1. Planting tobacco necessitates the use of fertilizers. The use of fertilizers create runoff leading to groundwater pollution. Tobacco is the primary input in the production of cigarettes which can cause second-hand smoke. Is there too little, too much or the correct amount of tobacco produced from a socially optimal perspective? Explain your answer. Support your explanation by drawing a graph representing the market for tobacco. Correctly identify private and social marginal benefit and marginal cost curves along with the market equilibrium price and quantity versus the socially optimal equilibrium price and quantity. Also, identify any deadweight loss in the graph if any.

Too much tobacco is produced relative to the socially optimal level. In the tobacco market, there are two externalities: a negative consumption externality when using the tobacco is consumed and a negative production externality when growing tobacco. Since these two effects are not internalized in the market, production and pricing of the good are based only on the private marginal cost and marginal benefit curves which are not equal to the social marginal cost and marginal benefit curves. Damages from the externality are not considered so too much of the good is produced resulting in a deadweight loss equal to the area abc below:
2. Members of a village in the Philippines called Tubig have open access to a groundwater reservoir. As long as an individual is a resident of Tubig, the individual can extract water from the reservoir. All non-residents of Tubig are not allowed to tap into the groundwater reservoir. Given this scenario, is the equilibrium level of water extracted “too much” or “too little” or equal to the social optimum? Explain why this occurs. Support your answer by drawing the private and social marginal cost curves and the marginal benefit curve for extracted water. Identify the deadweight loss area if any exists.

a. Too much water is extracted. In this scenario, the groundwater reservoir is an open access resource for residents of Tubig which means it is rival and non-excludable. There are private costs to extraction, which residents internalize in their decision making process such as the cost of the well, the labor cost, the cost of materials, etc. But there is also an external cost which members do not internalize: the cost of the reducing the groundwater reservoir itself. This leads to a difference in private and social marginal cost and overextraction of water. Note: you can also use an externality story here where your extraction reduces the available reservoir stock for other members. The deadweight loss is shown in area abc.
3. Acme Laundry provides laundry service in a competitive market. The laundry firm has the following cost function \( C(q) = 10 + 10q + q^2 \) where \( q \) is the quantity of laundry services provided. The price faced by firm is \( p \). The government noticed that laundry services led to increased water pollution in the nearby river and decided to impose an output tax \( t \) for every quantity of laundry service provided.

a. Derive the expression for the optimal quantity of laundry services to maximize profit. Set up the Acme Laundry’s problem properly and derive and interpret the first order condition.

b. Assume that the price \( p = 50 \) and tax \( t = 4 \). How much is the optimal output and profit with and without the tax? Given your result, how will Acme Laundry respond in terms of laundry service provided as the government tax rises?

a.

The objective of firm is:

\[
\max_{q} pq - tq - (10 + 10q + q^2)
\]

The first order condition is,

\[
p - t - (10 + 2q) = 0
\]

Simplifying,

\[
p = t + (10 + 2q)
\]

This is interpreted as price is equal to the tax rate plus marginal cost of production.

Thus, \( \frac{p-t-10}{2} = q^* \).

b. With the tax, \( \frac{50-4-10}{2} = q^* = 18 \). So profit is \( \Pi = (50 - 4)18 - (10 + 10(18) + (18)^2) = 828 - 514 = 314 \)

Without tax, \( \frac{50-10}{2} = q^* = 20 \). So profit is \( \Pi = (50)20 - (10 + 10(20) + (20)^2) = 1000 - 610 = 390 \)

As tax increases, quantity of services provided is reduced.
4. Andres is planning to set up a pizza store. He is deciding how many workers (W) and ovens (O) to buy. Given his economics background, he estimated the production function for pizza to be \( Q = 10W^{0.5}O^{0.5} \) where Q is number of pizza boxes.

a. Set up the objective function and constraints faced by Diego if he wishes to minimize the total cost, \( C \), of producing \( Q^* \) amount of pizza boxes a day by choosing the number of workers to hire and ovens to purchase. Define \( P_w \) and \( P_o \) as the price of workers and ovens, respectively.

b. Re-write (a) as an unconstrained problem using the Lagrangian equation. Define \( \lambda \) as the Lagrange multiplier.

c. Write the first order conditions that solve the Lagrangian equation and use them to derive Diego’s input demand equations for workers and ovens.

d. Assume the target number of pizza boxes that need to be produced per day is \( Q^* = 60 \) while input prices are \( P_w = $10 \) and \( P_o = $90 \). How many workers and how many ovens should Diego purchase? What is the lowest cost possible to produce 60 pizza boxes?

\[
a. \min_{W,L} C = P_w W + P_o O \\
\text{s.t. } Q^* = 10W^{0.5}O^{0.5}
\]

\[
b. \quad \mathcal{L} = P_w W + P_o O + \lambda (Q^* - 10W^{0.5}O^{0.5})
\]

c. \[
\frac{\partial \mathcal{L}}{\partial W} = P_w - \lambda 5W^{-0.5}O^{0.5} = 0 \\
\frac{\partial \mathcal{L}}{\partial O} = P_o - \lambda 5W^{0.5}O^{-0.5} = 0 \\
\frac{\partial \mathcal{L}}{\partial \lambda} = Q^* - 10W^{0.5}O^{0.5} = 0
\]

Using the first two FOCs we have the equilibrium, \( \frac{5W^{-0.5}O^{0.5}}{5W^{0.5}O^{-0.5}} = \frac{P_w}{P_o} \). The negative value of the left hand side is the MRTS between W and O. Simplifying, we arrive at \( O = \frac{P_w}{P_o} W \). Substitute the value of B into the last FOC. We derive,

\[
Q^* - 10W^{0.5} \left( \frac{P_w}{P_o} W \right)^{0.5} = 0.
\]

Solving for \( W^* \) we obtain input demand equation for \( W \):
\[ W^* = \frac{Q^*}{10} \left( \frac{P_o}{P_W} \right)^{0.5} . \]

The input demand for O is derived by taking the demand for W and plugging it into \( O = \frac{P_W}{P_o} W \).

We arrive at the following expression,

\[ O^* = \frac{Q^*}{10} \left( \frac{P_W}{P_o} \right)^{0.5} . \]

d. The optimal values for W and O are derived by substituting in the values for the parameters:

\[ W^* = \frac{Q^*}{10} \left( \frac{P_o}{P_W} \right)^{0.5} = \frac{60}{10} \left( \frac{90}{10} \right)^{0.5} = 18. \]
\[ O^* = \frac{Q^*}{10} \left( \frac{P_W}{P_o} \right)^{0.5} = \frac{60}{10} \left( \frac{10}{90} \right)^{0.5} = 2. \]

\[ \mathcal{C} = 90 \times 2 + 10 \times 18 = 360 \]

*Deadline: September 6, 2019 – 1:10pm. See syllabus for penalty due to late submissions.*