EconS 526

Problem Set 1. Unconstrained Optimization

Problem 1. For the following functions, find the critical point(s):

    (a) $y = 2x^3 - 0.5x^2 + 2$,
    (b) $y = 4x_1^2 - x_1 x_2 + x_2^2 - x_1^3$,
    (c) $y = 2x_1^2 + x_2^2 + 4x_3^2 - x_1 + 2x_3$.

Determine whether the critical points are local maximums, minimums or saddle points.

Problem 2. A publisher pays the author of a book a royalty of 15%. Demand for the book is $x = 200 - 5p$ and the production cost is $C = 10 + 2x + x^2$. Find the optimal sales from both the author's and the publisher's perspective. Prove that the level of sales that you find for each agent is a maximum.

Problem 3. A monopoly supplies its markets from two plants, with cost functions

$$C_1 = q_1^2, C_2 = 2q_2$$

where q_i is the output of the ith plant. The monopoly faces a linear demand curve

$$p = 70 - 2(q_1 + q_2).$$

Find the firm's profit maximizing output for each plant and the highest profit level that the monopolist can achieve. Does the profit maximizing output level for each plant satisfy the second order conditions for a maximum?

Problem 4. A monopoly seller is able to divide its overall market into two submarkets with the inverse demand functions,

$$p_1 = 100 - q_1.$$
$$p_2 = 120 - 2q_2.$$

It produces output at a single plant with the cost function, $C = 20(q_1 + q_2)$. There are no fixed costs and output in the two markets are indistinguishable in production. Solve for the optimal output and price levels in each submarket. Prove that at those levels, the monopolist is maximizing profit.