Problem Set 7. Game Theory

EconS 527

1. Suppose that Panasonic and Zenith are the only two firms that can produce a new type of 3D TV. The payoffs (in millions of dollars) from entering the product market are as follows: when both firms enter they both lose $40 million, if only one enters while the other does not, the entering firm receives $250 million while the non-entrant receives nothing and finally when both do not enter, they do not earn anything.
   a. Write the normal form representation of the game.
   b. Solve for all pure strategy Nash equilibrium(s) when both firms move simultaneously.
   c. Solve for the mixed strategy Nash equilibrium.
   d. If the US government gives a lump sum subsidy to Zenith of $50 million if it enters the market, how will it affect the pure strategy Nash equilibrium(s).

2. An open access fishery for king crab is available in open waters. There are n identical fishing boats. The total harvest revenue from all n boats is $Y = aX - bX^2$ where a and b are parameters while *X* is total inputs from all fishing boat such that $X = x_1 + x_2 + \cdots + x_n$. The revenue for an individual fishing boat is a proportion of total harvest revenue such that $y_i = Y\frac{x_i}{X}$ where x_i is the input by one fishing boat. Total cost by one boat is cx_i.
   a. Solve for the Nash equilibrium input level by one fishing boat. Given this individual level, solve for the aggregate level of input.
   b. Assume that there is a private owner of the entire fishery for king crab. Solve for their optimal level of input.
   c. Compare the level of inputs under Nash equilibrium and private ownership and given an intuitive explanation why the input levels are chosen as such.

3. There are two firms, an incumbent I and a new entrant E. In the first stage, the Entrant decides to enter or not. If it does not decide to enter, E receives nothing and I receives $2 million and the game ends. If E enters, both firms play a simultaneous move game where each has a choice of specializing in a large niche or small niche. If both firms choose large niche, they both lose $3 million and if they both choose small niche, they both lose $6 million. If one choose small and the other large, the firm choosing the large niche earns $1 million and the firm choosing a small niche loses $1 million.
   a. Draw the extensive form of the game.
   b. Determine all the Subgame perfect Nash equilibrium.

4. Two firms are in competition. They can both produce a large amount leading to a lower price and lower profits, i.e. $1 million each. Alternatively, they can reduce production to raise price and increase profit with each firm receiving $2 million. If one firm produces a lot but the other produces little, the firm producing a lot receives $3million and the firm producing a low output has 0 profit.
    a. Write the normal form representation of this game. Find the pure strategy Nash equilibrium.
    b. What is the Nash equilibrium result if the game is repeated for 10 time periods with a discount factor of δ.
    c. If the firms play this game repeatedly for an infinite period of time using a trigger strategy, derive the discount factor δ where the firms collude to sustain a profit $2million each in every time period.

Note: The problem set is due on November 25. See syllabus for penalty due to late submissions.