

# Multiple Signals in a Corporate Socially Responsible Equilibrium

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April 19, 2022

# Introduction

Corporate Social Responsibility (CSR), understood as companies integrating “social and environmental concerns in their business operations on a voluntary basis,” is generally promoted in both business and international trade; see ([European Commission, 2001](#); [U.S. Department of State, 2016](#)).

While firms face this trade-off between profits and social/environmental practices under complete information, this trade-off becomes even more involved when consumers cannot accurately observe firms’ responsible decisions (e.g., investment in abatement of recycling practices), which entails that observed signals are noisy or unreliable.

For this reason, policy makers have long debated how to induce businesses to act socially responsible in environments with noisy signals.

# Previous Literature / Research Question

Calveras and Ganuza, 2016 (CG) investigate signaling in a CSR framework, and assume a binary signaling mechanism where consumers only observe whether the firm invested in CSR or not.

While this type of signaling helps sustain separating equilibria that convey information to uninformed consumers, it is relatively restrictive.

## **Example:**

- When a news outlet reports about a firm investing in clean technologies, it provides different details about the dollar investment, the type of technology, or how it compares to other, more polluting, technologies.

## **Research Question:**

Theoretically, what does a CSR frame work with multiple signals look like, and how do equilibrium conditions change with respect to the exogenous parameters that govern the CSR equilibrium?

# Contribution / Preview of Results

We extend CG by introducing a setting with multiple signaling and provide examples of multiple signaling mechanisms that fit our generalization.

The model assumes that all details about CSR investment can be summarized by an index number, capturing the intensity of the firm's CSR practices, thus allowing for multiple (not necessarily binary) signals.

For generality, we allow for multiple signals in different probability structures, such as linear conditional probabilities, uniformly distributed signals, and conditional probabilities concentrated in two points.

## **Preview of Results:**

- We show that the presence of multiple signals first increases a firm's incentive to invest in CSR, and then decreases it until an equilibrium with complete information is reached.
- We show that as signal reliability or richness increases, a firms' incentive to invest in CSR increases and then decreases.
- We also show that richer signals reduce the firm's incentives to invest in CSR even when investments become more feasible.

# The Model: Overview

Perfectly Competitive Market where firms sell homogeneous goods.

One firm may differentiate by attaching a “credence attribute” to their good

- For simplicity, we will assume this credence good to be a cleaner production, yet more expensive, technology

# The Model: Consumers

Consider, a continuum of unit mass 1 consumers each with utility

$$u = v + \alpha g - p$$

$v$  is the valuation of the standard good

$g \in [0, G]$  is a consumer's valuation of the credence attribute

$\alpha$  determines the amount of "warm-glow" a consumer gets from consuming the credence good ([Andreoni, 1989](#)).

$\alpha$  is distributed over interval  $[0,1]$  and is assumed to have a CDF of  $H(\cdot)$ .

- $H(\cdot)$  is assumed to be a log-concave function where the reliability function can then be defined as  $\bar{H}(\cdot) = 1 - H(\cdot)$ .

# The Model: Firms and Technologies

## Firms Compete a la Bertrand

- As in the standard Bertrand Model, firms do not make any profits since they are all producing a homogeneous good.
- Market price ( $p$ ) and marginal cost ( $c$ ) are set to zero ( $c = p = 0$ )
- Firms operating at these assumed values are dubbed the “dirty” firms.

One firm, “The Firm”, has the ability to differentiate in the market by attaching a credence parameter to the good itself.

- The decision to attach a credence parameter to their good is done by choosing to either invest in the “clean” or “dirty” technology  $t \in \{c, d\}$ .
- Choosing the clean technology incurs a fixed cost of  $F \geq 0$
- Choosing the dirty technology incurs a fixed cost of  $F = 0$

# The Model: Firms and Technologies (*cont'*)

The firm can be one of three types depending on fixed costs

- With *ex ante* probability of  $\frac{(1-\theta)}{2}$  the firm has a fixed cost of  $F \rightarrow \infty$  of choosing the clean technology
- With a probability of  $\frac{(1-\theta)}{2}$  the firm has a fixed cost of  $F = 0$  of choosing the clean technology
- With probability of  $\theta$  the firm will choose to adopt a clean technology with with a fixed cost of  $0 < F < \infty$ .
- The firm learns their type before they decide to invest into the technology, and consumers do not observe firm's type.



# The Model: Signals and Information

The technology used by the firm is *not* directly observed by the consumers, but they know the firm has some prior probability of receiving a feasible fixed investment cost for the clean, CSR, technology.

Priors change according to the interval at which a feasible fixed cost is realized, yielding two different equilibria:

## The CSR equilibrium:

$$\Pr(C) = \frac{(1 - \theta)}{2} + \theta = \frac{(1 + \theta)}{2} \qquad \Pr(D) = \frac{1 - \theta}{2}$$

## The NCSR equilibrium:

$$\Pr(C) = \frac{1 - \theta}{2} \qquad \Pr(D) = \frac{(1 - \theta)}{2} + \theta = \frac{(1 + \theta)}{2}$$

# The Model: Signals and Information

Consumers and the firm both receive the same public signal concerning the technology used by the firm,  $s_i \in \{s_1, \dots, s_n\}$ .

The realization of the public signal depends on the technology the firm is using s.t.

$$P(s_1|C) = 1 \quad \text{and} \quad P(s_i|C) = 0 \quad \forall i \neq 1$$

$$P(s_i|D) = \gamma_i \quad \forall i \in \{1, \dots, N\}$$

where  $\gamma_i$  represents the level of “unreliability” of the public signal the consumers and the firm receive.

This generalization allows for several multiple signaling settings, and we illustrate some of these setting with examples.

# The Model: Signal and Information Examples

## Example 1 (Two signals and CG):

$N = 2$ ,  $\gamma_1 = 1 - \gamma$  and  $\gamma_2 = \gamma$

$$\Pr(s_1|C) = 1 \quad \text{and} \quad \Pr(s_1|D) = 1 - \gamma$$

$$\Pr(s_2|C) = 0 \quad \text{and} \quad \Pr(s_2|D) = \gamma,$$

where parameter  $\gamma$  can be understood as the degree of “signal precision,” or as CG defines it more explicitly, “market transparency.”

## Example 2 (Linear conditional probabilities):

$$\Pr(s_i|D) = (1 - \gamma) + (i - 1)b$$

where if  $\sum_{i=1}^N [(1 - \gamma) + (i - 1)b] = 1$ , then  $b = \frac{2[1 - (1 - \gamma)N]}{N(N - 1)}$ , implying that

$$\Pr(s_i|D) = (1 - \gamma) + (i - 1) \frac{2[1 - (1 - \gamma)N]}{N(N - 1)} \quad \text{where } 1 \geq \gamma \geq 0.$$

- $\Pr(s_i|D)$  is increasing in signal “dirtiness” ( $i \rightarrow N$ ) if  $\gamma \geq \frac{N-1}{N}$ , otherwise it is decreasing.
- $\Pr(s_i|D)$  decreases in  $N$  if and only if  $\gamma$  is relatively high,  $\gamma \geq \frac{2N-1}{2N+1}$ .

# The Model: Signal and Information Examples

## Example 3 (Uniform distribution / Infinite signals):

If, in the context of Example 2, conditional probabilities are constant,  $b = 0$ , we obtain that  $\Pr(s_i|D) = 1 - \gamma$  for all  $i$ , which yields

$$\Pr(s_i|D) = \frac{1}{N}$$

since  $\gamma = \frac{N-1}{N}$  when solving for  $\sum_{i=1}^N \Pr(s_i|D) = 1$ .

## Example 4 (Linear conditional probabilities):

Assuming the general form for clean signals where  $P(s_1|C) = 1$  and  $P(s_i|C) = 0$  for all  $i \neq 1$ , and specifying  $\gamma_N = \gamma$  and  $\gamma_i = \frac{1-\gamma}{N-1}$  for all  $i \neq N$ , then

$$P(s_i|D) = \frac{1-\gamma}{N-1} \quad \forall i \neq N \quad \text{and} \quad P(s_N|D) = \gamma.$$

- $\Pr(s_i|D)$  is decreasing in signal “reliability”  $\gamma$  for all  $i \neq N$ .
- $\Pr(s_i|D)$  decreases in signal “richness”  $N$ .

# The Benchmark Model: Timing

- 1) Nature chooses the type of firm (i.e. its investment cost  $F$ ).
- 2) The firm chooses its appropriate technology  $t \in \{c, d\}$ , and rest of firms produce and sell the standard “dirty” good.
- 3) Nature chooses the signal realization  $s_i \in \{s_1, \dots, s_n\}$  according to the aforementioned information structure. Consumers and the firm simultaneously observe this realization.
- 4) The firm sets its price ( $p$ ).
- 5) Each consumer decides whether to buy or not buy from the firm, and if the consumer does not buy from the firm, then they buy from the “dirty” firm.
- 6) The firm’s profits ( $\Pi$ ) are realized.

# The Model: Market Equilibrium

Where, the firm's price will only identify the consumers s.t.

$$v + \alpha^* P(C|s)G - p = v$$

→ Optimal Price set by the firm is:

$$p^*(s) = G\alpha^* P(C|s)$$

→ Optimal Profits to the firm are:

$$\pi^*(s) = P(C|s)\Pi = (1 - H(\alpha^*))G\alpha^* P(C|s)$$

where,

$$\Pi = (1 - H(\alpha^*))G\alpha^* \quad \text{and} \quad \alpha^* \equiv \frac{[1 - H(\alpha^*)]}{h(\alpha^*)}$$

**Lemma 1:** The price set by the firm is  $p^*(s)$ , whereas firm's profits (gross of fixed costs, if any) are then  $\pi^*(s)$

# The Corporate Socially Responsible (CSR) Equilibrium

We know in the CSR Equilibrium the incentive compatibility condition must be

$$\mathbb{E}[\pi(t = C)] \geq \mathbb{E}[\pi(t = D)]$$

Let,  $\pi_{CSR}^*(s_i) \equiv \Pi P(C|s_i)$  be the firm's profits in the case in which consumers anticipate that the strategic type chooses the clean technology.

Using Bayes rule we obtain:

$$\begin{aligned}\pi_{CSR}^*(s_i) &= \Pi P(C|s_i) \\ &= \Pi \frac{P(C)P(s_i|C)}{P(C)P(s_i|C) + P(D)P(s_i|D)}\end{aligned}$$

which yields,

$$\pi_{CSR}^*(s_1) = \Pi \frac{(1 + \theta)}{(1 + \theta) + (1 - \theta)\gamma_1} \quad \text{and} \quad \pi_{CSR}^*(s_i) = 0 \quad \text{for all } i \neq 1.$$

**Lemma 2.** Profit  $\pi_{CSR}^*(s_i)$  is weakly decreasing in signal  $s_i$  and in  $\gamma_1$ , and weakly increasing in  $\theta$ .

Note that  $\gamma_1$  can be a function of other exogenous parameters such as signal reliability  $\gamma$  and richness  $N$ .

# The Corporate Socially Responsible (CSR) Equilibrium

In the CSR Equilibrium, expected profits are represented as

$$\begin{aligned}\mathbb{E} [\pi_{CSR}(t = C)] &= \sum_{i=1}^N \Pr(s_i|C) \pi_{CSR}^*(s_i) - \hat{F} \\ &= \Pi \frac{(1 + \theta)}{(1 + \theta) + (1 - \theta)\gamma_1} - \hat{F}, \text{ and}\end{aligned}$$

$$\begin{aligned}\mathbb{E} [\pi_{CSR}(t = D)] &= \sum_{i=1}^N \Pr(s_i|D) \pi_{CSR}^*(s_i) \\ &= \Pi \frac{(1 + \theta)\gamma_1}{(1 + \theta) + (1 - \theta)\gamma_1} \text{ where,}\end{aligned}$$

**Proposition 1.** *In a CSR equilibrium, the expected profits of investing in the clean (dirty) technology are decreasing (increasing) in  $\gamma_1$ , and both are increasing in  $\theta$ .*

Similar to the results presented in Lemma 2,  $\gamma_1$  can be a function of other exogenous parameters such as signal reliability  $\gamma$  and richness  $N$ .



# The Corporate Socially Responsible (CSR) Equilibrium

With that said, for the CSR Equilibrium to exist with multiple signaling the incentive compatibility condition and expected profits are formulated as

$$\mathbb{E}[\pi(t = C)] \geq \mathbb{E}[\pi(t = D)]$$

$$\sum_{i=1}^n P(s_i|C)\pi_{CSR}^*(s_i) - \hat{F} \geq \sum_{i=1}^n P(s_i|D)\pi_{CSR}^*(s_i)$$

$$\begin{aligned} \rightarrow \Delta\pi_{CSR} &= \sum_{i=1}^n (Pr(s_i|C) - Pr(s_i|D)) \pi_{CSR}^*(s_i) \geq \hat{F} \\ &= \Pi \frac{(1+\theta)(1-\gamma_1)}{(1+\theta) + (1-\theta)\gamma_1} \geq \hat{F}. \end{aligned}$$

**Corollary 1.** *In a CSR equilibrium, the difference in expected profits of investing in the clean technology is decreasing in  $\gamma_1$ , but increasing in  $\theta$ .*

As in Proposition 1, if  $\gamma$  and  $N$  have a negative relationship with  $\gamma_1$ , then the difference in expected profits  $\Delta\pi_{CSR}$  increases in  $\gamma$  and  $N$ .

# The Not Corporate Socially Responsible (NCSR) Equilibrium

We know in the NCSR Equilibrium the incentive compatibility condition must be

$$\mathbb{E}[\pi(t = C)] \leq \mathbb{E}[\pi(t = D)]$$

Let,  $\pi_{NCSR}^*(s_i) \equiv \Pi P(C|s_i)$  be the firm's profits in the case in which consumers anticipate that the strategic type chooses the clean technology.

Using Bayes rule we obtain:

$$\begin{aligned}\pi_{NCSR}^*(s_i) &= \Pi \frac{P(C|s_i)}{P(C|s_i) + P(D|s_i)} \\ &= \Pi \frac{P(C)P(s_i|C)}{P(C)P(s_i|C) + P(D)P(s_i|D)}\end{aligned}$$

which yields,

$$\pi_{NCSR}^*(s_1) = \Pi \frac{(1 - \theta)}{(1 - \theta) + (1 + \theta)\gamma_1} \quad \text{and} \quad \pi_{NCSR}^*(s_i) = 0 \quad \text{for all } i \neq 1.$$

**Lemma 3.** Profit  $\pi_{NCSR}^*(s_i)$  is weakly decreasing in signal  $s_i$ ,  $\gamma_1$ , and  $\theta$ .

As in the CSR Equilibrium,  $\gamma_1$  can be a function of other exogenous parameters such as signal reliability  $\gamma$  and richness  $N$ .

# The Not Corporate Socially Responsible (NCSR) Equilibrium

In the NCSR Equilibrium, expected profits are represented as

$$\begin{aligned}\mathbb{E}[\pi_{NCSR}(t = C)] &= \sum_{i=1}^N \Pr(s_i|C) \pi_{NCSR}^*(s_i) - \hat{F} \\ &= \Pi \frac{(1 - \theta)}{(1 - \theta) + (1 + \theta)\gamma_1} - \hat{F}, \text{ and}\end{aligned}$$

$$\begin{aligned}\mathbb{E}[\pi_{NCSR}(t = D)] &= \sum_{i=1}^N \Pr(s_i|D) \pi_{NCSR}^*(s_i) \\ &= \Pi \frac{(1 - \theta)\gamma_1}{(1 - \theta) + (1 + \theta)\gamma_1} \text{ where,}\end{aligned}$$

**Proposition 2.** *In a NCSR equilibrium, expected profits of investing in the clean (dirty) technology are decreasing (increasing) in  $\gamma_1$ , and both are decreasing in  $\theta$ .*

Similar to the results presented in Lemma 3, if  $\gamma$  and  $N$  have a negative relationship with  $\gamma_1$ , then the effect on expected profits is opposite.

# The Not Corporate Socially Responsible (NCSR) Equilibrium

With that said, for the NCSR Equilibrium to exist with multiple signaling the incentive compatibility condition and expected profits are formulated as

$$\mathbb{E}[\pi(t = C)] \leq \mathbb{E}[\pi(t = D)]$$

$$\sum_{i=1}^n P(s_i|C) \pi_{NCSR}^*(s_i) - \hat{F} \leq \sum_{i=1}^n P(s_i|D) \pi_{NCSR}^*(s_i)$$

$$\begin{aligned} \rightarrow \Delta \pi_{NCSR} &= \sum_{i=1}^n (Pr(s_i|C) - Pr(s_i|D)) \pi_{NCSR}^*(s_i) \leq \hat{F} \\ &= \Pi \frac{(1 - \theta)(1 - \gamma_1)}{(1 - \theta) + (1 + \theta)\gamma_1} \leq \hat{F}. \end{aligned}$$

**Corollary 2.** *In a NCSR equilibrium, the difference in expected profits of investing in the clean technology is decreasing in both  $\gamma_1$  and  $\theta$ .*

As in Proposition 2, if  $\gamma$  and  $N$  have a negative relationship with  $\gamma_1$ , then the difference in expected profits  $\Delta \pi_{NCSR}$  increases in  $\gamma$  and  $N$ .

# Results: Equilibrium Conditions

In summary, the firm adopts the clean technology if and only if  $\hat{F}$  satisfies

$$\underbrace{\Pi \frac{(1-\theta)(1-\gamma_1)}{(1-\theta) + (1+\theta)\gamma_1}}_{\Delta\pi_{CSR}} \geq \hat{F} \geq \underbrace{\Pi \frac{(1-\theta)(1-\gamma_1)}{(1-\theta) + (1+\theta)\gamma_1}}_{\Delta\pi_{NCSR}}$$

or, alternatively, solving for the unreliability parameter  $\hat{\gamma}_1$ ,

$$\underbrace{\frac{(\Pi - \hat{F})(1+\theta)}{\Pi(1+\theta) + \hat{F}(1-\theta)}}_{\bar{\gamma}_1} \geq \hat{\gamma}_1 \geq \underbrace{\frac{(\Pi - \hat{F})(1-\theta)}{\Pi(1-\theta) + \hat{F}(1+\theta)}}_{\underline{\gamma}_1},$$

where  $\bar{\gamma}_1$  and  $\underline{\gamma}_1$  represent the upper and lower bounds for signal unreliability.

Cutoffs  $\bar{\gamma}_1$  and  $\underline{\gamma}_1$  are both increasing in  $\Pi$  and decreasing in  $\hat{F}$ , but  $\bar{\gamma}_1$  ( $\underline{\gamma}_1$ ) is increasing (decreasing) in  $\theta$ .

# Results: Evaluating Equilibrium Conditions

To evaluate changes in equilibrium conditions, we use the difference in profit gains

$$\begin{aligned}\Delta\pi &= \Delta\pi_{CSR} - \Delta\pi_{NCSR} \geq \widehat{F} \\ &= \sum_{i=1}^n (Pr(s_i|C) - Pr(s_i|D)) (\pi_{CSR}^*(s_i) - \pi_{NCSR}^*(s_i)) \geq \widehat{F} \\ &= \Pi \frac{\gamma_1(1 - \gamma_1) [(1 + \theta)^2 - (1 - \theta)^2]}{[(1 + \theta) + (1 - \theta)\gamma_1][(1 - \theta) + (1 + \theta)\gamma_1]} \geq \widehat{F}.\end{aligned}$$

**Corollary 3.** *The difference in profit gains,  $\Delta\pi$ , is increasing in  $\gamma_1$  if and only if  $\gamma_1 < \tilde{\gamma}_1$ , where cutoff  $\tilde{\gamma}_1 \equiv \frac{\theta^2 + 2\sqrt{1 - \theta^2} - 1}{3 + \theta^2}$  is decreasing in  $\theta$ . This implies that  $\Delta\pi$  is increasing in  $N$  if and only if  $N < \lfloor \tilde{N} \rfloor$ , where  $\lfloor \tilde{N} \rfloor$  is the smallest integer solving  $\gamma_1 < \tilde{\gamma}_1$ . Furthermore,  $\Delta\pi$  is unambiguously increasing in  $\theta$ , but  $\frac{\partial \Delta\pi}{\partial \theta}$  is decreasing in the number of signals  $N$ .*

# Results: Evaluating Equilibrium Conditions

## Example 1 (Two signals and CG)

$$\Delta\pi = \Pi \frac{\gamma(1-\gamma)[(1+\theta)^2 - (1-\theta)^2]}{[(1+\theta) + (1-\theta)(1-\gamma)][(1-\theta) + (1+\theta)(1-\gamma)]} \geq \hat{F},$$

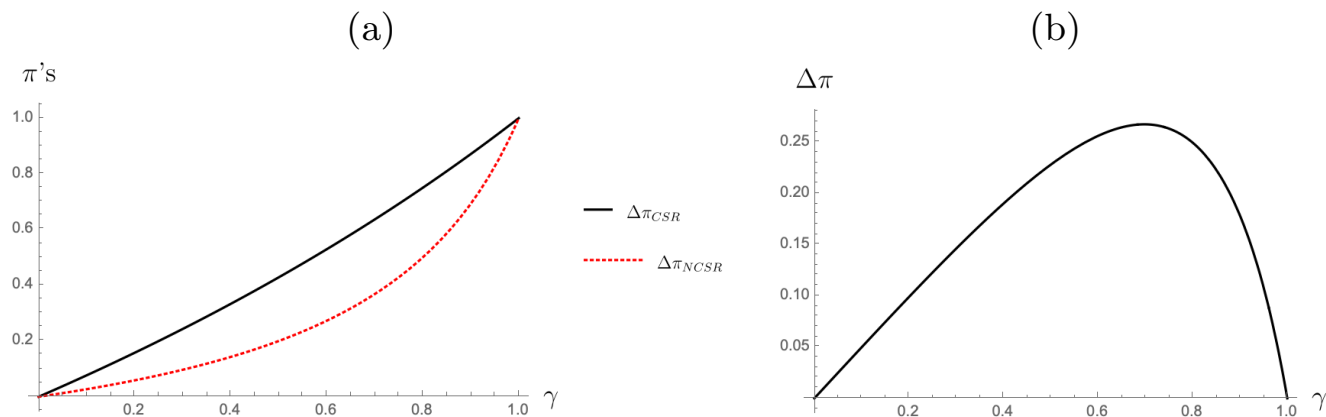


Figure 1: Profit Gains (a) and Difference in Profit Gains (b)

## Example 2 (Linear conditional probabilities).

Our results are analogous to that of Example 1 with the exception of  $N - 2$  other profits equating to zero factor into the equation.

# Results: Evaluating Equilibrium Conditions

## Example 3 (Uniform distribution / Infinite signals)

$$\Delta\pi = \Pi \frac{(N-1)[(1+\theta)^2 - (1-\theta)^2]}{[(1+\theta)N + (1-\theta)][(1-\theta)N + (1+\theta)]} \geq \hat{F},$$

(a)

(b)

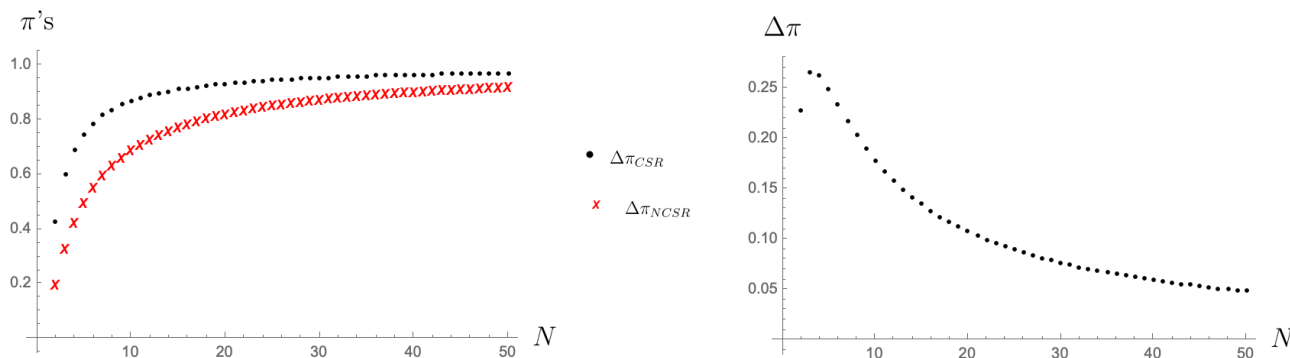


Figure 2: Profit Gains (a) and Different in Profit Gains (b)



# Results: Evaluating Equilibrium Conditions

## Example 4 (Extreme cases)

$$\Delta\pi = \Pi \frac{(N + \gamma)(1 - \gamma)[(1 + \theta)^2 - (1 - \theta)^2]}{[(1 + \theta)(N - 1) + (1 - \theta)(1 - \gamma)][(1 - \theta)(N - 1) + (1 + \theta)(1 - \gamma)]} \geq \widehat{F},$$

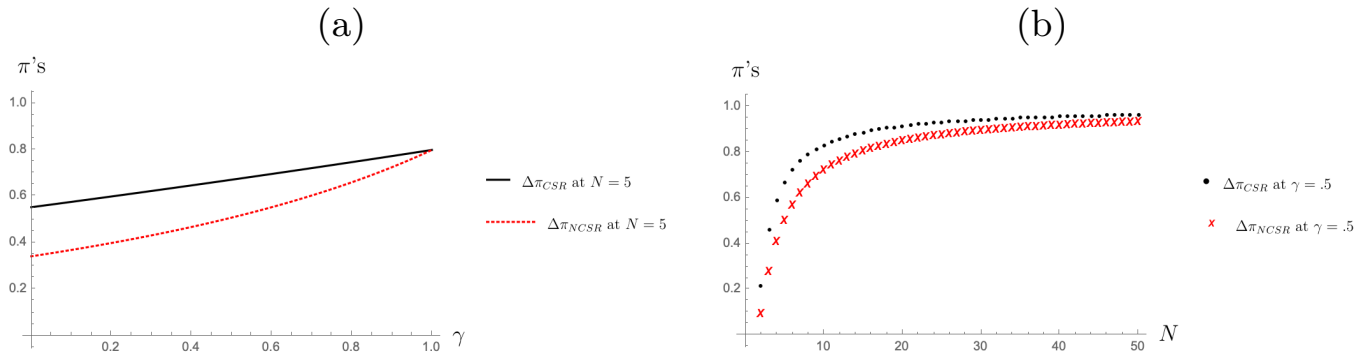


Figure 3: Profit Gains over Signal Reliability  $\gamma$  (a) and Richness  $N$  (b)

# Results: Evaluating Equilibrium Conditions

## Example 4 (Extreme cases)

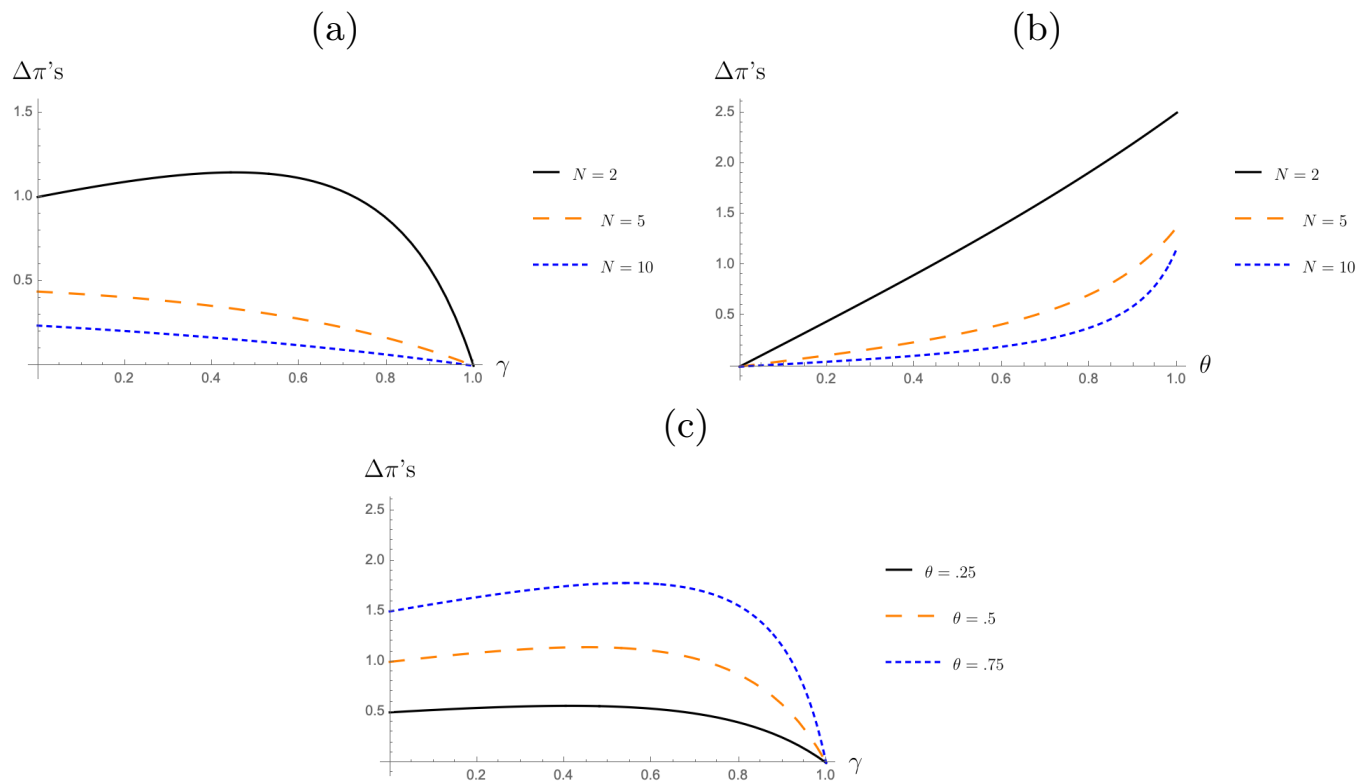


Figure 4: Simulated Comparative Statics with All Parameters

# Conclusions / Discussions

We extend CG's binary signaling setting by allowing for more signals. In this setting, we can still support the equilibrium where it is profitable for a firm to invest in CSR when fixed costs are feasible (i.e.  $\Delta\pi \geq \hat{F} \geq 0$ ).

This result informs policy makers that firms still have incentives to invest in CSR, even when operating in an environment with multiple signals.

We find a level of signals  $\tilde{N}$  which maximizes the profit gains  $\Delta\pi$  for a firm choosing to practice CSR. This finding entails that there is an optimal number of signals that provides the firm with the most incentive to invest in CSR.

We find that profit gains from practicing CSR are increasing as fixed costs become more feasible (higher  $\theta$ ), but also find that as the number of signals  $N$  increases, the effect is diminished. Our result, then, entails that even though more feasible fixed costs induces firms to invest in CSR, the number of signals implicitly dampens this effect, ultimately, reducing firm's incentive to invest in CSR.

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