EconS 510 – Takehome Quiz # 3

1. Intelligent Electronics, Inc. manufactures monochrome liquid crystal display (LCD) notebook computer screens. The number of hours an LCD screen functions until failure is represented by the outcome of a random variable X having range R(X) = $[0,\infty)$ and probability density function

$$f(x) = .01e^{-x/100} I_{[0,\infty)}(x)$$

The value of x is measured in 1000's of hours. The company has a one year warranty on its LCD screen during which time the LCD screen will be replaced free of charge if it fails to function.

a. Assuming that the LCD screen is left on constantly, what is the probability that the firm will have to perform warranty service on an LCD screen within the one year warranty period?

b. What is the probability that the screen functions for at least 100,000 hours?

c. What is the probability that the screen functions for at least 100,000 hours *given* that the screen has already functioned for 50,000 hours?

a. What is the probability that the screen functions *for an additional* 100,000 hours *given* that the screen has already functioned for 50,000 hours?

2. People Power, Inc. is a firm that specializes in providing temporary help to various businesses. Job applicants are administered an aptitude test that evaluates mathematics, writing, and manual dexterity skills. Analyzing thousands of job applicants who have come in and taken the test, it was found that the scores on the three tests could be viewed as outcomes of random variables with the joint density function below.

$$f(x_1, x_2, x_3) = \left(\frac{8}{3}\right)\left[x_1 + 2x_2\right] x_3^3 \prod_{i=1}^{3} I_{[0,1]}(x_i)$$

The tests are graded on a 0-1 scale, with 0 the lowest score and 1 the highest, and the $x_i\text{'s},\ i = 1,2,3$ refer, in order, to the mathematics, writing, and manual dexterity skills test outcomes):

a. A job opening has occurred for an office manager. People Power Inc. requires scores of > .75 on both the mathematics and writing tests for a job applicant to be offered the position. Define the marginal density function for the mathematics and writing scores. Use it to define a probability space in which probability questions concerning events for the mathematics and writing scores can be answered. What is the probability that a job applicant who has just entered the office to take the test will qualify for the office manager position?

b. A job opening has occurred for a warehouse worker. People power Inc. requires a score of >.80 on the manual dexterity test for a job applicant to be offered the position. Define the marginal density function for the dexterity score. Use it to define a probability space in which probability questions concerning events for the dexterity score can be answered. What is the probability that a job applicant who has just entered the office to take the test will qualify for the warehouse worker position?

c. Find the conditional density of the writing test score, given that the job applicant achieves a score of >.75 on the mathematics test. Given that the job applicant scores >.75 on the mathematics test, what is the probability that she scores >.75 on the writing test?

3. The weekly average price (in dollars/gallon) and total quantity sold (measured in 1000's of gallons) of unleaded gasoline sold by the Colfax ClamShell gas station can be viewed as the outcome of the bivariate random variable (P,Q) having the joint density function:

$$2pe^{-pq}I_{[3.50,4.00]}(p)I_{(0,\infty)}(q)$$

a. What is the probability that total dollar sales in a week will be less than $2000?

b. Find the marginal density of price. What is the probability that price will exceed $3.75 per gallon?

c. Find the conditional density of quantity, given price = $3.75. What is the probability that > 10,000 gallons will be sold in a given week?

d. Find the conditional density of quantity, given price = $4.00. What is the probability that > 10,000 gallons will be sold in a given week?
Compare this result to your answer in c. Does this make economic sense? Explain.

4. The joint probability density of the bivariate random variable (X,Y) is given by

$$f(x,y) = 6x^2yI_{[0,1]}(x)I_{[0,1]}(y)$$

a. Find the joint cumulative distribution function of (X,Y). Use the CDF to find the probability that $x \leq .5$ and $y \leq .75$.

b. Find the marginal cumulative distribution function of X. What is the probability that $x \leq .5$?

c. Find the marginal density of X from the marginal cumulative distribution function of X.

5. The Imperial Electric Co. makes high quality portable compact disc players for sale in international and domestic markets. The company operates two plants in the United States, where one plant is located in the Pacific Northwest and one is located in the South. At either plant, once a disc player is assembled, it is subjected to a stringent quality control inspection, at which time the disc player is either approved for shipment, or else the disc player is sent back for adjustment before it is shipped. On any given day, the proportion of the units produced at each plant that are shipped without adjustment, and the total production of disc players at the company's two plants, is an outcome of a trivariate random variable, with the following joint probability density function:

$$f(x,y,z) = \left(\frac{3}{7}\right)\left[x + y^2 + 2z\right] e^{-x} I_{[0,\infty)}(x) I_{[0,1]}(y) I_{[0,1]}(z)$$

where x = total production of disc players at the two plants, measured in thousands of units,
y = proportion of the units produced at the Pacific Northwest plant that are shipped without adjustment, and
z = proportion of the units produced in the Southern plant that are shipped without adjustment.

a. In this application, the use of a ***continuous*** trivariate random variable to represent proportions and total production values must be viewed as only an ***approximation*** to the underlying real-world situation. Why? In the remaining parts of the question, assume the approximation is acceptably accurate, and use the approximation to answer questions where appropriate.

b. What is the probability that less than 50% of the disc players produced in each plant will be shipped without adjustment, and that production will be less than one thousand units on a given day?

c. Derive the marginal probability density function for the total production of disc players at the two plants. What is the probability that less than one thousand units will be produced on a given day?

d. Derive the marginal probability density function for the bivariate random variable (Y, Z). What is the probability that less than 50% of the disc players will be shipped without adjustment from each plant?

e. Derive the conditional density function for X, ***given*** that 50% of the disc players are shipped from the Pacific Northwest plant without adjustment. What is the probability that $\leq$ 1000 disc players will be produced by the Imperial Electric Co. on a day for which 50% of the disc players are shipped from the Pacific Northwest plant without adjustment?

6. ACE Rentals, a small car-rental company, rents three types of cars: compacts, mid-size sedans, and large luxury cars. Let (x_1, x_2, x_3) represent the number of compacts, mid-size sedans, and luxury cars that ACE rents per day, respectively. Let the sample space for the possible outcomes of (X_1, X_2, X_3) be given by

$$S = \{(x_1, x_2, x_3) : x_1, x_2, \text{and } x_3 \in \{0, 1, 2, 3\}\}$$

where ACE has an inventory of 9 cars, evenly distributed among the 3 types of cars.

The discrete probability density function associated with (X_1, X_2, X_3) is given by

$$f(x_1, x_2, x_3) = \left(\frac{1}{576}\right)(x_1 + 2x_2 + 3x_3)\prod_{i=1}^{3} I_{\{0,1,2,3\}}(x_i)$$

The compact car rents for \$20/day, the mid-size sedan rents for \$30/day, and the luxury car rents for \$50/day.

a. Derive the marginal density function for X_3. What is the probability that all 3 luxury cars are rented on a given day?

b. Derive the marginal density function for (X_1, X_2). What is the probability of more than 1 compact and more than 1 mid-size sedan being rented on a given day?

c. Derive the conditional density function for X_1, given $x_2 = 2$.

What is the probability of renting no more than 1 compact given that 2 or more mid-size sedans are rented?

e. Derive the conditional density function for (X_1, X_2), given that $x_3 = 0$. What is the probability of renting more than 1 compact and more than 1 mid-size sedan given that no luxury cars are rented?

f. If it costs \$150/day to operate ACE Rentals, define a random variable that represents the daily profit made by the company. Define an appropriate density function for this random variable. What is the probability that ACE Rentals makes a positive daily profit on a given day?