EconS 510 – Takehome Quiz # 4

1. The weekly number of luxury and compact cars sold by "Honest" Abe Smith at the Auto Mart, a local car dealership, can be represented as the outcome of a bivariate random variable (X,Y), where X is the number of luxury cars and Y is the number of compact cars sold. The non-zero values (i.e., the support) of the joint density function for this bivariate random variable is defined as follows:

Y-values

		0	**1**	**2**	**3**
X-values	**0**	.15	.10	.05	.05
	1	.10	.10	.05	.04
	2	.05	.05	.05	.04
	3	.05	.04	.04	.04

Abe receives a base salary of $400/week from the dealership, and also receives a commission of $100 for every compact car sold and $200 for every luxury car sold.

a. What is the expected value of the weekly commission that Abe obtains from selling cars? What is the expected value of his total pay received for selling cars?

b. What is the expected value of his commission from selling compact cars? What is the expected value of his commission from selling luxury cars?

c. *Given* that Abe sells 2 compact cars, what is the expected value of his commission from selling luxury cars?

d. If 25% of Abe's total pay goes to federal and state taxes, what is the expected value of his pay after taxes?

2. The yield, in bushels per acre, of a certain type of feed grain in the midwest can be represented as the outcome of the random variable Y defined by

$$Y = 3x_L^{.6}x_K^{.3}e^U$$

where X_L and X_K are the per acre units of labor and capital utilized in production, and U is a random variable with probability density function given by

$$f(u) = 3e^{-3u}I_{[0,\infty)}(u)$$

The price received for the feed grain is $4/bushel , labor price per unit is $10, and capital price per unit is $15.

a. What is the expected yield per acre?

b. What is the expected level of profit per acre if labor and capital are each applied at the rate of 10 units per acre?

c. Define the levels of input usage that maximize expected profit. What is the expected maximum level of profit?

d. The acreage can be irrigated at a cost of \$125 per acre, in which case the yield per acre is defined by

$$Y = 5x_L^{.6}x_K^{.3}e^u$$

If the producer wishes to maximize expected profit, should she irrigate?

3. The daily price/gallon and quantity sold (measured in *millions* of gallons) of unleaded gasoline on the wholesale spot market of a major commodity exchange is the outcome of a bivariate random variable (P,Q) having the joint probability density function

$$f(p,q) = 2pe^{-pq} I_{[3.50,4.00]}(p)\, I_{[0,\infty)}(q)$$

a. Define the regression curve of q on p.

b. Graph the regression curve that you have defined in a.)

c. What is the expected value of the quantity of gasoline sold, given that price is equal to \$3.75 per gallon?

d. What is the expected value of total dollar sales of gasoline on a given day?

4. Stanley Statistics, an infamous statistician, wants you to enter a friendly wager with him. For \$2,500, he will let you play the following game. He will continue to toss a fair coin until the first head appears. Letting x represent the number of times the coin was tossed to get the first head, Stanley will then pay you $\$2^x$.

a. Define a probability space for the experiment of observing how many times a coin must be tossed in order to observe the first head.

b. What is the expected payment that you will receive if you play the game?

c. Do you want to play the game? Why or why not?

5. The manager of a bakery is considering how many chocolate cakes to bake on any given day. The manager knows that the number of chocolate cakes that will be demanded by customers on any given day is a random variable whose probability density is given by

$$f(x) = \frac{x+2}{27} I_{\{0,1,2,3,4,5\}}(x)$$

The bakery makes a profit of \$1.50 on each cake that is sold. If a cake is not sold on a given day, the cake is thrown away (because of lack of freshness), and the bakery loses \$1. If the manager

wants to maximize expected daily profit from the sale of chocolate cakes, how many cakes should be baked?

6. A fruit processing firm is introducing a new fruit drink, "Peach Passion," into the domestic market. The firm faces uncertain output prices in the initial marketing period and intends to make a short run decision by choosing the level of production that maximize the expected value of utility:

$$E\left[u\left(\Pi\right)\right] = E\left(\Pi\right) - \alpha \operatorname{var}\left(\Pi\right)$$

Profit is defined by $\Pi = Pq - C\left(q\right)$, p is the price received for a unit of Peach Passion, u is utility, the cost function is defined by $C(q) = .5q^{1.5}$, α is a risk aversion parameter, and the PDF of the uncertain random output price is given by

$$f(p) = .048(5p - p^2)\, I_{[0,5]}(p)$$

a. If the firm were risk neutral, i.e., $\alpha = 0$, find the level of production that maximizes expected utility.

b. Now consider the case where the firm is risk averse, i.e., $\alpha > 0$. Graph the relationship between the optimal level of output and the level of risk aversion (i.e., the level of α). How large does α have to be for optimal $q = 1$?

c. Assume that $\alpha = 1$. Suppose that the Middle Slobovian Dept. of Agriculture were to *guarantee* a price to the firm. What *guaranteed* price would induce the firm to produce the same level of output as in the case where price was uncertain?