EconS 510

Takehome Quiz #4 Answer Key

Problem 1.

a) The following table shows the commission Abe could receive for selling different numbers of luxury and compact cars.

commission(X,Y)	*Y*=0	1	2	3
X=0	0	100	200	300
1	200	300	400	500
2	400	500	600	700
3	600	700	800	900

Therefore, the expected weekly commission is

$$E(commission) = \sum_{X,Y \in \{0,1,2,3\}} P(X,Y) * commission(X,Y) = \$354.$$

The expected total pay is $354 + 400 = \$754.$

b) The following tables present the marginal density functions for *X* and *Y*, respectively.

X	*f(X)*	*Commission(X)*
0	0.35	0
1	0.29	200
2	0.19	400
3	0.17	600

Y	*f(Y)*	*Commission(Y)*
0	0.35	0
1	0.29	100
2	0.19	200
3	0.17	300

Therefore, the expected value of his commission from selling compact cars is

$$E(commission(Y)) = \sum_{Y \in \{0,1,2,3\}} P(Y) * commission(Y) = \$118.$$

The expected value of his commission from selling luxury cars is

$$E(commission(X)) = \sum_{X \in \{0,1,2,3\}} P(X) * commission(X) = \$236.$$

c) The following table shows the conditional density function.

X|Y=2	*f(X|Y=2)*
0	0.263158
1	0.263158
2	0.263158
3	0.210526

Therefore, $E(commission(X)\,|\,Y=2) = \sum_{X\in\{0,1,2,3\}} P(X\,|\,Y=2) * commission(X\,|\,Y=2) = \$284.21$

d) $E(pay\ after\ tax) = 0.75*754 = \$565.5$

Problem 2

a) $E(Y) = 3x_L^{.6}x_K^{.3}\int_0^\infty e^u\left(3e^{-3u}\right)du = -\frac{9}{2}x_L^{.6}x_K^{.3}e^{-2u}\Big|_0^\infty = \frac{9}{2}x_L^{.6}x_K^{.3}$

b) $E(\pi) = 4*\frac{9}{2}10^{0.9} - 10*10 - 15*10 = -\$107.02$

c) $\max E(\pi) = 18x_L^{.6}x_K^{.3} - 10x_L - 15x_K$

First order conditions are $10.8x_L^{-.4}x_K^{.3} = 10$, and $5.4x_L^{.6}x_K^{-.7} = 15$. Therefore, the optimal levels of labor and capital inputs are 0.07996 and 0.02665 per acre, respectively.

The optimal profit per acre is $\pi^* = \$1.33$.

d) π' = profit per acre after irrigated

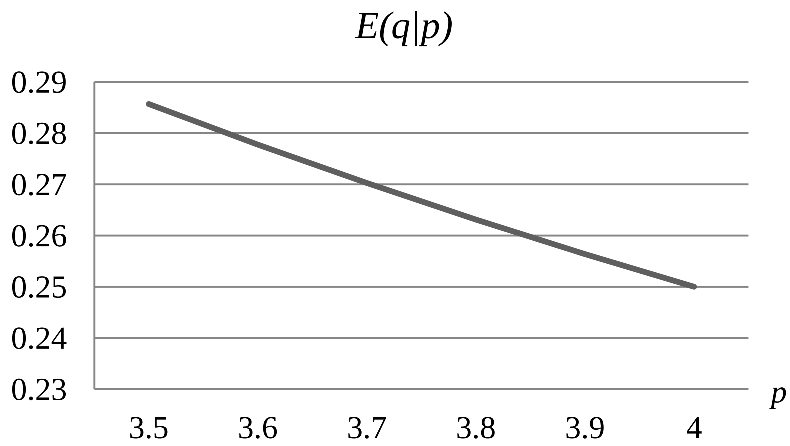
$E(\pi') = 30x_L^{.6}x_K^{.3} - 10x_L - 15x_K - 125$

She should not irrigate, since $\pi'^* = -\$102.96$.

Problem 3

a) $E\left(q\,|\,p\right) = \int_0^\infty q*P\left(q\,|\,p\right)dq = \int_0^\infty pqe^{-pq}dq = p^{-1}$ for $p\in\left[3.5, 4\right]$

b)



c) $E\left(q \mid p=3.75\right)=1/3.75=0.2667$

d) $E\left(pq\right)=\int_{3.5}^{4}\int_{0}^{\infty}2p^2qe^{-pq}dq=2*0.5=1$ million dollars

Problem 4

a) $P\left(x\right)=0.5^x I_{\{1,2,3,4...\}}\left(x\right)$

b) $E\left(payment\right)=\sum_x 2^x 0.5^x=\infty$

c) No. The expected payment seems to be infinite, but it is because the expectation does not exist in this case. In order to benefit from the game (2^{12}>2500), the coin has to be tails for the first 11 times. The probability is very small (about 0.0005).

Problem 5

If 0 cake is baked, $E\left(\text{Profit}\right)=0$.

If 1 cake is baked, $E\left(\text{Profit}\right)=-1*P(x=0)+1.5*P(x\geq 1)=\frac{-2+37.5}{27}=\$1.31$.

If 2 cakes are baked, $E\left(\text{Profit}\right)=-2*P(x=0)+0.5*P(x=1)+3*P(x\geq 2)=\$2.35$.

If 3 cakes are baked,
$E\left(\text{Profit}\right)=-3*P(x=0)-0.5*P(x=1)+2*P(x=2)+4.5*P(x\geq 3)=\$3.02$.

If 4 cakes are baked,
$E(\text{Profit}) = -4*P(x=0) - 1.5*P(x=1) + 1*P(x=2) + 3.5*P(x=3) + 6*P(x \geq 4) = \$3.22.$

If 5 cakes are baked,
$E(\text{Profit}) = -5*P(x=0) - 2.5*P(x=1) + 0*P(x=2) + 2.5*P(x=3) + 5*P(x=4) + 7.5*P(x=5)$
$= \$2.87.$
Baking 4 cakes yields to the highest expected profit.

Problem 6

a) $E(u(\Pi)) = E(\Pi) = q\int_0^5 p*0.048*(5p - p^2)dp - 0.5q^{1.5} = 2.5q - 0.5q^{1.5}$

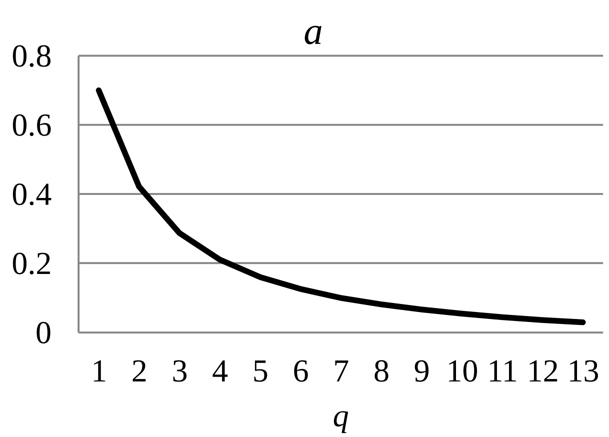
$$q* = \frac{100}{9}$$

b)

$$\text{var}(\Pi) = \int_0^5 \left(pq - 0.5q^{1.5} - 2.5q + 0.5q^{1.5}\right)^2 0.048*(5p - p^2)dp$$
$$= 0.048q^2 \int_0^5 (p - 2.5)^2 (5p - p^2)dp$$
$$= 1.25q^2$$

$$E(u(\Pi)) = 2.5q - 0.5q^{1.5} - \alpha 1.25q^2$$

$$2.5 - 0.75q^{0.5} - 2.5\alpha q = 0 \Rightarrow \alpha = \frac{2.5 - 0.75q^{0.5}}{2.5q}$$



$q = 1 \Rightarrow \alpha = 0.7$

c) Price is uncertain:

$$E\left(u\left(\Pi\right)\right) = 2.5q - 0.5q^{1.5} - 1.25q^{2}$$
$$\Rightarrow q^* = 0.7416$$

Price is set to p' :

$$\Pi' = p'q - 0.5q^{1.5} \Rightarrow p' - 0.75q^{.5} = 0$$
$$\Rightarrow p' = 0.75 * 0.7416^{.5} = 0.6459$$