EconS 510

Takehome Quiz #1 Answer Key

Due Thursday, September 11, 2014

Problem 1

a) $i.\ P(A) \geq 0$ for $\forall A \subset S$.

$ii.\ P(S) = (1+2+3+4+5)/15 = 1.$

$iii.\ P\left(\bigcup_{i \in I} A_i\right) = \sum_{i \in I} P(A_i)$ when A_i are disjoint events contained in *S*.

Therefore, *P* is a probability set function.

b) $i.\ P(A) \geq 0$ for $\forall A \subset S$.

$ii.\ P(S) = -\frac{1}{2}e^{-2x}\Big|_0^{\infty} = \frac{1}{2}$

Therefore, *P* is <u>not</u> a probability set function.

c) $i.\ P(A) \geq 0$ for $\forall A \subset S$.

$ii.\ P(S) = \frac{N(S)}{N(S)} = 1$

$iii.\ P\left(\bigcup_{i \in I} A_i\right) = \sum_{i \in I} P(A_i)$ when A_i are disjoint events contained in *S*.

Therefore, *P* is a probability set function.

d) $i.\ P(A) \geq 0$ for $\forall A \subset S$.

$ii.\ P(S) = \ln x\Big|_1^e = 1$

$iii.\ P\left(\bigcup_{i \in I} A_i\right) = \sum_{i \in I} P(A_i)$ when A_i are disjoint events contained in *S*.

Therefore, *P* is a probability set function.

Problem 2

a) $S = \{(x, y) : x, y \in \{0,1,2,3,4\}\}$

b) $i.\ P(A) \geq 0$ for $\forall A \subset S$.

$ii.\ P(S) = \sum_{x\in\{0,1,2,3,4\}} \sum_{y\in\{0,1,2,3,4\}} P(x, y) = 1$

$iii.\ P\left(\bigcup_{i\in I} A_i\right) = \sum_{i\in I} P(A_i)$ when A_i are disjoint events contained in S.

Therefore, the information provided can be used to define a probability set function.

c) $P(x > 2) = 0.02 + 0.01 + 0.05 + 0.05 + 0.05 + 0.02 + 0.01 + 0.05 + 0.10 + 0.0\,5 = 0.41$

$P(y > 2) = 0.02 + 0.01 + 0.05 + 0.05 + 0.10 + 0.02 + 0.01 + 0.02 + 0.05 + 0.05 = 0.38$

d) $P(y > 2 | x > 2) = \dfrac{P(x > 2, y > 2)}{P(x > 2)} = \dfrac{0.25}{0.41} = 0.61$

e) $P(x > 2, y > 2) = 0.25$

f) $P(x = 0, y = 0) = 0.1$

$P(y = 0 | x = 0) = \dfrac{P(x = 0, y = 0)}{P(x = 0)} = \dfrac{0.1}{0.1 + 0.05 + 0.02 + 0.02 + 0.02} = 0.48$

Problem 3

a) $P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.6 + 0.8 - 1 = 0.4$

b) $P(C) = 1 - P(B) = 1 - 0.8 = 0.2$

c) No. $P(D) < P(C) = 0.2$, since $D \subset C$.

d) $P(E) = P(A) - P([0,2]) = 0.5$

e) $P([0,3) \cup (4,5]) = 1 - P([3,4]) = 1 - P(A \cap B) = 0.6$

Problem 4

a) $P(A_1 \cap A_2) = P(A_1) + P(A_2) - P(A_1 \cup A_2) = 0.55 + 0.6 - 0.82 = 0.33$

$P(A_1) * P(A_2) = 0.55 * 0.6 = 0.33 = P(A_1 \cap A_2)$

$P(A_1 \cap A_3) = P(A_1) + P(A_3) - P(A_1 \cup A_3) = 0.55 + 0.45 - 0.7525 = 0.2475$

$P(A_1) * P(A_3) = 0.55 * 0.45 = 0.2475 = P(A_1 \cap A_3)$

$P(A_2 \cap A_3) = P(A_2) + P(A_3) - P(A_2 \cup A_3) = 0.6 + 0.45 - 0.78 = 0.27$

$P(A_2) * P(A_3) = 0.6 * 0.45 = 0.27 = P(A_2 \cap A_3)$

Therefore, the three events are pairwise independent.

b) $P(A_1 \cap A_2 \cap A_3) = P(A_2 \cap A_3 | A_1) * P(A_1) = 0.2 * 0.55 = 0.11$

$P(A_1) * P(A_2) * P(A_3) = 0.55 * 0.6 * 0.45 = 0.1485 \neq 0.11$

Therefore, the three events are <u>not</u> jointly independent.

c) $P(A_1 \cap A_2 | A_3) = \dfrac{P(A_1 \cap A_2 \cap A_3)}{P(A_3)} = \dfrac{0.11}{0.45} = 0.244 \neq P(A_1 \cap A_2) = 0.33$

d)

$$\begin{aligned} P(A_1 \cup A_2 \cup A_3) &= P(A_1) + P(A_2) + P(A_3) - P(A_1 \cap A_2) - P(A_1 \cap A_3) - P(A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_3) \\ &= 0.55 + 0.6 + 0.45 - 0.33 - 0.2475 - 0.27 + 0.11 = 0.8625 \end{aligned}$$

Problem 5

a) $i.\ P(A) \geq 0$ for $\forall A \subset S$.

$ii.\ P(S) = (1 + 2 + 3 + 4 + 5 + 6 + 5 + 4 + 3 + 2 + 1) / 36 = 1.$

$iii.\ P\left(\bigcup_{i \in I} A_i\right) = \sum_{i \in I} P(A_i)$ when A_i are disjoint events contained in S.

Therefore, P is a probability set function.

b) $P(A) = (1 + 2 + 3 + 4 + 5) / 36 = \dfrac{5}{12}$

c) $P(B)=(5+4+3+2+1)/36=\frac{5}{12}$

d) $P(A\cap B)=P(\varnothing)=0\neq P(A)*P(B)$. Therefore, they are <u>not</u> independent events.

e) Yes, the probability space applies to the outcome of this experiment.

The probabilities of the 36 possible combinations of outcomes are assumed to be equal.

Problem 6

a) $S=\{HHH,HHT,HTH,HTT,THH,THT,TTH,TTT\}$

b) $A=\{HHH,HHT,HTH,HTT,THH,THT,TTH\}$

$P(A)=N(A)/N(S)=7/8=0.875$

c) $B=\{HTT,THT,TTH,TTT\}$

$P(B)=N(B)/N(S)=4/8=0.5$

d) $A\cap B=\{HTT,THT,TTH\}$

$P(A\cap B)=N(A\cap B)/N(S)=3/8=0.375$

e) *A* and *C* are disjoint events. *B* and *C* are not disjoint events. This is because,

$C=\{TTT\};A\cap C=\varnothing;B\cap C=\{TTT\}\neq\varnothing.$

f) $A\cup C=S\Rightarrow P(A\cup C)=P(S)=1$

$B\cup C=B\Rightarrow P(B\cup C)=P(B)=0.5$

$A\cup B=S\Rightarrow P(A\cup B)=P(S)=1$

Problem 7

a) $i.\ P(A)\geq 0$ for $\forall i$

$ii.\ P(S)=\sum_{i\in\{1,2,3,4\}}P(A_i)=1$ is possible when the four events are disjoint.

$iii.\ P\left(\bigcup_{i\in\{1,2,3,4\}} A_i\right) = \sum_{i\in\{1,2,3,4\}} P(A_i)$ is possible when A_i are disjoint events.

Therefore, the assignment of probabilities is possible.

b) $i.\ P(A) \geq 0$ for $\forall i$

$ii.\ P(S) = 1$ is possible.

$iii.\ P\left(\bigcup_{i\in I} B_i\right) = \sum_{i\in I} P(B_i)$ is possible when A_i are disjoint events.

Eg. Let $P(A_2) = P(A_3) = P(A_4) = 1/3$; A_2, A_3, A_4 are disjoint events;

and $A_1 \subset A_2$, which means $A_1, A_2 - A_1, A_3, A_4$ are disjoint events.

Therefore, the assignment of probabilities is possible.

c) $P(A_1 \cup A_2) = 0.7 + 0.6 - 0.1 = 1.2 \Rightarrow P(S) \neq 1$

Therefore, the assignment of probabilities is <u>not</u> possible.

d) $A_1 \subset A_1 \cup A_2 \Rightarrow P(A_1) \leq P(A_1 \cup A_2)$ and $A_2 \subset A_1 \cup A_2 \Rightarrow P(A_2) \leq P(A_1 \cup A_2)$

Since above inequalities are not satisfied, the assignment of probabilities is <u>not</u> possible.

e) $A_1 \subset A_2 \subset A_3 \subset A_4 \Rightarrow P(A_1) \leq P(A_2) \leq P(A_3) \leq P(A_4)$

Since above inequalities are not satisfied, the assignment of probabilities is <u>not</u> possible.

f) $P(A_1 \cap A_2) = P(A_1) + P(A_2) - P(A_1 \cup A_2) = 0.4 + 0.3 - 0.5 = 0.2 \neq 0 = P(\varnothing)$

Therefore, the assignment of probabilities is <u>not</u> possible.