in Figure 3. This flowfield can affect the propulsion operation. Even if the nozzle flow is choked, the external flowfield can influence the location and size of the sonic region as well as deflection of the exhaust plume, as shown in Figure 3. The aircraft angle of attack influences this external flowfield and therefore the engine operation. Legacy mission codes may not be able to read an engine table that explicitly captures this type of dependence.

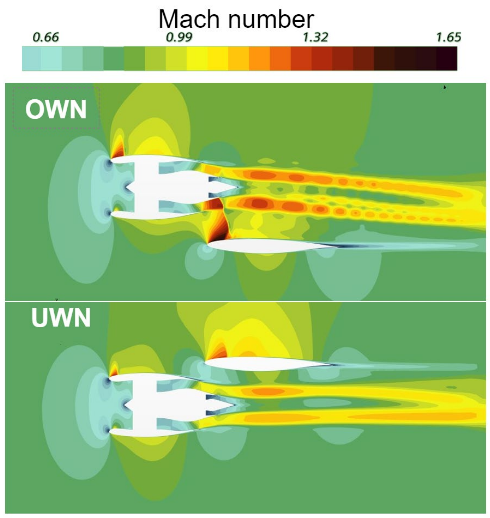

Figure 3. The external flowfield influences the engine cycle; these effects may not be captured in legacy mission analysis codes. OWN: over-wing nacelle; UWN: under-wing nacelle.

Several efforts in the field toward modern conceptual sizing and synthesis are currently underway, including LEAPS (Capristan et al., 2020), GASPy (Marfatia & Bergeson, 2021), SUAVE (MacDonald et al., 2017), TASOPT (Drela, 2016), MARLi (Druot et al., 2019), and Environmental Design Space (Nunez et al., 2021). Many of these efforts rely on traditional thrust and drag bookkeeping to account for propulsive and aerodynamic effects. The prevalence of novel aircraft concepts, which leverage coupled dynamic effects such as aero-propulsive interactions, has been found to produce considerable uncertainty within the traditional vehicle sizing and synthesis processes upon which these methods are often based. In our work, we began to reformulate the aircraft sizing and synthesis processes to leverage a more general kinematic expression of air vehicle performance.

Traditional sizing and synthesis methods within the conceptual and preliminary design phases, as presented in seminal works by Anderson (2000), Roskam (2005), and Raymer (2006), generally use an iterative approach to air vehicle design that begins with an initial vehicle layout or specification followed by disciplinary analysis. The results of these analyses are then leveraged to conduct performance analyses of the design, often as a combination of point performance measures and performance in a selected design scenario. These performance results are compared with corresponding requirements and used to iteratively modify the vehicle layout and configuration. Similar design methods, such as the energy-based constraint analysis of Mattingly et al. (2002), use a similar layout-analysis-performance design cycle that emphasizes the tuning of scaling factors within the design process. In these traditional sizing and synthesis approaches, the linkage between disciplinary analyses and vehicle-level performance is most evident through the utilized equations of motion. Most commonly, the aircraft equations of motion are posed as Eq. (1) and Eq. (2):

$$T \cos \epsilon - D - W \sin \gamma = m \frac{dV}{dt} \tag{1}$$

$$L + T \sin \epsilon - W \cos \gamma = m \frac{V^2}{r_c} \tag{2}$$

In these equations, ϵ is the engine mounting angle, γ is the aircraft flight path angle, r_c is the radius of curvature in the vertical plane, and the thrust, lift, drag, weight, and velocity are represented as $T, L, D, W,$ and V, respectively. These equations represent two-dimensional translational motion of a point-mass system, a common representation of air vehicles in the conceptual design phase. Disciplinary analysis traditionally focuses on estimating the "forces of flight" as distinct phenomena. That is, conceptual or preliminary aircraft analysis provides estimates of lift and drag that are considered as independent from the thrust forces estimated via propulsive analysis. In later steps, performance analysis typically modifies Eq. (1) and Eq. (2) to consider particular point performance conditions or uses these equations to assess design mission performance. In these cases, estimates of the forces of flight are leveraged from upstream modeling and analysis efforts.

In the development of novel vehicle concepts and configurations, the fundamental layout-analysis-performance cycle is largely preserved. However, the nature of these concepts tends to dictate more nuanced disciplinary analysis, often characterized by the estimation of coupled dynamic effects. The transition toward design processes enriched with highly coupled analysis has been enabled by emerging MDAO methods. However, the consequence of this enrichment is a deviation from the traditional forces-of-flight formulation upon which the sizing and synthesis processes are based.

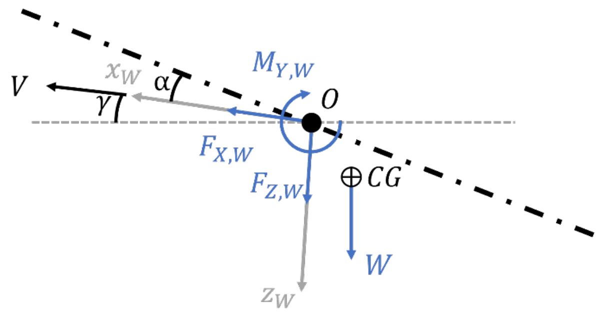

Figure 4. Point-mass representation free-body diagram.

To apply equations of motion of the form of Eq. (1) and Eq. (2) for highly coupled forces, one must implement a bookkeeping scheme that separates coupled forces into the traditional force representation. This process is likely to produce some degree of representation error, which may be on the order of 10%. While this error may often be acceptable within some aspects of conceptual design studies, it can lead to spurious conclusions when assessing the relative benefits of proposed configurations in terms of overall fuel burn.

The OWN mission and trajectory analysis addressed some of these sources of error within the design process by reformulating the basis of performance analysis to leverage a more general kinematic formulation of the equations of motion. With this approach, one can represent a wider variety of force and moment representations, which can include both traditional force-of-flight approaches or the results of highly coupled methods. Let us consider the point-mass system shown in Figure 1, which represents notional forces on a vehicle in the wind-axis reference frame along the X-Z plane. The translational equations of motion for this system are provided as Eq. (3) and Eq. (4), which pose external forces on the vehicle in general kinematic terms with no specific distinction or delineation of disciplinary forces.

$$F_{x,W} - mg\sin\gamma = m\frac{dV}{dt} \tag{3}$$

$$F_{z,W} + mg\cos\gamma = -mV\frac{d\gamma}{dt} \tag{4}$$

Because the kinematic formulation is a more general expression of rigid body equations of motion, more specific expressions of external forces can be readily incorporated in Eq. (3) and Eq. (4). For instance, traditional aerodynamic forces may be expressed as follows:

$$F_{x,aero} = -D\,, \quad F_{z,aero} = -L \tag{5}$$

Similarly, traditional propulsive forces may be expressed as follows:

$$F_{x,prop} = T\cos\epsilon\,, \quad F_{z,prop} = T\sin\epsilon \tag{6}$$

Summation of the forces expressed in Eq. (5) and Eq. (6) along each axis and substitution in Eq. (3) and Eq. (4) directly yield the traditional equations of motion expressed as Eq. (1) and Eq. (2).

In addition to supporting traditional forces of flight, a more general kinematic formulation of the equations of motion provides a convenient interface for coupled forces common to novel aircraft concepts. The resultant forces that arise from coupled analyses need only be resolved into the appropriate reference frame. Thus, this approach avoids additional representation errors that may arise from bookkeeping coupled forces in an intermediate disciplinary representation.

The benefits of utilizing a more general kinematic formulation of the equations extend beyond their utility for accommodating diverse external forces. A similar treatment of the equations of motion leveraged by Chakraborty and Mishra (2021) eschewed the traditional forces-of-flight formulation in favor of a kinematic-based expression of the equations in order to synthesize a variety of vehicles in disparate classes, including fixed-wing general aviation aircraft, electric vertical takeoff and landing vehicles, and rigid airships. These various vehicle categories share a common performance analysis formulation, which by nature enables the consideration of diverse external forces that may diverge from the traditional forces of flight.

Within an MDAO framework such as NASA's OpenMDAO, the proposed approach provides the basis for a flexible performance analysis. Kinematic equations of motion may be utilized with minimal modification to upstream analyses. For instance, the existing aerodynamic analysis, which provides estimates of lift and drag throughout the flight envelope, may be interfaced with the kinematic equations of motion by implementing Eq. (5), requiring little to no modification of existing methods or techniques.

We used OpenMDAO as well as NASA's Dymos optimal controls package to optimize trajectories for climb and cruise. In the spirit of controlled experimentation, the OWN and UWN aircraft began takeoff with the same gross weight and fuel. Note that a fully converged mission analysis for a target range would involve iterative solution of a boundary value problem, and the converged takeoff gross weight would be an output. Instead, we deliberately did not enforce this closure and simply allowed the aircraft to cruise to a fixed distance (no descent or landing modeled).

Our preliminary results reveal a need for further research. In particular, the trajectory optimization is highly under-constrained such that we find many solutions with unrealistic trajectories, including wildly oscillating throttle/power code settings or angles of attack over time. Further, if we impose physically realistic yet arbitrary constraints (such as limiting the derivatives of control variables), the resulting fuel burn is highly sensitive to our arbitrary settings. Simply put, the fuel burn for a single aircraft concept varies so greatly with these arbitrary constraints that the variation likely exceeds the difference between OWN and UWN designs. We will address this challenge in ongoing 2023 work.

Milestone(s)
Initial theory and software development

Major Accomplishments
- Theoretical development of a more general drag- and thrust-free mission and trajectory analysis method
- Initial implementation in OpenMDAO and Dymos software

Publications
None.

Outreach Efforts
Met and collaborated with the NASA GASPy and OpenMDAO development team.

Awards
None.

Student Involvement
M.S. students Anish Vegesna, Marc Koerschner, James Van der Linden, and Sam Crawford contributed to system/mission analysis formulation.

References

Capristan, F.M., Caldwell, D., Condotta, R., Petty, B. (2020). *Aircraft Analysis Using the Layered and Extensible Aircraft Performance System* (Report No. TM-2020-220558). NASA Scientific and Technical Information Program.

Marfatia, K. J., & Bergeson, J. D. (2021). *Aircraft flaps modeling in openMDAO.* https://ntrs.nasa.gov/citations/20210019908

Drela, M. (2016). *TASOPT 2.16 Transport Aircraft System Optimization*. Cambridge, MA.

MacDonald, T., Clarke, M., Botero, E. M., Vegh, J. M., & Alonso, J. J. (2017). SUAVE: an open-source environment enabling multi-fidelity vehicle optimization. In 18th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference (p. 4437).

Druot, T. Y., Belleville, M., Roches, P., Gallard, F., Peteilh, N., & Gazaix, A. (2019, June 17). A multidisciplinary airplane research integrated library with applications to partial turboelectric propulsion. *AIAA Aviation 2019 Forum*. AIAA Aviation 2019 Forum, Dallas, Texas. https://doi.org/10.2514/6.2019-3243

Salas Nunez, L., Tai, J. C., & Mavris, D. N. (2021, January 11). The environmental design space: Modeling and performance updates. *AIAA Scitech 2021 Forum*. AIAA Scitech 2021 Forum, VIRTUAL EVENT. https://doi.org/10.2514/6.2021-1422

Anderson, J. D. (2012). *Introduction to flight* (7th ed). McGraw Hill.

Roskam, J. (2005). *Airplane design*. DARcorporation.

Raymer, D. P. (2006). *Aircraft design: A conceptual approach* (4th ed). American Institute of Aeronautics and Astronautics.

Mattingly, J. D., Heiser, W. H., & Pratt, D. T. (2002). *Aircraft Engine Design* (2nd ed). American Institute of Aeronautics and Astronautics.

Chakraborty, I., & Mishra, A. A. (2021). Generalized energy-based flight vehicle sizing and performance analysis methodology. *Journal of Aircraft*, *58*(4), 762–780. https://doi.org/10.2514/1.C036101