EconS 510 – Takehome Quiz # 6

1. Scott Willard, a famous weatherman on national TV, states that the temperature on a typical late fall day in the Upper Midwest, measured in terms of both the Celsius and Fahrenheit scales, can be represented as the outcome of the bivariate random variable (C, F) such that

$$E\begin{bmatrix} C \\ F \end{bmatrix} = \begin{bmatrix} 5 \\ 41 \end{bmatrix} \quad \text{and} \quad Cov(C,F) = \begin{bmatrix} 25 & 45 \\ 45 & 81 \end{bmatrix}$$

a. What is the correlation between C and F?

b. To what extent is there a linear relationship between C and F? Define the appropriate linear relationship if it exists.

c. Given that the temperature is 45 degrees Fahrenheit, can you determine the temperature measured in Celsius *with probability 1*? If so, do it.

d. Is (C, F) a degenerate bivariate random variable? Is this a realistic result? Why or why not?

2. The annual rate of return per dollar invested for two different investment instruments is the outcome of a bivariate random variable (X_1, X_2) with the following mean vector and covariance matrix:

$$E(X) = \begin{bmatrix} .04 \\ .11 \end{bmatrix} \text{ and } Cov(X) = \begin{bmatrix} .0001 & -.0004 \\ -.0004 & .0025 \end{bmatrix}$$

a. Find the correlation matrix of (X_1 ,X_2). Do the outcomes of X_1 and X_2 satisfy a linear relationship $x_1 = \alpha_1 + \alpha_2 x_2$?

b. If an investor wishes to invest $1000 in a way that maximizes her expected dollar return on the investment, how should she distribute her investment dollars between the two projects? What is the variance of dollar return on this investment portfolio?

c. Suppose the investor wants to minimize the variance of her dollar return. How should she distribute the $1000? What is the expected dollar return on this investment portfolio?

d. Suppose the investor's utility function with respect to her investment portfolio is $U(M) = 5M^b$, where M is the dollar return on her investment of $1000. The investor's objective is to maximize the expected value of her utility. If b = 1, define the optimal investment portfolio.

e. Repeat d), but let b = 2.

f. Interpret the investment behavior differences in d) and e) in terms of investor attitude toward risk.

3. The daily quantity demanded of three flavors of fruit juices that your company produces can be viewed, during the summer months, as a trivariate random variable having the following mean vector and covariance matrix:

$$E\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix} = \begin{bmatrix} 100 \\ 250 \\ 50 \end{bmatrix} \qquad \text{and} \qquad Cov\left(Q_1, Q_2, Q_3\right) = \begin{bmatrix} 36 & 25 & 10 \\ 25 & 100 & 5 \\ 10 & 5 & 25 \end{bmatrix}$$

where the quantities are measure in 100's of gallons.

a. What is the expected total quantity of fruit juice that will be sold on a given day?

b. What is the standard deviation associated with the total quantity of fruit juice sold on a given day?

c. If the fruit juices each sell for $2.50 per gallon, what is the expected value and standard deviation of daily total revenue realized on any given day from the sale of the fruit juices?

d. Define an interval event for daily total revenue, centered on the mean value of revenue that has at least a .90 probability of occurring.

e. Define the correlation matrix between the three quantities demanded. Are the quantities demanded strongly linearly related ? Explain.

f. Based on the moment information given above, if it is know that the quantity demanded of one of the fruit juices is substantially decreased from the day before, what would you anticipate will happen to the quantity demanded that will be observed for the other two fruit juices? Explain.

4. The yield, in bushels per acre, of a certain type of high-yield feed grain in the Midwest grown under specific cultivation practices can be represented as the outcome of the random variable Y defined by the following stochastic Quadratic Production Function:

$$Y = \left(50 + 20x - 2x^2\right)\exp\left(\varepsilon\right) \text{ , where } x \in \left[0,10\right]$$

where x is the per acre units of fertilizer utilized in production, and ε is a random variable with probability density function given by

$$f\left(\varepsilon\right) = 2e^{-2\varepsilon} I_{(0,\infty)}\left(\varepsilon\right) .$$

The price received for the feed grain is \$6/bushel, and fertilizer price is \$10/acre per unit applied.

a. Define the *expected* yield per acre function (i.e., as a function of the level of fertilizer). What is the expected level of *profit*, net of fertilizer costs, per acre if fertilizer is applied at the rate of 5 units per acre?

b. What is the standard deviation of yield per acre as a function of the level of fertilizer? What is the standard deviation of profit, net of fertilizer costs, per acre if fertilizer is applied at the rate of 5 units per acre?

c. Define the level of fertilizer that should be applied in order to maximize the level of *expected* profits, net of fertilizer costs. Define the level of fertilizer that should be applied in order to minimize the standard deviation of profits, net of fertilizer costs. Are maximizing expected profits and minimizing variability of profits compatible objectives?

d. Define the median level of profit as a function of the fertilizer level. What is the median level of profit, net of fertilizer cost, if fertilizer is applied at the level that maximizes expected profit, net of fertilizer cost?