding toll on route A is:

$$\tau_A = N_{1A}c'_{1A}(N_{1A}) \equiv MEC_{1A}$$

This toll is just the difference of social and private marginal cost on that road; the social cost including just the users of group 1. There are no route spill-overs in fully separated equilibrium: that is, road A is treated just as in the FB policy.

*Case SE*1. The corresponding Lagrangian is:

$$
\begin{aligned}
L = {} & \int_0^{N_{1A}} P_1(t)\,dt + \int_0^{N_{2A}+N_{2B}} P_2(t)\,dt \\
& - N_{1A}c_{1A}(N_{1A}+N_{2A}) - N_{2A}c_{2A}(N_{1A}+N_{2A}) - N_{2B}c_{2B}(N_{2B}) \\
& - \lambda_1\big[P_1(N_{1A}) - c_{1A}(N_{1A}+N_{2A}) - P_2(N_{2A}+N_{2B}) \\
& \qquad\qquad + c_{2A}(N_{1A}+N_{2A})\big] \\
& - \lambda_2\big[P_2(N_{2A}+N_{2B}) - c_{2B}(N_{2B})\big]
\end{aligned}
$$

where the constraints (A.1a, b) have been rewritten using (A.1f) as an equality in order to eliminate τ_A as a variable. The Lagrangian multiplier λ_1 represents the shadow price of not price discriminating on road A, that is, it represents the increase of social welfare that could be achieved by charging type 1 users more than type 2 users, since the latter have a suboptimally priced substitute (road B). This problem can be solved for N_{1A}, N_{2A}, N_{2B}, λ_1, and λ_2. The toll which decentralizes the solution allocation is then determined by (A.1a) as:

$$
\tau_A = N_{1A}c'_{1A} + N_{2A}c'_{2A} - \left[\frac{P'_2 N_{2B}c'_{2B}\cdot(P'_1 - c'_{1A} + c'_{2A})}{P'_1P'_2 - P'_1c'_{1B} - P'_2c'_{2B}}\right]
$$

The toll on route A equals marginal external cost minus a positive adjustment term which depends on the slope of demand function and cost function.

*Case SE*2. The Lagrangian is:

$$
\begin{aligned}
L = W & - \lambda_2\big[P_2(N_{2B}) - c_{2B}(N_{1B}+N_{2B})\big] \\
& - \gamma_1\big[P_1(N_{1A}+N_{1B}) - c_{1B}(N_{1B}+N_{2B})\big]
\end{aligned}
$$

where (A.1e) has been used as an equality with Lagrangian multiplier γ_1 which represents the ‘‘shadow price’’ of not being able to price discriminate on road B.

Again, we solve and use (A.1a) to determine the toll on route A as:

$$
\tau_A = N_{1A}c'_{1A} - \left[\frac{(N_{1B}c'_{1B} + N_{2B}c'_{2B})P'_2P'_1}{P'_1P'_2 - P'_1c'_{2B} - P'_2c'_{1B}}\right]
$$

The toll here equals the marginal congestion cost plus an adjustment term which depends on the slopes of the demand and cost functions. When the users are identical, so that $c'_{1B} = c'_{2B}$ and $P'_1 = P'_2$, this formula reduces to Eq. (2) of Verhoef *et al*. [26].

It is difficult to judge analytically whether these solutions for cases SE1 and SE2 are unique, because of the nonlinear form of the constraints. In the simulation study, we use different initial values to show that in these cases no more than one equilibrium solution can be found.

The TB policy is the same as the SB policy except that we add an extra constraint (6), which we check separately rather than include in the Lagrangian.

A.2.3. *PM Policy*

The maximizing problem here has the same constraints as the ones in the SB policy. The only difference is that the objective function now is:

$$R = (N_{1A})[P_1(N_{1A}) - c_{1A}(N_{1A} + N_{2A})] + N_{2A}[P_2(N_{2A} + N_{2B}) - c_{2a}(N_{1A} + N_{2A})]$$

Case SE. The solution of this case must be unique for the same reason as the SE case in SB policy. The toll which maximizes revenue is found to be:

$$\tau_A = N_{1A}[c'_{1A}(N_{1A}) - P'_1]$$

The toll is set at marginal social cost plus a monopolistic mark-up which is inversely related to the demand elasticity of group 1 (compare Small [21, Eq. (4.41)]). Equivalently, this equation can be written as $\tau_A + N_{1A}P'_1 = N_{1A}c'_{1A}$, that is, marginal revenue equals marginal cost.

*Case SE*1. The toll is found to be:

$$\tau_A = N_{1A}c'_{1A} + N_{2A}c'_{2A} - N_{1A}P'_1 + \left[\frac{(N_{2A}P'_2c'_{2B} + N_{1A}P'_1P'_2 - N_{1A}P'_1c'_{2B})(P'_1 - c'_{1A} + c'_{2A})}{P'_1P'_2 - P'_1c'_{2B} - 2(P'_2)^2 + P'_2c'_{2B}}\right]$$

Again the toll equals marginal congestion cost plus a monopolistic mark-up.

*Case SE*2. The revenue-maximizing toll on route A is:

$$\tau_A = N_{1A}c'_{1A} - N_{1A}P'_1 + \left[\frac{N_{1A}(P'_1)^2(P'_2 - c'_{2B})}{P'_1(P'_2 - c'_{2B}) - c'_{1B}P'_2}\right]$$

Again, the uniqueness of equilibrium solution for case SE1 and SE2 is proved numerically.

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