EconS 510 – STAT 511 MID-TERM EXAM
October 19, 2011

(30 pts) **I.** **<u>True-False</u>**: For each question below, indicate whether the statement is true or false. If you indicate ***false***, provide a brief justification for why the statement is false. If you answer ***true***, you do ***not*** need to provide a justification.

T F **1.** Let $P(A) = .5$, and $P(B) = .4$. It is possible that $A \subset B$.

T F 2. Let $P(A) = .75$, $P(B) = .2$, and $P(A \mid B) = .15$. Then events A and B are independent events.

T F 3. The set function $P(A) = \sum_{x \in A} .25(.75)^{x-1}$ can serve as a *probability* set function on the event space $\Upsilon = \{A : A \subset S\}$ with sample space $S = \{1, 2, 3, ...\}$.

T F **4.** The random variable X is such that $E\left(\left(X - E(X)\right)^3\right) = 25$. Then the mean and variance of the random variable exists, and the probability density function of the random variable is not symmetric about its mean.

T F 5. The following is a valid probability density function for some random variable X:

$$f(x) = 5x^3 I_{(0,1)}(x) .$$

T F **6.** The moment generating function for a random variable X is given by $M_X(t) = \frac{e^t(t-1)+1}{.5t^2}$ *for* $t \neq 0$. Then the probability density function for the random variable is given by $f(x) = 2xI_{[0,1]}(x)$. (Hint: $\int xe^{xt}dx = \frac{e^{xt}(xt-1)}{t^2}$).

T F 7. The random variables X and Y are such that $\text{var}(X) = 9$, $\text{var}(Y) = 16$ and $\text{cov}(X,Y) = -6$. It follows that the correlation between X and Y is $\rho_{XY} = \frac{-6}{(9)(16)} = -.04166$, so that only 4.166% of the variation in Y is represented by the best linear predictor in terms of X.

T F 8. The regression function for Y on X is given by

$$E(Y \mid x) = 2x^2 - x + 1$$

and the probability density function of X if given by

$$f(x) = 3x^2 I_{[0,1]}(x) .$$

Then $E(X) = .75$ and $E(Y) = 2(E(X))^2 - E(X) + 1 = 1.375$.

T F 9. The joint probability density function of the pair of random variables (X,Y) is given by

$$f(x,y)=1.5y^2e^{-.5x}I_{[0,\infty)}(x)I_{[0,1]}(y)$$

Then the random variables are independent.

T F 10. The cumulative distribution function of the random variable X is given by

$$F(b)=\left\{\begin{array}{c}1\\ \dfrac{b^2+5b}{150}\\ 0\end{array}\right\} \text{ if } b\left\{\begin{array}{c}>10\\ \in[0,10]\\ <0\end{array}\right\}.$$

Then the probability density function for X is given by

$$f(x)=\frac{2x^3+15x^2}{900}I_{[0,10]}(x).$$

(**35 pts. ea**) Answer any ***TWO*** of the next three questions. Do ***not*** answer all three questions. Only the first two answers that you provide will be graded. **SHOW YOUR WORK**, being sure to justify all of the answers you provide.

II. Your company manufactures a product that depends on the quantities of labor, l, and capital, k, used and the production function is stochastic, defined as follows:

$$Y = 10l^{.4}k^{.1}e^{V}, \text{ where } V \sim f(v) = \frac{3}{16}v^2 I_{[-2,2]}(v).$$

a) If 9 units each of capital and labor are applied, what is the expected level of production that will occur? (HINT: $\int v^2 e^v dv = e^v\left(v^2 - 2v + 2\right)$).

b) At the same levels of capital and labor applied in a), what is the MEDIAN level of production that will occur?

c) What is the variance of production, σ_Y^2, given the levels of capital and labor applied in a)? (HINT: $\int v^2 e^{2v} dv = .25e^{2v}\left(2v^2 - 2v + 1\right)$).

d) For given values of capital and labor, are Y and $\varepsilon = e^V$ a pair of *degenerate* random variables? What is the correlation between Y and ε?

III. The daily price of ethanol, in dollars per gallon, and quantity sold, in 1000's of gallons, in a large east coast market in the summer months is given by a pair of random variables (P,Q) that have the following joint moment generating function:

$$M_{P,Q}(t_P,t_Q) = \exp\left(3.50t_P + 1000t_Q + .02t_P^2 + 1250t_Q^2 - 5t_Pt_Q\right).$$

a) What is the expected daily quantity of ethanol sold?

b) What is the expected daily total sales of ethanol, in dollars?

c) If ethanol costs \$2.75/gallon to produce, what is the expected daily return above production costs?

d) What is the covariance and the correlation between price and quantity? Does this make economic sense?

IV. The AJAX Computer Components Company assembles computer disk drives at a large manufacturing facility in Seattle. It has outsourced the manufacture of the wiring harness used in making the drives to three smaller companies that together provide the materials to AJAX for final assembly of the drives. The plants are located in Portland, Dallas, and Mexico City. The plants produce 15, 25, and 60 percent of the company's wiring harness needs, respectively. The harnesses carry the labels "PORT," "DAL," or "MEX" attached to the underside of the harnesses in order to identify the plant in which a wiring harness was manufactured.

Regarding quality control, the manufacturing defect rates of harnesses manufactured in the three plants are known to be .01, .03, and .02 for the Portland, Dallas, and Mexico City plants, respectively. In the process of manufacturing and testing, a harness is found to have a serious defect and you want the quality control team to follow up with the company that manufactured the harness to correct the potentially serious problem in the manufacture of the harness. However, the company label is missing. Answer the following questions.

a) Which of the three smaller companies is the *most* probable company to have manufactured the defective wiring harness?

b) Which of the three smaller companies is the *least* probable company to have manufactured the defective wiring harness?

c) What is the probability that a wiring harness installed in disk drives manufactured by AJAX will be a defective wiring harness?

d) Given that a wiring harness installed in a disk drive is *not* defective, what is the probability that the harness was manufactured in Mexico City?