Modeling Supersonic Jet Noise Reduction with Global Resolvent Modes
Project 059C

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ASCENT

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Modeling Supersonic Jet Noise Reduction with Global Resolvent Modes

University of Illinois Urbana-Champaign

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PM: Sandy Liu and Muni Majjigi

Cost Share Partner(s): Boom, Gulfstream

Objective:

Develop a rapid capability, using physics-based models, to estimate changes in jet take-off noise due to changes in nozzle design and engine cycle

Project Benefits:

Reduce sound environmental impact due to anticipated return of supersonic civilian transport aircraft

Improved workflows and reliability for industry partners to make noise-related design decisions

Research Approach:

Utilize input-output (resolvent) descriptions of the jet aeroacoustics to link nozzle design and engine cycle choices to their impact on the radiated noise.

Envisioned usage:

1. Compute RANS of baseline nozzle with identified cycle and design parameters
2. Compute input-output operator and its gain sensitivities wrt design parameters
3. Select new cycle and/or nozzle design parameters that reduce gains of far-field noise
4. Return to 1.) with new nozzle and repeat

Major Accomplishments (to date):

Resolvent-based JNR design workflow complete

Demonstrated application on GTRI nozzle with variable mixer length

Future Work / Schedule:

- Complete input/output "local resolvent" calibration development
- Complete linearized operator development in PyFR
- Perform LES of Plug20 nozzle jet flows
- Apply resolvent-based JNR design workflow to Plug20 nozzles

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Motivation

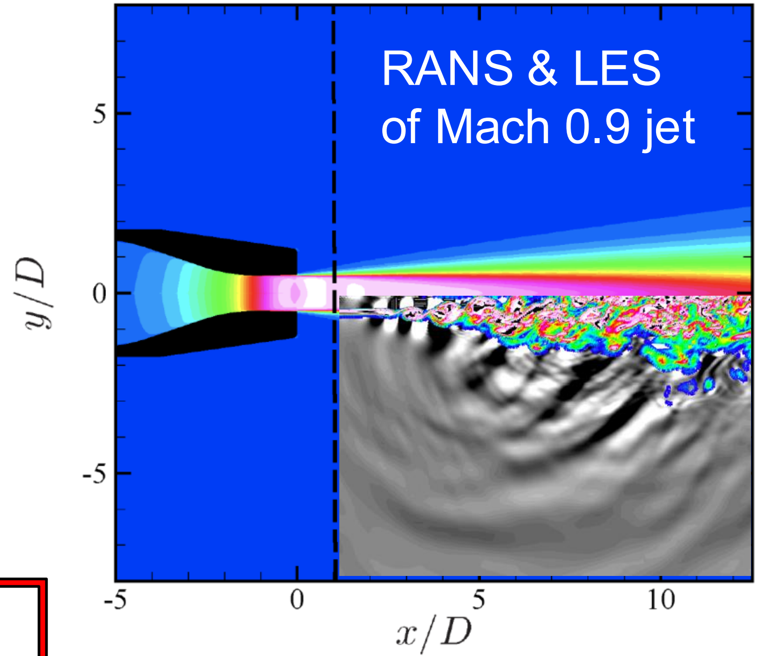
- Return of supersonic civilian transport aircraft highly anticipated
- Likely jet engine parameters are different from subsonic transport:
  - Bypass ratio ~ 2 turbofans
  - Mixed fan and core streams
  - Jet exit Mach number ~ 0.9
- Jet take-off noise key environmental barrier to community acceptance (subsonic & supersonic)
- Need means to accurately assess engine design choices on radiated take-off noise

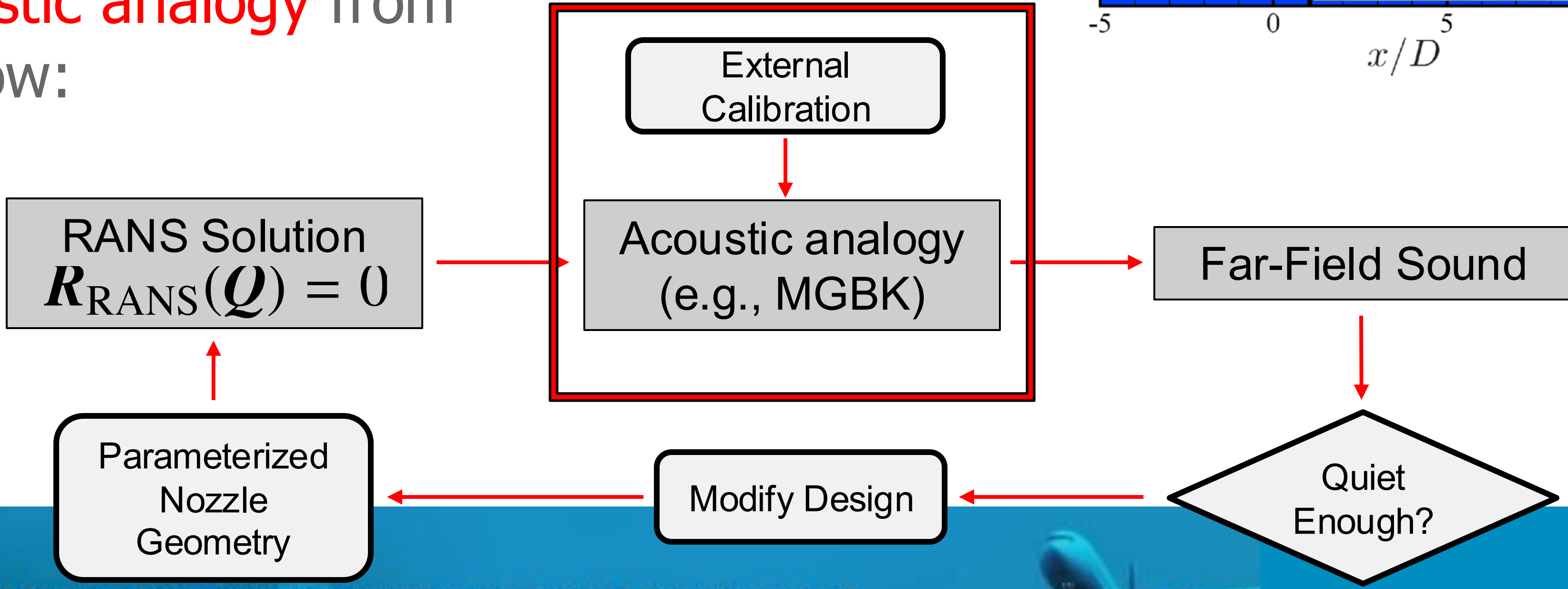
Credit: Boom

Credit: Gulfstream

PAST: RANS-Estimated JNR Sensitivity

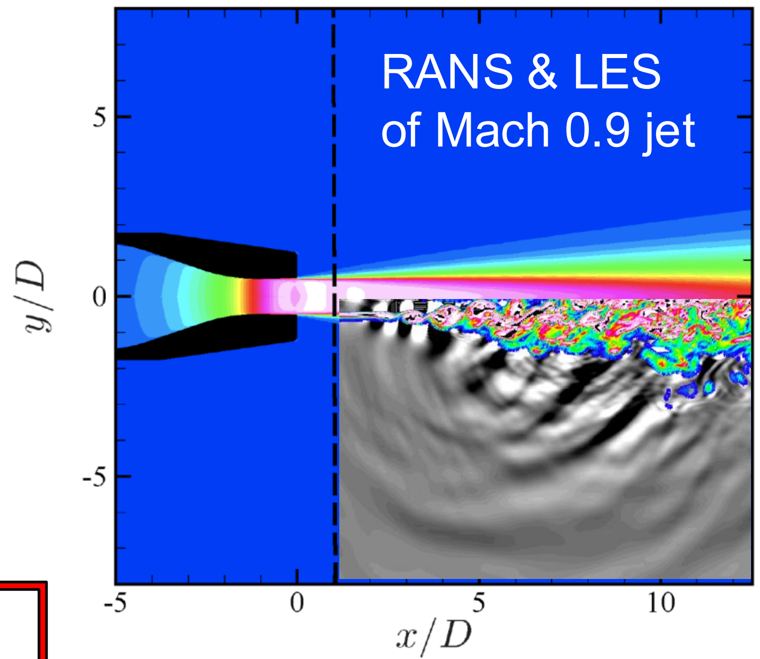
- RANS calculations are inexpensive, but lack acoustic field
- Large-eddy simulations (LES) are expensive, include acoustic field
- Prior: approximate noise field with output of acoustic analogy from RANS mean-flow:

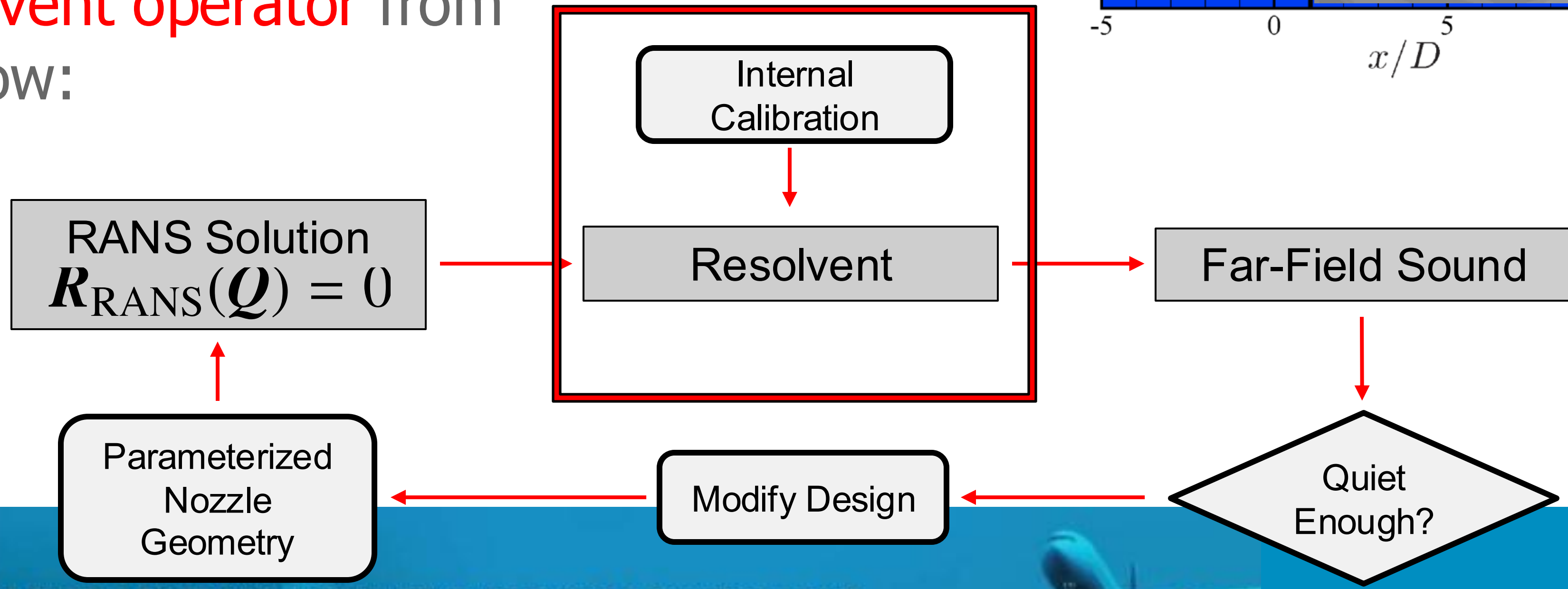




NEW: Resolvent-Estimated JNR Sensitivity

- RANS calculations are inexpensive, but lack acoustic field
- Large-eddy simulations (LES) are expensive, include acoustic field
- New: approximate noise field with output of resolvent operator from RANS mean-flow:





Resolvent-Based JNR Estimation

- Start with RANS-estimated mean flow $\boldsymbol{R}_{\mathrm{RANS}}(\boldsymbol{Q}) = 0 \longrightarrow \boldsymbol{Q}$
- Hypothesize a linearization about the mean

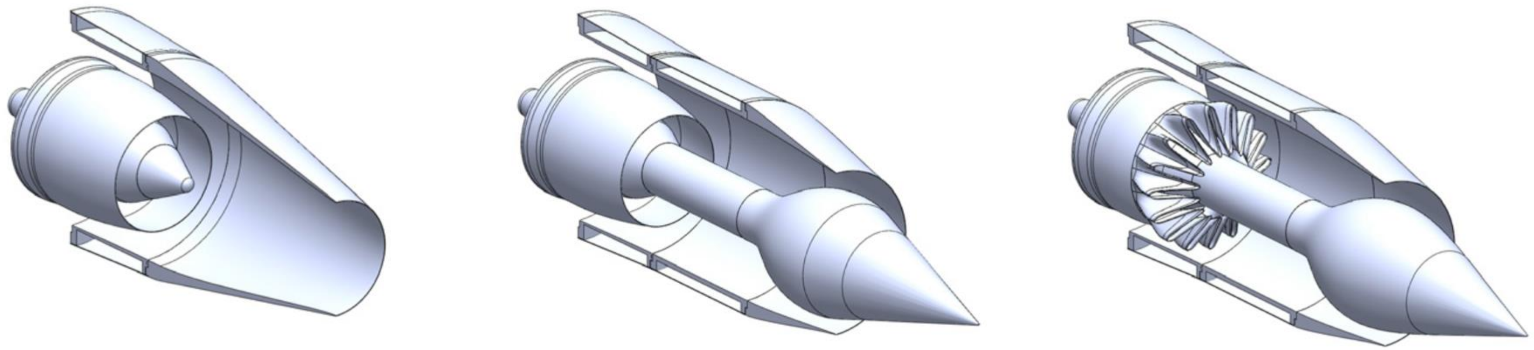
$$\frac{\partial \boldsymbol{q}}{\partial t} = \mathrm{A}[\boldsymbol{Q}]\boldsymbol{q} + \mathrm{B}[\boldsymbol{Q}]\boldsymbol{f}$$
$$p' = \mathrm{C}[\boldsymbol{Q}]\boldsymbol{q}$$

- Convert to the frequency domain $\boldsymbol{q}(\boldsymbol{x}, t) = \hat{\boldsymbol{q}}(\boldsymbol{x})e^{i\omega t}$

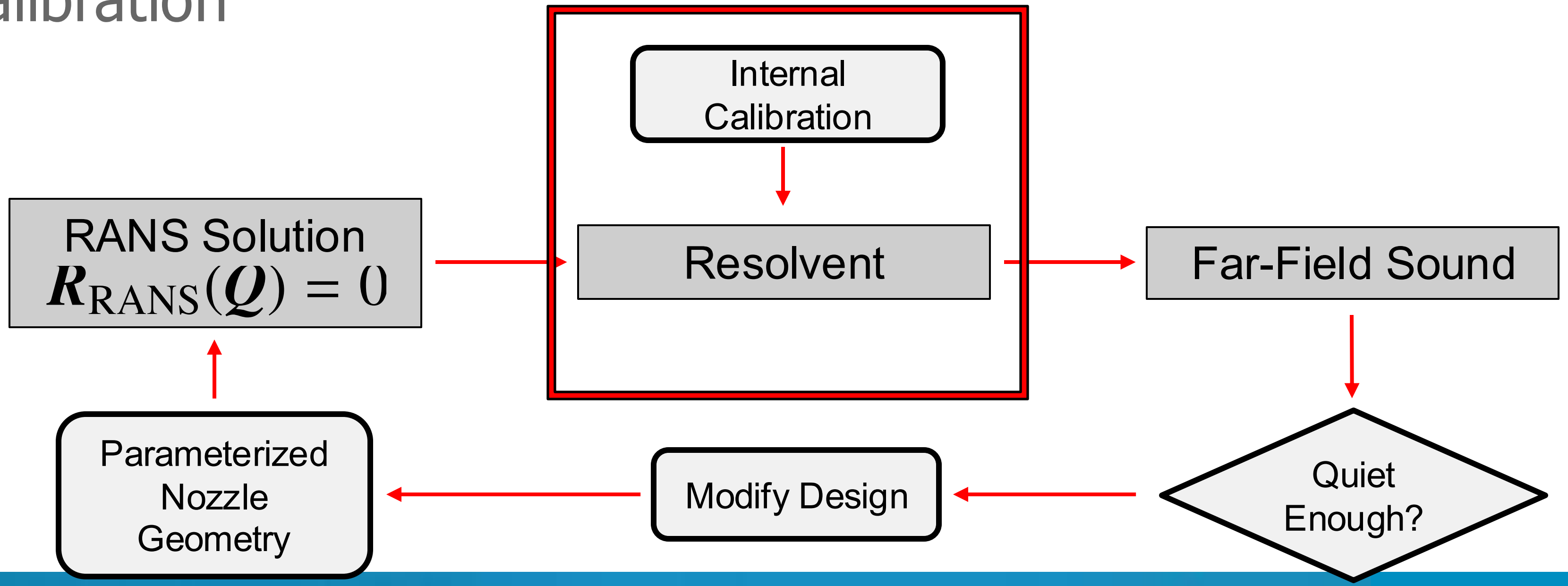
$$\hat{p} = \underbrace{\mathrm{C}\overbrace{(i\omega\mathrm{I} - \mathrm{A})^{-1}}^{\mathrm{R}(\omega)\ \text{(Resolvent Operator)}}\mathrm{B}}_{\mathrm{H}(\omega)\ \text{(Input/Output Operator)}}\hat{\boldsymbol{f}} \longrightarrow \boxed{\hat{p} = \mathrm{H}(\omega)\hat{\boldsymbol{f}}}$$

Current Focus

- NASA complex nozzles (Bridges et al., TM-20210010291)



- Internal calibration



Towards More Complex Nozzles

- Computing the RANS- and LES-based flows through these nozzles is feasible with existing tools (e.g., PI-modified OpenFOAM and CharLES)
- We planned to use CharLES to also compute the linearized operators
- However, during our contract negotiations with Cascade

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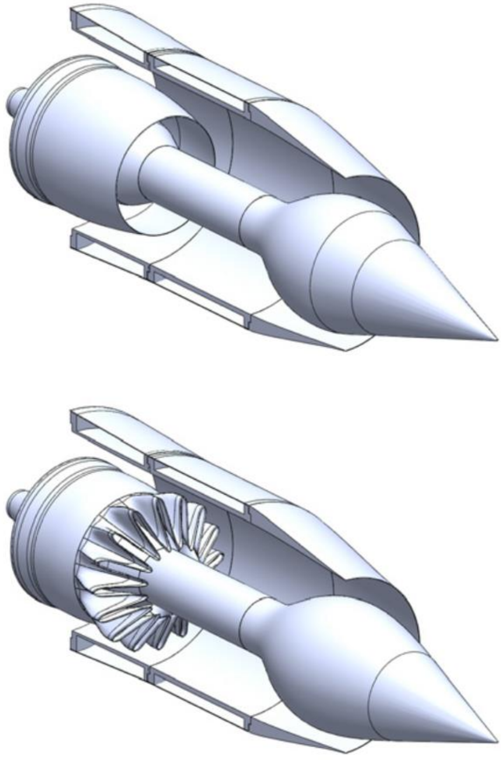
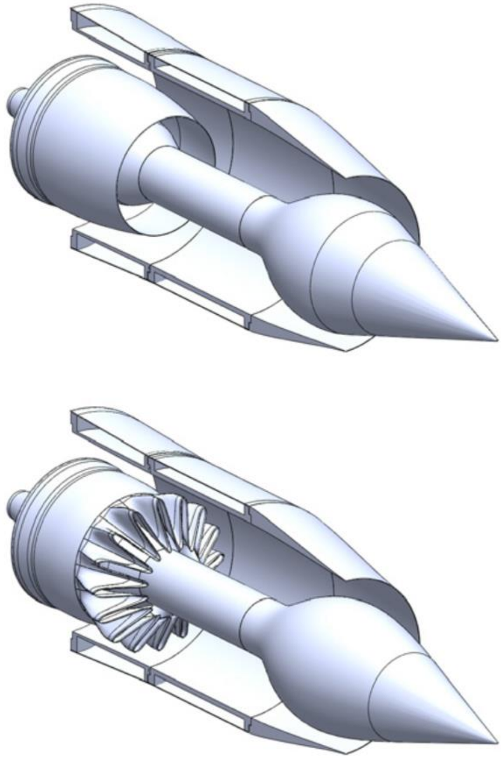
and the source code was no longer available



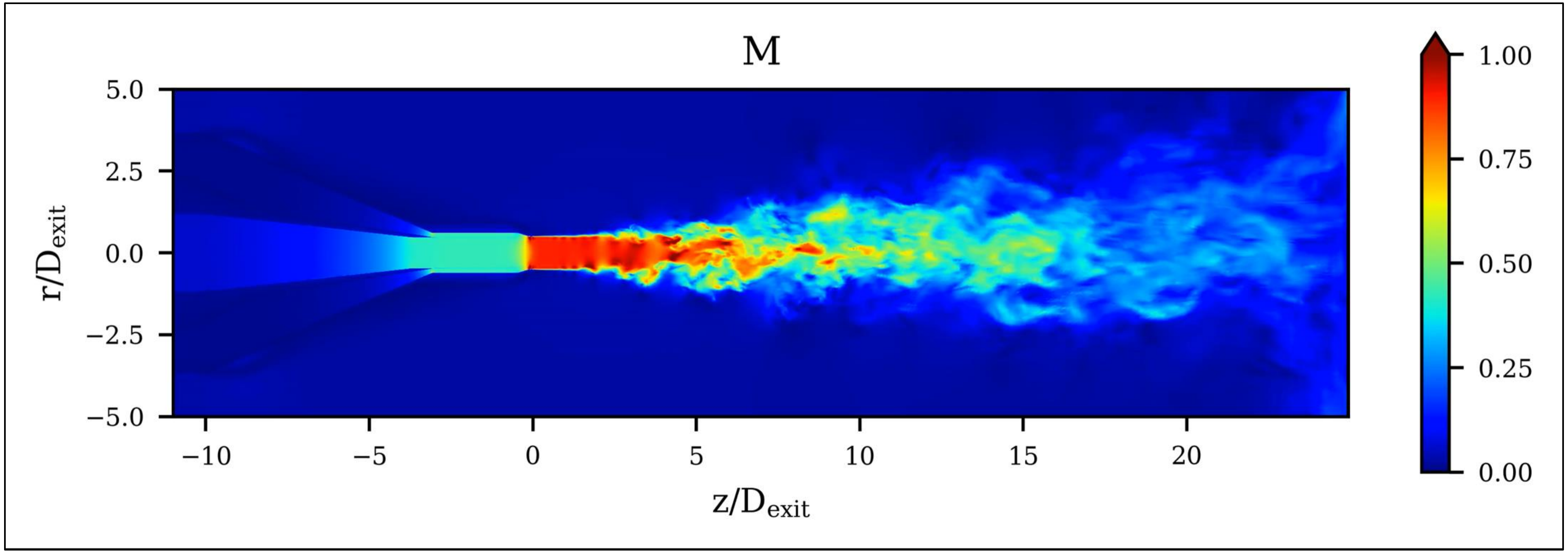
Towards More Complex Nozzles

- We therefore pivoted to PyFR an open-source, compressible Navier-Stokes solver that is well-supported
  - Python front end
  - Just-in-time compilation for execution on CPU, GPU, Arm architectures
  - High-order flux reconstruction
  - Unstructured mesh
- We are adding:
  - Dynamic slip vel. Wall Model (in-progress)
  - Extraction of linearized operator (in-progress)
- We have measured $\approx 10\times$ speed-up on GPUs



Towards More Complex Nozzles

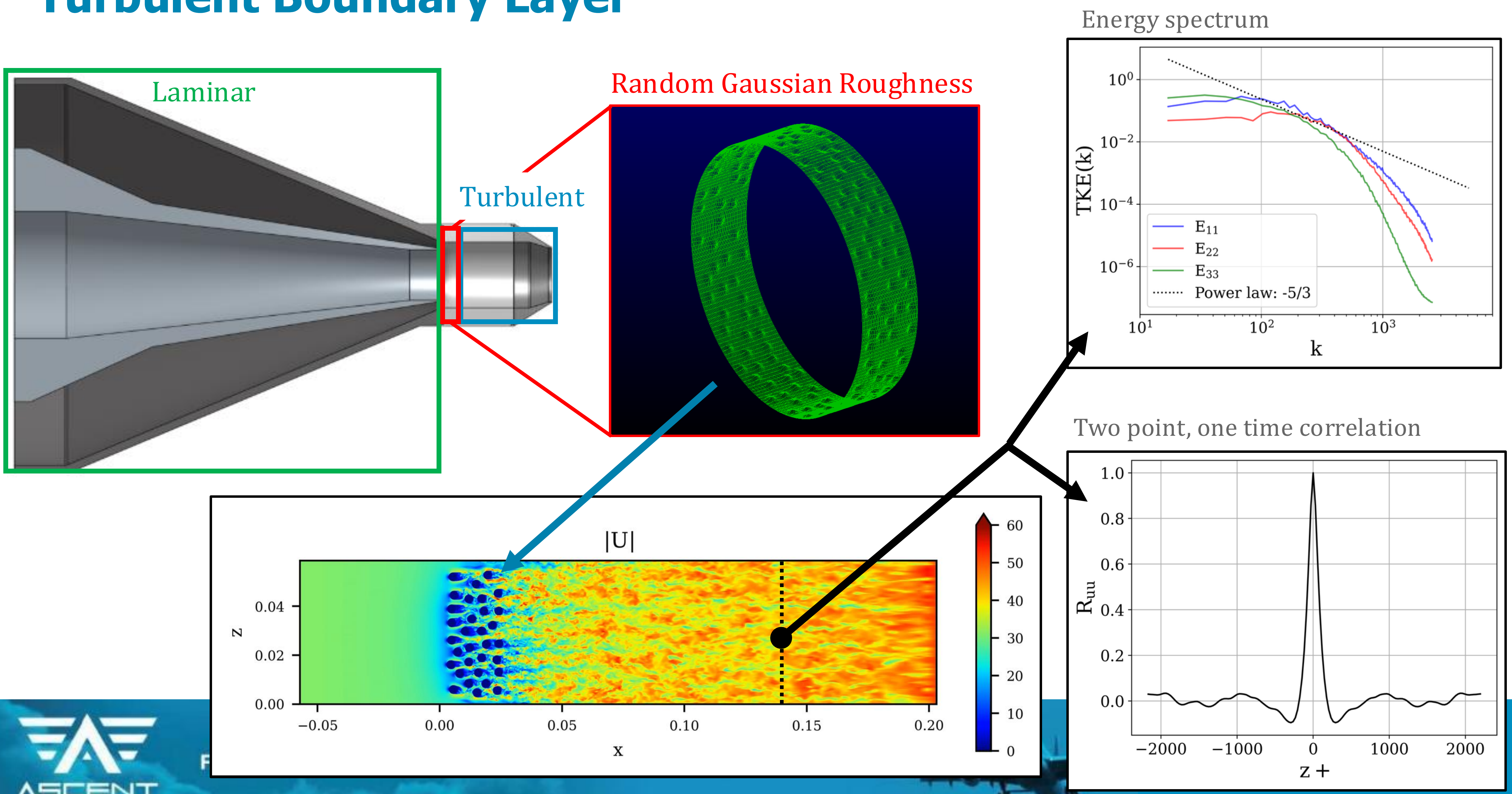
- Demonstration of PyFR on GTRI Nozzle (under-resolved, no wall-model)



NPR_p	ER	L/D_e	TTR_p	TTR_s
1.6913	1.0	3.0	1.0	1.0

Reference Case (M = 0.9)

Turbulent Boundary Layer



Internal Calibration $\boxed{\hat{p} = \mathrm{H}(\omega)\hat{f}}$

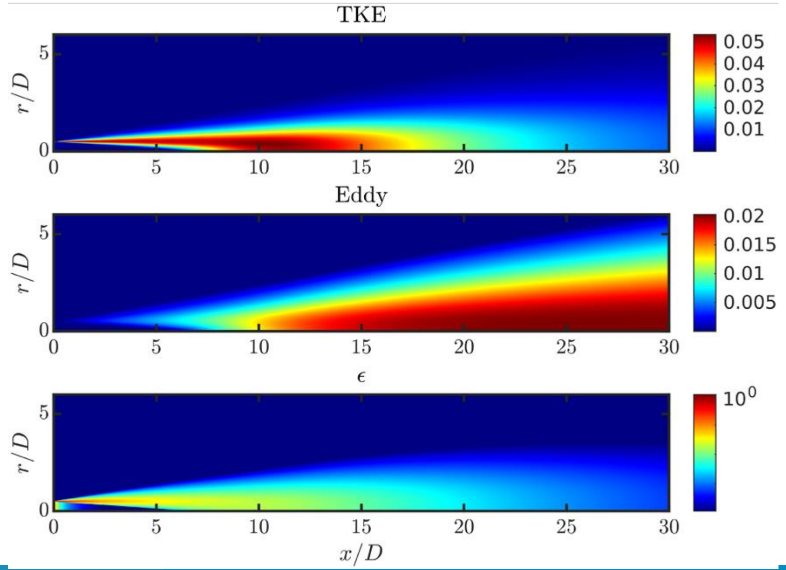
- Recall from earlier the input/output operator $\hat{p} = \mathrm{U\Sigma V}^H \hat{f} = \sum_{i=1}^{N} \sigma_i \boldsymbol{u}_i \underbrace{\boldsymbol{v}_i^H \hat{f}}$

“Generation”

Propagation

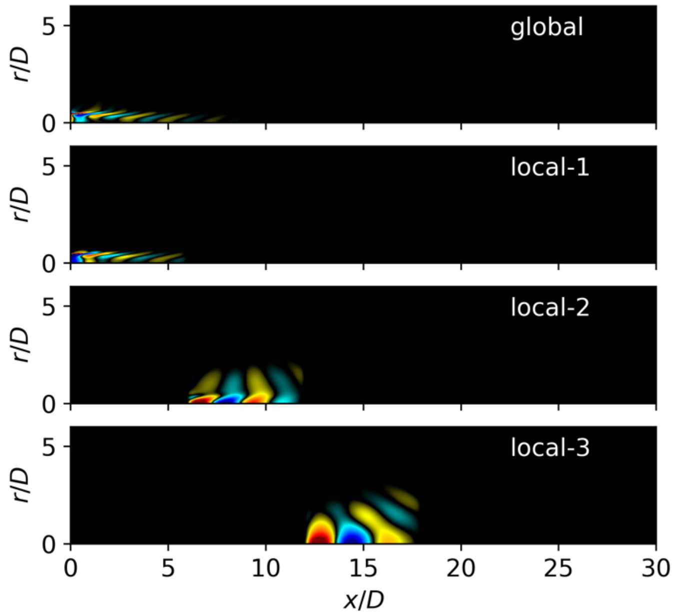
Amplification

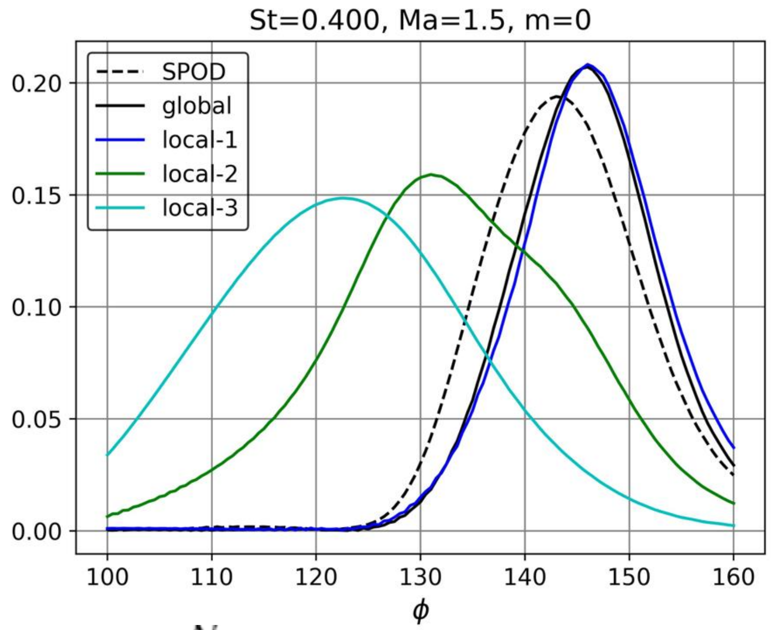
- The “Generation” step converts the natural jet’s unsteadiness into a “forcing amplitude” of each propagation mode
- Knowing the “forcing amplitudes” is necessary to make quantitative jet noise predictions
- Hypothesis: the “forcing amplitudes” can be learned with RANS-available quantities (e.g., mean flow, TKE, etc.)



Internal Calibration

- Approach: localize the forcing through the input matrix $H(\omega) = C\,(i\omega I - A)^{-1}\,\boxed{B}$



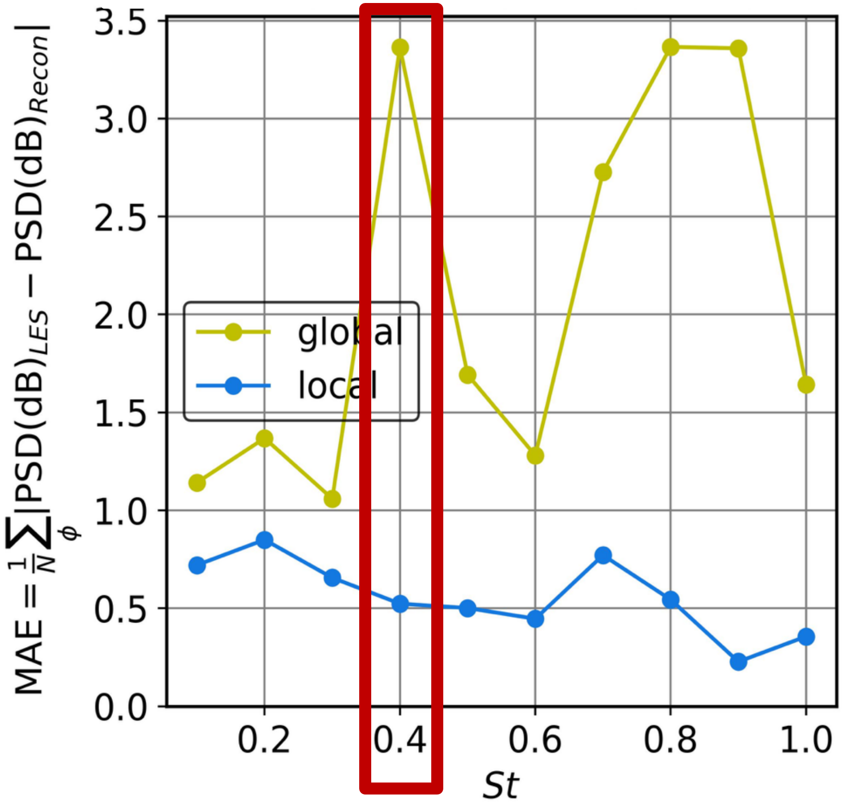


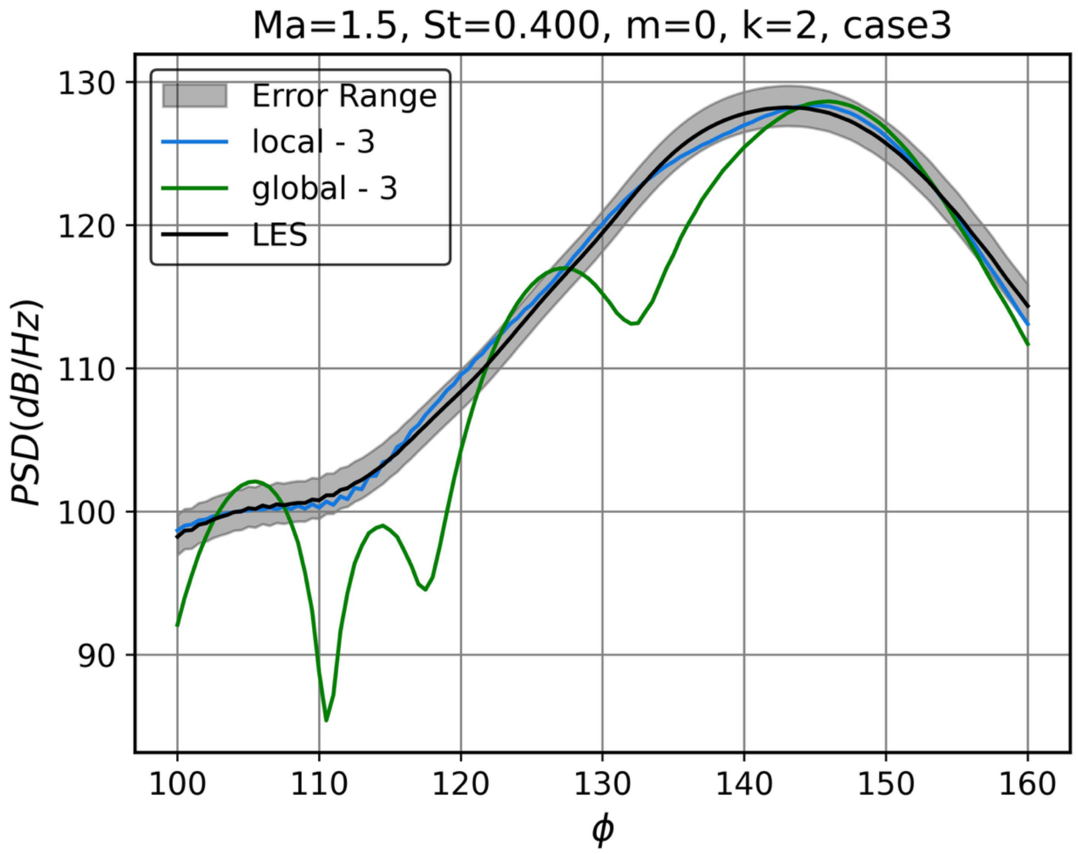
- Then approximate $\hat{p} \approx \sum_{j=1}^{N} \hat{p}^{(j)}$ where $\hat{p}^{(j)} = \sum_{i=1}^{N_j} \sigma_i^{(j)} \hat{\boldsymbol{u}}_i^{(j)} \hat{\boldsymbol{v}}_i^{(j)} \hat{\boldsymbol{f}}$
- Learn the forcing amplitudes $\hat{\boldsymbol{v}}_i^{(j)} \hat{\boldsymbol{f}}$ using $\hat{p}$

Internal Calibration

- Demonstration of localized resolvent using higher-speed jet data (leveraging ONR support)

$$m = 0$$

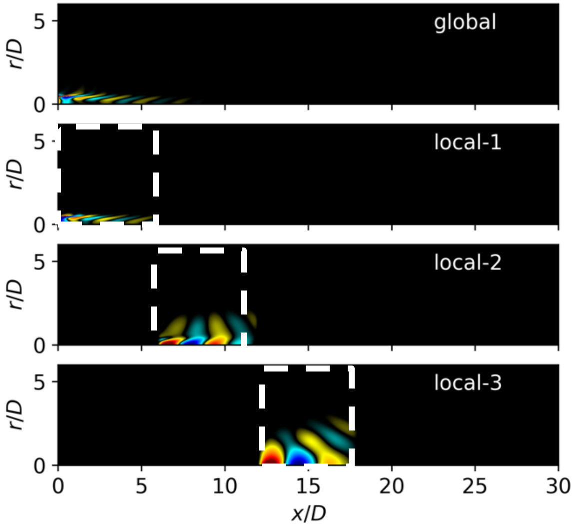


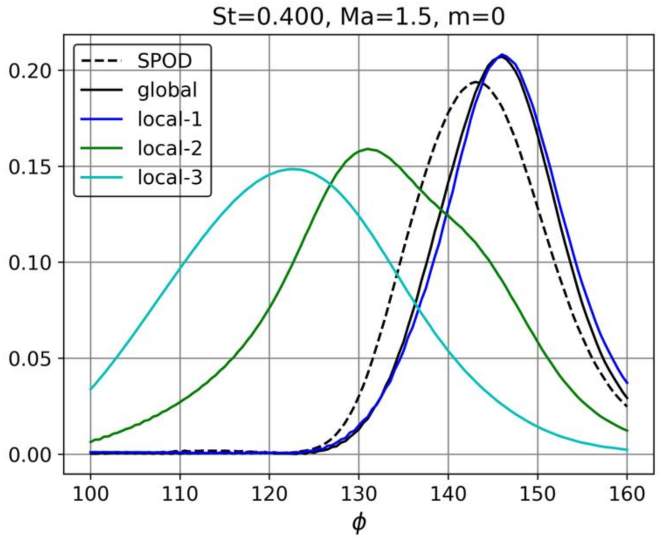


Internal Calibration

- Approach: localize the forcing through the input matrix

$$\mathsf{H}(\omega) = \mathsf{C}\,(i\omega\mathsf{I} - \mathsf{A})^{-1}\,\boxed{\mathsf{B}}$$





- Then approximate $\langle \hat{p}, \hat{p}^* \rangle \approx \sum_{k=1}^{N_s} \hat{p}_s^{(k)} \hat{p}_s^{(k)*}$ where $\hat{p}_s^{(k)} = \sum_{j=1}^{N_\ell} \sum_{i=1}^{N_m} \sigma_i^{(j)} \hat{\boldsymbol{u}}_i^{(j)} \hat{\boldsymbol{v}}_i^{(j)} \hat{\boldsymbol{f}}$
- Learn the forcing amplitudes $\hat{\boldsymbol{v}}_i^{(j)} \hat{\boldsymbol{f}}$ using $\hat{p}$

Internal Calibration

Solution methods:

CSD minimization:

$$\min \| \langle \hat{p}, \hat{p}^* \rangle - \sum_{k=1}^{N_s} \hat{p}_s^{(k)} \hat{p}_s^{(k)*} \|$$

CSD minimization + dB loss:

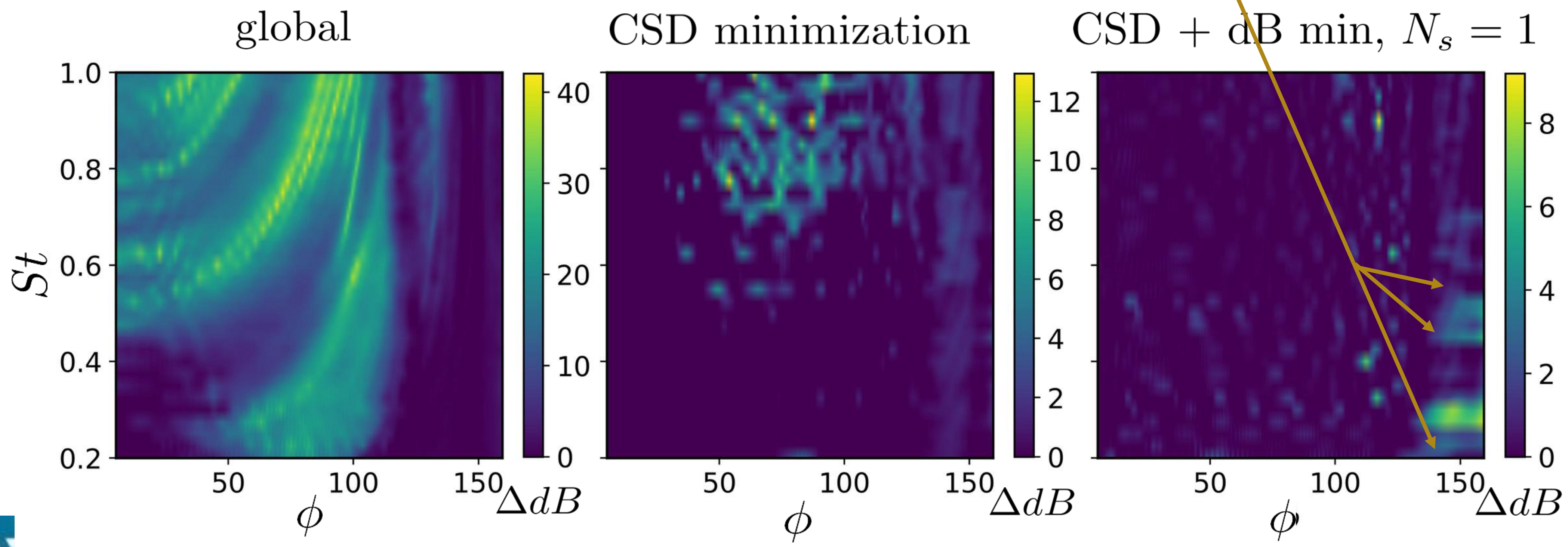
$$\min \| \langle \hat{p}, \hat{p}^* \rangle - \sum_{k=1}^{N_s} \hat{p}_s^{(k)} \hat{p}_s^{(k)*} \| + \lambda \left| \log_{10}(\langle \hat{p}, \hat{p} \rangle) - \log_{10}\Big(\sum_{k=1}^{N_s} \hat{p}_s^{(k)} \hat{p}_s^{(k)} \Big) \right|$$

Internal Calibration

- Demonstration of localized resolvent using higher-speed jet data (leveraging ONR support)

$$m = 0$$

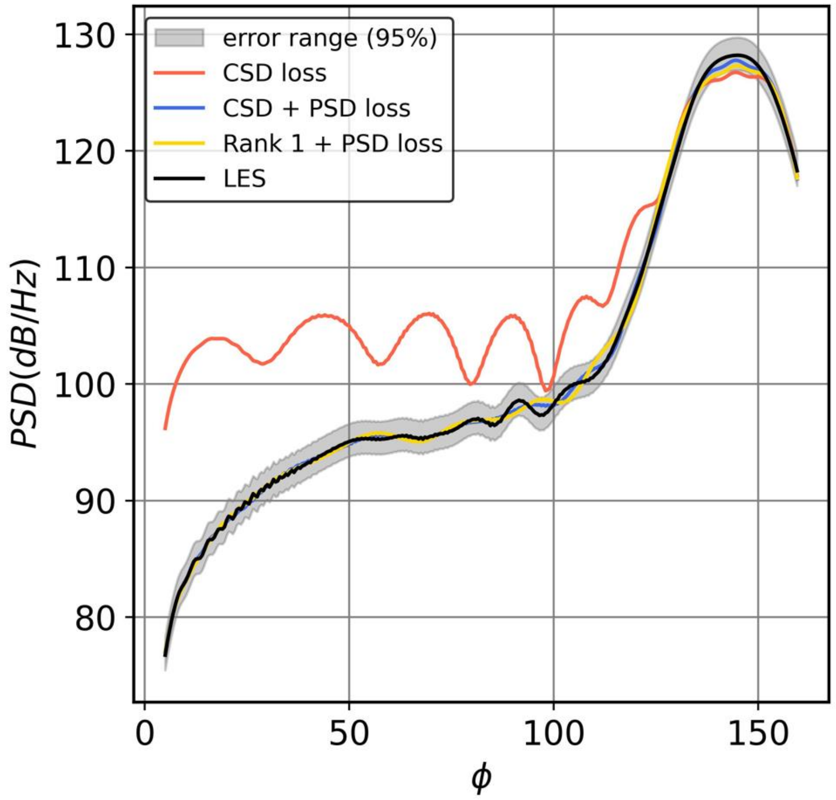
Dominant punk modes



Internal Calibration

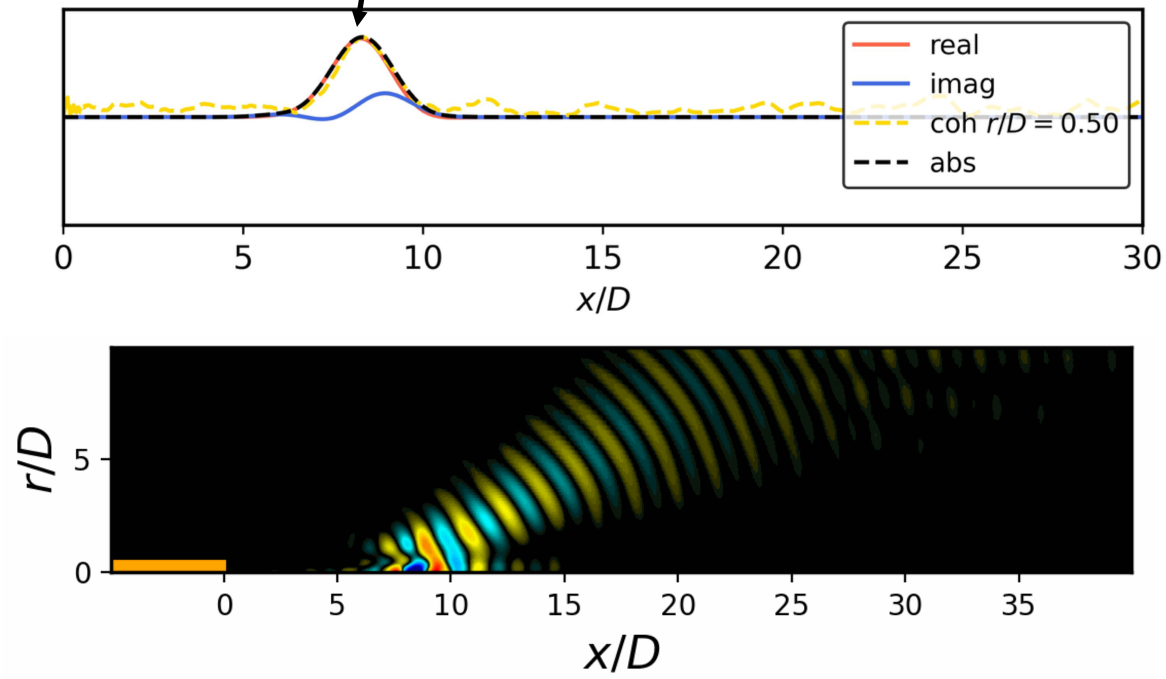
$$\mathbf{CSD + dB\ min,\ N_s = 1, N_m = 1}$$

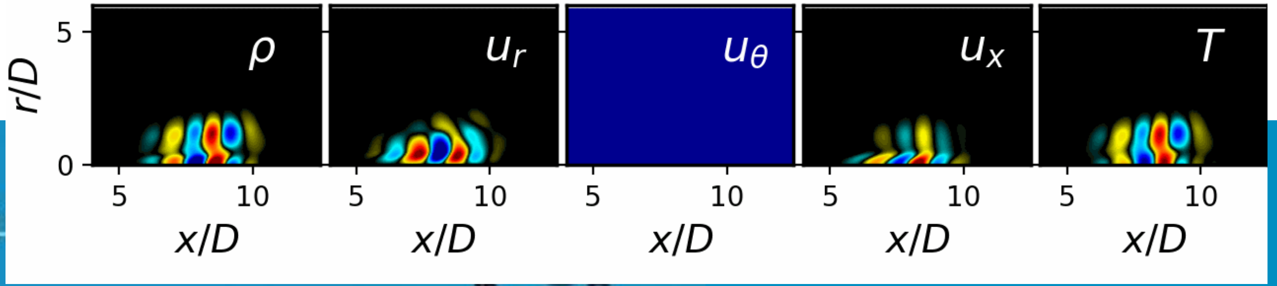
$$\mathbf{Ma = 1.5, St = 0.40, m = 0}$$



$$\sum_j \boldsymbol{w}_j \hat{\boldsymbol{v}}^{(j)} \hat{\boldsymbol{f}}$$

$\boldsymbol{w}_j - j^{th}$ local spline





wavepacket undergoes axial amplification, saturation and downstream decay

Summary

- STATUS
  – Resolvent-based JNR design workflow complete
  – Demonstrated application on GTRI nozzle with variable mixer length
- TODO
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