Homework 6. Cost and Duality

EconS 527

1. Solve for the following conditional factor demand functions and the cost function. Also, derive the effect of output on total cost and the demand for input x.
   a. \( \min_{x,z} w_1 x + w_2 z \) s. t. \( y = x^{0.2} z^{0.4} \)
   b. \( \min_{x,z} w_1 x + w_2 z \) s. t. \( y = (x^p + z^p)^{1/p} \)

2. Solve for the production functions from the following cost functions. In your solution, make sure to derive the conditional factor demands. A factor of production, \( x_i \), is inferior if the conditional demand for that factor decreases as output increases. Are any inputs inferior? Prove it.
   a. \( c(w_1, w_2, y) = \alpha y^b (w_1^r + w_2^r)^{1/r} \)
   b. \( c(w_1, w_2, y) = \min \left[ \frac{w_1}{a}, \frac{w_2}{b} \right] y \)

3. A manager of a plant wishes to achieve an output level \( q \) by minimizing their cost of production. They are given a technology that results in the following production function: \( q = R^a (L + bK)^c \) where \( a > 0, b > 0 \) and \( c > 0 \) and \( a + c < 1 \). The input prices for R, L and K are \( v, w \) and \( r \), respectively.
   a. Solve for the conditional factor demand functions and the cost function of the manager.
   b. How does an increase in \( v \) affect demand for L and total cost of the firm?

4. A firm has two plants. The manager of the two plants estimate the cost function of plant 1 as \( c_1(y_1) = 3w_1^{0.5}w_2^{0.5}y_1^2 \) and the cost function of plant 2 is \( c_2(y_2) = w_1^{0.5}w_2^{0.5}y_2^2 \).
   a. What is the cost function of the entire firm if total aggregate output target is \( Y \)?
   b. Show how individual output in each plant responds if the aggregate output target decreases because of a recession in the country.

5. A beer company has the following production function: \( q = 1.52L^{0.4}K^{0.5} \). In the short run, capital, \( K \), is 100. The rental rate of capital is 8 and labor cost is 24.
   a. Find the fixed cost, variable cost and total cost equations.
   b. Find the average fixed cost, average variable cost, average total cost and marginal cost equations.
   c. What quantity will average cost reach a minimum?
   d. If a 400 lump sum tax is in place, will it affect your answer in (c)? If so, what is the new level?

Deadline: November 2, 2018, 2pm. See syllabus for penalty due to late submissions.