EconS/Fin 510
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Takehome Quiz #2

1. Which of the following are valid probability density functions? Justify your answer.
   a. $f(x) = (.4)^x (.6)^{1-x} I_{\{0,1\}}(x)$
   b. $f(x) = 3x^2 I_{(0,1)}(x)$
   c. $f(x) = (x^2 - 2x + 1) I_{[0,1]}(x)$
   d. $f(x) = .5^x I_{\{1,2,3,...\}}(x)$
   e. $f(x) = e^{-x} I_{(0,\infty)}(x)$

2. Graph each of the functions in 1) that are *probability density* functions.

3. The daily quantity demanded of unleaded gasoline in a regional market can be represented as the outcome of a random variable defined by

$$Q = 250 - 3p + \varepsilon$$

where ε is a random variable having the probability density function

$$f(\varepsilon) = (.25 + .0625\varepsilon) I_{[-4,0]}(\varepsilon) + (.25 - .0625\varepsilon) I_{(0,4]}(\varepsilon).$$

Quantity demanded, Q, is measured in 1000's of gallons (i.e., Q = 1 indicates 1000 gallons), and price, p, is measured in dollars.

a. What is the probability that quantity demanded will be greater than 240,000 gallons if price is set equal to \$4? What if price is set equal to \$3?

b. If the variable cost of supplying Q-amount of unleaded gasoline is given by $C(Q) = 2.75Q$, define a random variable that can be used to represent the daily profit above variable cost from the sale of unleaded gasoline. If price is set equal to \$3, what is the probability that daily profit above variable cost will exceed \$60,000?

c. Setting ε to the outcome that is assigned the highest density weighting by its probability density function, what would be the profit-maximizing choice of the price of unleaded gasoline? What would be the associated level of maximized profit realized? If price were actually set to the profit-maximizing choice you just calculated, what is the probability that daily profit actually realized on any given day would equal the associated maximized profit level you just calculated?

d. Regarding the demand function identified above, consider the rate of change in quantity demanded with respect to price, $\frac{\partial Q}{\partial p}$, and the elasticity of quantity demanded with respect to price, $\frac{\partial Q}{\partial p}\frac{p}{Q}$. Which if either of these two concepts is a random variable? Why? Given that p is set to \$3, calculate the values of these measures (*if you can* …).

4. Ace Digital Receivers, Inc, a small electronics store, sells two different types of television cable box receivers. One is a standard definition television (SDTV) receiver box, and the other is a high definition television (HDTV) receiver box.

The STDV box sells for \$150 and costs \$125 to build. The HDTV box costs \$300 and costs \$250 to build. The probability space $\{S, \Upsilon, P\}$ associated with the number of each type of the boxes sold on a given day is defined by

$$S = \{(x, y) : x \text{ and } y \in \{0,1,2,3,4,5\}\}, \Upsilon = \{A : A \subset S\}$$

and $$P(A) = \sum_{(x,y)\in A} \frac{14.0625}{x!y!(5-x)!(5-y)!}$$

where x is the number of STDV boxes and y is the number of HDTV boxes.

a. Define a random variable that represents the amount of profit realized in a day from the sale of these two types of television receivers. What is the range of this random variable?

b. Define the discrete probability density function associated with the amount of profit realized in a day.

c. What is the probability that the store makes at least \$200 in profit on a given day?

d. What is the probability that the store makes at least \$400 in profit on a given day?

e. What is the probability that the store makes \$375 in profit on a given day?

5. For each of the functions below, determine which are cumulative distribution functions (CDFs), and for those that *are* CDFs, determine the probability density functions that are associated with the CDFs.

a. $F(b) = \left(1 - e^{-.5b}\right) I_{(0,\infty)}(b)$

b. $F(b) = b^3 I_{[0,1]}(b)$

c. $F(b) = \left(1 - .5^{trunc(b)+1}\right) I_{[0,\infty)}(b)$

6.. The AJAX Microchip Company produces memory chips for personal computers. The company's entire production is generated from 2 assembly lines, labeled "I" and "II." Assembly line "I" uses more rapid assembly techniques and produces 75% of the company's output, while assembly line "II" produces the remaining 25% of the output. The probability that a memory chip produced on assembly line "I" is defective is .04 while the corresponding probability for assembly line"II" is .02.

a) A memory chip is chosen at random from a bin containing a day's production. Given that the chip is found to be defective, what is the probability that the chip was made on assembly line "II"?

The management of the AJAX Microchip Company is interested in increasing quality control at the plant, and is considering the purchase of a testing device that can determine when a memory chip is faulty. In particular, the specifications on the device are as follows:

$$P(A \mid B) = P(\bar{A} \mid \bar{B}) = .99$$

where A is the event that the testing device *indicates* that a memory chip is faulty, and B is the event that the memory chip *really is* faulty.

b. What is the probability that the testing device indicates that a memory chip is not faulty, given that the memory chip really is faulty?

c. If the testing device is applied to the memory chips produced by AJAX's assembly line "I", what is P(B|A), i.e., the probability that a chip really is faulty given that the testing device indicates the chip is faulty.

d. Suppose AJAX management wants P(B|A) to be .99. Is there a value of $r = P(A \mid B) = P(\bar{A} \mid \bar{B})$ that will ensure this testing accuracy if the test is applied to the chips produced on assembly line "I"? If so, what is it?