EconS 510

Takehome Quiz #2 Answer Key

Problem 1.

a) $i. f(x) \geq 0$ for $\forall x.$

$ii. \sum_{x \in \{0,1\}} f(x) = 0.4 + 0.6 = 1.$

Therefore, $f(x)$ is a valid pdf.

b) $i. f(x) \geq 0$ for $\forall x.$

$ii. \int_0^1 f(x)dx = x^3\Big|_0^1 = 1.$

Therefore, $f(x)$ is a valid pdf.

c) $\int_0^1 f(x)dx = \frac{1}{3}x^3 - x^2 + x\Big|_0^1 = \frac{1}{3} \neq 1.$

Therefore, $f(x)$ is <u>not</u> a valid pdf.

d) $i. f(x) \geq 0$ for $\forall x.$

$ii. \sum_{x \in \{1,2,3,...\}} f(x) = 0.5^1 + 0.5^2 + 0.5^3 + ... = \frac{1}{0.5} - 1 = 1.$

Therefore, $f(x)$ is a valid pdf.

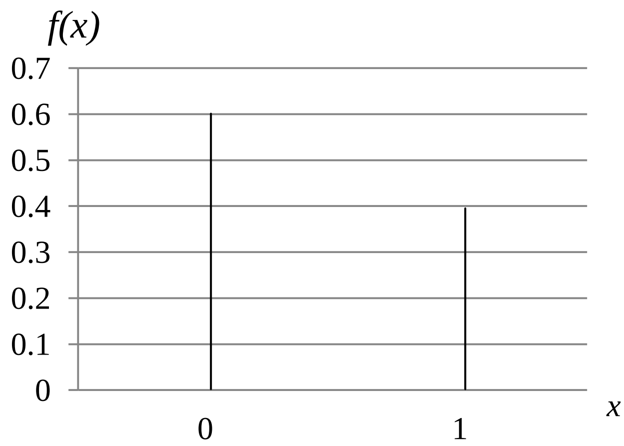
e) $i. f(x) \geq 0$ for $\forall x.$

$ii. \int_0^\infty f(x)dx = -e^{-x}\Big|_0^\infty = 1.$

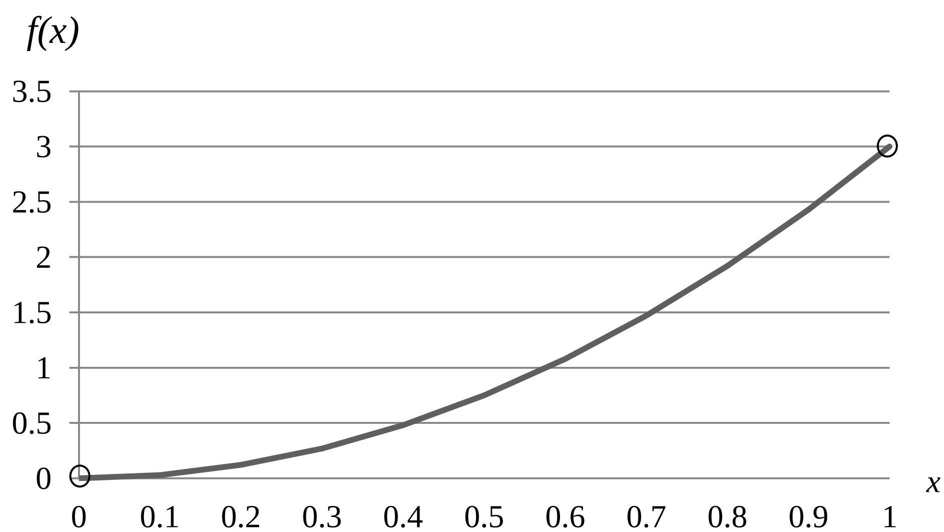
Therefore, $f(x)$ is a valid pdf.

Problem 2

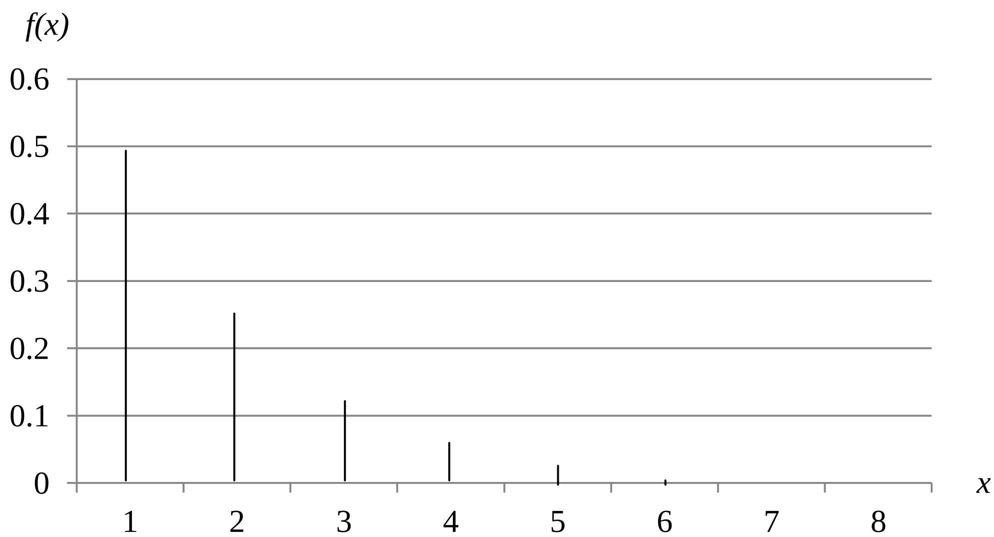
1. a) The following graph shows the probability density function $f(x) = (.4)^x (.6)^{1-x} I_{\{0,1\}}(x)$.



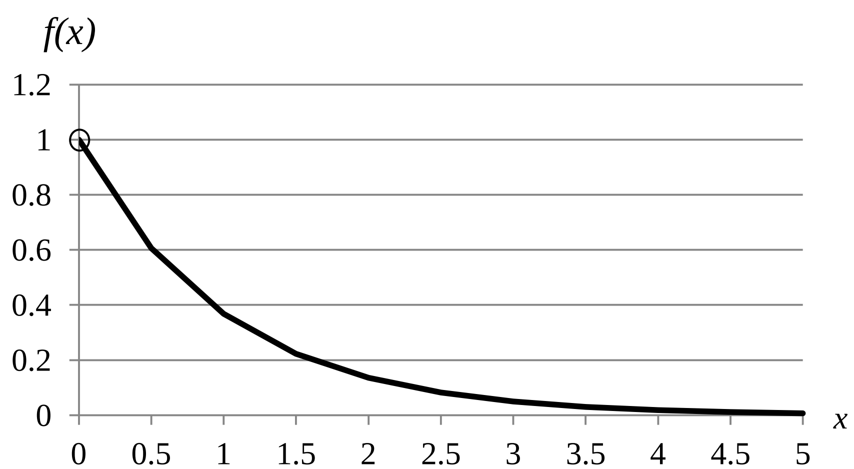
1. b) The following graph shows the probability density function $f(x) = 3x^2 I_{(0,1)}(x)$.



1. d) The following graph shows the probability density function $f(x) = .5^x I_{\{1,2,3,...\}}(x)$.



1. e) The following graph shows the probability density function $f(x) = e^{-x} I_{(0,\infty)}(x)$.



Problem 3

a) When $p = 4$,

$$P(Q > 240) = P(238 + \varepsilon > 240) = P(\varepsilon > 2)$$
$$= 0.25\varepsilon\Big|_2^4 - 0.0625/2 * \varepsilon^2\Big|_2^4$$
$$= 0.5 - 0.375 = 0.125$$

When $p = 3$,

$$P(Q > 240) = P(241 + \varepsilon > 240) = P(\varepsilon > -1)$$
$$= 0.25\varepsilon\Big|_{-1}^0 + 0.0625/2 * \varepsilon^2\Big|_{-1}^0 + 0.5$$
$$= 0.25 - 0.03125 + 0.5 = 0.71875$$

b) Daily profit above variable cost = $\pi = pQ - 2.75Q = (p - 2.75)(250 - 3p + \varepsilon)$ where,

$$f(\varepsilon) = (.25 + .0625\varepsilon) I_{[-4,0]}(\varepsilon) + (.25 - .0625\varepsilon) I_{[0,4]}(\varepsilon).$$

Therefore, when $p = 3$, $P(\pi > 60) = P(0.25 * 241 + 0.25 * \varepsilon > 60) = P(\varepsilon > -1) = 0.71875.$

c) $\varepsilon = 0$ is the outcome that is assigned the highest density weighting by its pdf.

$\frac{\partial \pi}{\partial p} = 258.25 - 6p = 0 \Rightarrow p = 43.04$ is the profit-maximizing choice of the price.

$\pi = (43 - 2.75)(250 - 3*43) = 4870$. The maximized profit is about $4870,000.

Since ε is a continuous random variable, the probability at a single point is 0. Therefore, $P(\pi = 4870) = P(\varepsilon = 0) = 0$.

d) $\frac{\partial Q}{\partial p}\frac{p}{Q} = \frac{-9}{241 + \varepsilon}$. The elasticity of quantity demanded with respect to price is a random variable, since it has the random component ε . The value depends on the random term.

$\frac{\partial Q}{\partial p} = -3$ is a constant. Therefore, it is not a random variable.

Problem 4

a) $\pi = 25x + 50y$

$\pi = \{0, 25, 50, 75, 100, ..., 375\}$

b) $P(\pi) = \sum_{\pi = 25x + 50y, (x,y) \in A} \frac{14.0625}{x!\, y!(5-x)!(5-y)!}$, or

π	$P(\pi)$	π	$P(\pi)$
0	0.000977	200	0.151367
25	0.004883	225	0.131836
50	0.014648	250	0.098633
75	0.034180	275	0.063477
100	0.063477	300	0.034180
125	0.098633	325	0.014648
150	0.131836	350	0.004883
175	0.151367	375	0.000977

c) $P(\pi \geq 200) = 0.5$

d) $P(\pi \geq 400) = 0$

e) $P(\pi = 375) = P(x = 5, y = 5) = 0.000977$

Problem 5

a) $F(-\infty) = 0$, $F(\infty) = 1$, $f(b) = 0.5e^{-0.5b} \geq 0$ for $b \in (0, \infty)$

Therefore, $F(b)$ is a CDF. And $f(b) = 0.5e^{-0.5b} I_{(0,\infty)}(b)$ is the corresponding pdf.

b) $F(-\infty) = 0$, $F(\infty) = 0$

Therefore, $F(b)$ is <u>not</u> a CDF.

c) $F(-\infty) = 0$, $F(\infty) = 1$, $f(b) \geq 0$

Therefore, $F(b)$ is a CDF. And $f(x) = F(x) - F(x-1) = 0.5^{x+1} I_{\{0,1,2,...\}}(x)$ is the corresponding pdf.

Problem 6

a) $C =$ the chip is from assembly line "II", $P(C) = 0.25$

$D =$ the chip is defective, $P(D|\mathrm{C}) = 0.02$, $P(D|\bar{C}) = 0.04$

$$P(C|D) = \frac{P(D|C)*P(C)}{P(D|C)*P(C) + P(B|\bar{C})*P(\bar{C})} = \frac{0.005}{0.035} = \frac{1}{7} = 0.1429$$

b) $P(\bar{A}|B) = 1 - P(A|B) = 0.01$

c) $P(B|A) = \dfrac{P(A|B)P(B)}{P(A|B)P(B) + P(A|\bar{B})P(\bar{B})} = \dfrac{0.99*0.04}{0.99*0.04 + 0.01*0.96} = \dfrac{0.0396}{0.0492} = 0.8049$

d) $P(B|A) = 0.99 = \dfrac{P(A|B)P(B)}{P(A|B)P(B) + P(A|\bar{B})P(\bar{B})} = \dfrac{0.04r}{0.04r + (1-r)0.96}$

$$\Rightarrow 0.04r = 0.0396r + 0.9504 - 0.9504r$$

$$\Rightarrow r = \frac{0.9504}{0.9508} = 0.9996$$