Repeated games - Extensions

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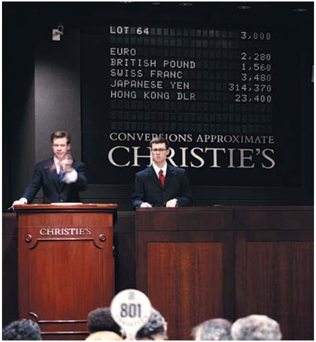
Repeated games - Extensions

- **Temporary punishment:**
  - Temporary (rather than permanent) reversion to the psNE of the unrepeated game.
- **Nasty punishments:**
  - How to sustain cooperation by the design of punishments in which players obtain a lower payoff than in the psNE of the unrepeated game.
- **More equitable punishments:**
  - The cheated party obtains a lower average payoff in the standard GTS than the cheater. We will fix that with a modified GTS.
- **Imperfect monitoring:**
  - Players may not observe cheaters instantaneously.

Temporary punishment

- Two main auction houses in the world of fine art: Sotheby's and Christie's
- Their competition is in the commission they charge.



Temporary punishment

- The following figure illustrates their competition, with payoffs in millions of dollars.

		Sotheby's 4%	6%	8%
Christie's	*4%*	0, 0	2, 1	4, -1
	6%	1, 2	4, 4	7, 1
	8%	-1, 4	1, 7	5, 5

- Unique psNE in the unrepeated game: $(6\%, 6\%)$ with equilibrium profits $(4, 4)$

Temporary punishment

- But, can these two auction houses increase their profits?

		Sotheby's 4%	*Sotheby's* 6%	*Sotheby's* 8%
Christie's	*4%*	0, 0	2, $\underline{1}$	4, -1
	6%	$\underline{1}$, 2	$\underline{4}$, $\underline{4}$	$\underline{7}$, 1
	8%	-1, 4	1, $\underline{7}$	5, 5

- Sure! If they coordinate towards the cooperative outcome $(8\%, 8\%)$ their profits become $(5, 5)$.
  - Is this legal? No. Did it occur? Yes, over the course of 7 years.
  - Were they caught? Yes, Sotheby's chairman went to jail and his company had to pay $7.5 million in fines.

Temporary punishment

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Charity Seel $47.5 Million fo Building

LAW

Ex-Sotheby's Chairman Is Sentenced To Year in Prison, Fined $7.5 Million

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By JERRY MARKON Staff Reporter of The Wall Street Journal
Updated April 23, 2002 12:01 a.m. ET

NEW YORK -- A. Alfred Taubman was sentenced to a year and a day in prison and fined $7.5 million by a federal judge who criticized the former Sotheby's Holdings Inc. BID -0.13% chairman for his lack of contrition for his role in a price-fixing scheme that rocked the art world.

While praising Mr. Taubman for his charitable work, Judge George Daniels called him a leader in the conspiracy to fix fees charged to auction clients in the 1990s. And the judge said that, by refusing to acknowledge responsibility or show remorse, Mr. Taubman, who remains Sotheby's controlling shareholder, "has continued the conspiracy of denial."

Popular Now What's This?

Temporary punishment

- In order to understand how this price-fixing can be sustained, let's first examine a standard **GTS in this setting:**
  - In period $t = 1$, cooperate by charging 8%.
  - In period $t > 1$,
    - charge 8% if both auction houses charged 8% in previous periods, or
    - charge 6% if one or both auction houses did not charge 8% in all previous periods.
  - (Note that charging 6% is the usual reversion towards the psNE of the unrepeated game that we discussed in other games.)

Temporary punishment

- At any given time period t, for which all players cooperated in all precious rounds, the profits of any auction house *i* from charging 8% are

$$5 + \delta 5 + \delta^2 5 + \ldots = \frac{5}{1-\delta}$$

- If, instead, it unilaterally denotes towards its most profitable deviation (charging 6% yields a payoff of 7, while changing 4% only yields a payoff of 4), 6% its profits become

$$\underbrace{7}_{\text{current gain}} + \delta 4 + \delta^2 4 + \ldots = 7 + \frac{\delta}{1-\delta} 4$$

Temporary punishment

- Hence, for each auction house to have incentives to cooperate in the collusive agreement (charging 8%) in the SPNE of the infinitely repeated game, we need that

$$\frac{5}{1-\delta} \geq 7 + \frac{\delta}{1-\delta}4$$

Multiplying both sides by $(1-\delta)$, yields

$$5 \geq 7(1-\delta) + \delta 4$$

and solving for δ, we have

$$5 \geq 7 - 7\delta + 4\delta \implies \delta \geq \frac{2}{3}$$

Temporary punishment

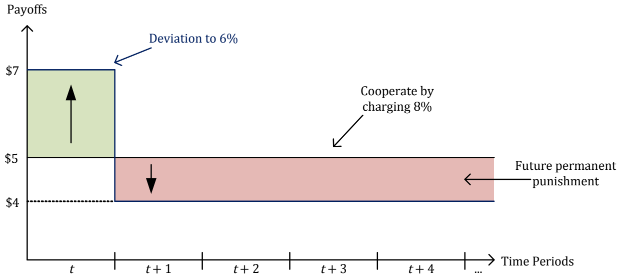
- Why don't we modify the usual grim-trigger strategy in order to have a reversion to moderate rates (6%, which is the psNE of the stage game) during *just a few periods*? **Temporary reversion**.

Temporary punishment

- **Temporary reversion in the Grim-Trigger strategy:**
  1. In period $t = 1$: choose 8% (cooperative outcome).
  2. In period $t > 1$: choose 8% as long as both auction houses charged 8%, or
     - choose 6% during *three* periods if one (or both) auction houses did not charge 8% in the previous period. Then return to 8%.
- You can, of course, consider other temporary reversions for more than 3 periods. For less than three periods the GTS might not be able to sustain cooperation (check, as a practice).

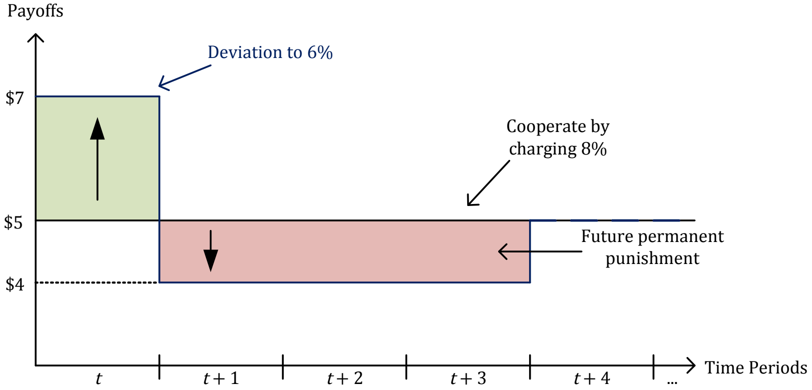
Christie's and Sotheby's Game

- Permanent Reversion:



Christie's and Sotheby's Game

- Temporary Reversion:



Christie's and Sotheby's Game

- Let's redo the previous example with **temporary reversion**:
- At any period t at which all players cooperated in all previous rounds, the profits from charging 8% are:

$$5 + \delta 5 + \delta^2 5 + \delta^3 5 + \delta^4 5 + \delta^5 5 + \ldots$$

- In contrast, the profits from deviating towards the "best deviation" of 6% are:

$$\underbrace{7}_{\text{Immediate profit from cheating}} + \underbrace{\delta 4 + \delta^2 4 + \delta^3 4}_{\text{Temporary reversion to 6\% by other firm (and me!)}} + \underbrace{\delta^4 5 + \delta^5 5}_{\text{Return to cooperation, 8\%}} + \ldots$$

Christie's and Sotheby's Game

- Hence, a firm has incentives to stick to this modified GTS with temporary reversion if:

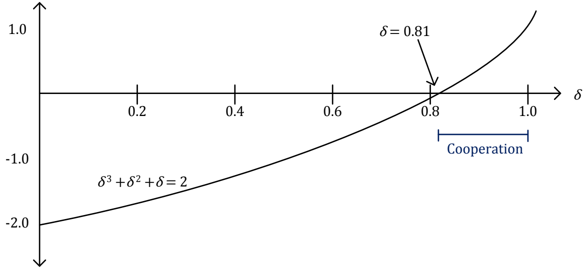
$$
\begin{aligned}
& 5 + \delta 5 + \delta^2 5 + \delta^3 5 + \cancel{\delta^4 5 + \delta^5 5 + \ldots} \\
\geq \quad & 7 + \delta 4 + \delta^2 4 + \delta^3 4 + \cancel{\delta^4 5 + \delta^5 5 + \ldots}
\end{aligned}
$$

Rearranging,

$$
\begin{aligned}
\Longrightarrow \quad & (5-4)\delta + (5-4)\delta^2 + (5-4)\delta^3 \geq 7 - 5 \\
\Longleftrightarrow \quad & \delta + \delta^2 + \delta^3 \geq 2 \implies \delta \geq 0.81
\end{aligned}
$$

Temporary reversion:

- Plot of equation $\delta^3 + \delta^2 + \delta = 2$, for any discount factor $\delta \in (0, 1)$.



- Its only root in the range of admissible $\delta \in (0, 1)$ is $\delta = 0.81$.

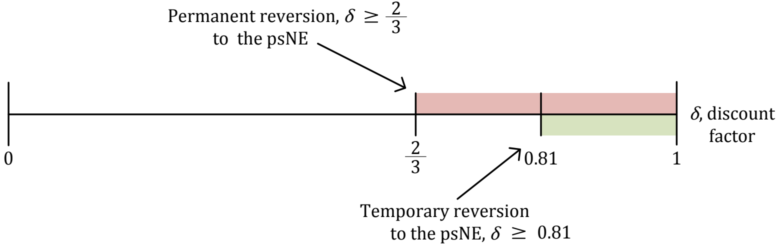
Comparing Permanent and Temporary reversion:

- **Comparison**:

1. Under *Permanent reversion* (charging 6% forever if cheating is detected), cooperation can be supported if $\delta \geq \frac{2}{3} \simeq 0.67$.
2. Under *Temporary reversion* (charging 6% during three periods if cheating is detected), cooperation can be supported only if players are more about future payoffs, i.e., $\delta \geq 0.81$.
3. **Intuition:**
   1. Cheating has a weaker punishment under temporal reversion (payoff of 4 during three periods) than under permanent reversion (payoff of 4 forever).
   2. As a consequence, players have to value future payoffs a lot in order for them to not be tempted to deviate from the cooperative outcome (8%).

Comparing Permanent and Temporary reversion:

- Range of discount factors supporting cooperation under permanent and temporary reversion.



What if we temporally revert to something worse than the NE of the stage game?

- Can cooperation be more easily sustained?
- Consider the following modified grim-trigger strategy:

1. In period $t = 1$: charge 8% (cooperative outcome).
2. In period $t > 1$: charge 8% if either:
   1. Both auction houses charged 8% in the previous period, or
   2. Both auction houses charged 4% in the previous period.
   3. Otherwise, charge 4%.
3. Intuition... (next slide)

What if we temporally revert to something worse than the NE of the stage game?

Intuition:

1. Both auction houses start setting 8%, and if either house deviates from 8% then both auction houses revert to 4%.
    1. Finally, if and only if both auction houses charged 4% in the previous period, then they both return to 8%.
2. 8%$\longrightarrow$If one (or both) deviate$\longrightarrow$Then both set 4% (price war)$\longrightarrow$Then, both return to 8%.
3. Cheating is punished by starting a **price war** that lasts *only one period*.

What if we temporally revert to something worse than the NE of the stage game?

- At any period t at which we have been cooperating in all previous rounds, charging 8% yields:

$$5 + \delta 5 + \delta^2 5 + \delta^3 5 + \ldots$$

- In contrast, deviating towards the "best deviation" of 6% yields:

$$\underbrace{7}_{\substack{\text{Today's profit} \\ \text{from deviation}}} + \underbrace{\delta 0}_{\substack{\text{Price-war} \\ \text{(4\% by both firms)} \\ \text{"Stick"}}} + \underbrace{\delta^2 5 + \delta^3 5}_{\substack{\text{Returning} \\ \text{to 8\%} \\ \text{"Carrot"}}} + \ldots$$

What if we temporally revert to something worse than the NE of the stage game?

Sotheby's (columns) / *Christie's* (rows)

	4%	*6%*	*8%*
4%	0, 0		
6%			7, 1
8%			5, 5

Price-war

deviation

Return to cooperation

What if we temporally revert to something worse than the NE of the stage game?

- Hence, charging 8% can be supported if

$$5 + \delta 5 + \cancel{\delta^2 5 + \delta^3 5 + \ldots} \geq 7 + \delta 0 + \cancel{\delta^2 5 + \delta^3 5 + \ldots}$$

rearranging,

$$5 + 5\delta \geq 7 \implies 5\delta \geq 2 \implies \delta \geq \frac{2}{5}$$

- Finally, we must check that houses will go through with the threatened punishment:

1. If neither auction house charged 8%, nor charged 4%, then the prescribed rate is 4%,

$$\underbrace{0}_{\substack{\text{The other house}\\ \text{acts accordingly,}\\ \text{setting 4\%}}} + \underbrace{\delta 5 + \delta^2 5 + \delta^3 5}_{\substack{\text{Both houses}\\ \text{return to 8\%}}} + \ldots$$

2. The best deviation that this auction house can have to setting this price war of 4% is 6% (payoff of 1) rather than 8% (payoff of -1). The payoff from 6% is:

$$\underbrace{1}_{\substack{\text{Today's}\\ \text{profit}}} + \underbrace{\delta 0}_{\substack{\text{Price}\\ \text{war}}} + \underbrace{\delta^2 5 + \delta^3 5}_{\substack{\text{Both houses}\\ \text{return to 8\%}}} + \ldots$$

3. Comparing,

$$0 + \delta 5 + \cancel{\delta^2 5 + \delta^3 5 + \ldots} \geq 1 + \delta 0 + \cancel{\delta^2 5 + \delta^3 5 + \ldots}$$

$$\implies \delta 5 \geq 1 \implies \delta \geq \frac{1}{5}$$

What if we temporally revert to something worse than the NE of the stage game?

Therefore, this strategy profile can be supported as a SPNE of the infinitely repeated game when:

1. $\delta \geq \frac{2}{5}$, which ensures that an auction house wants to cooperate (charging 8%), and
2. $\delta \geq \frac{1}{5}$, which ensures that an auction house is willing to engage in a punishing price war when needed.

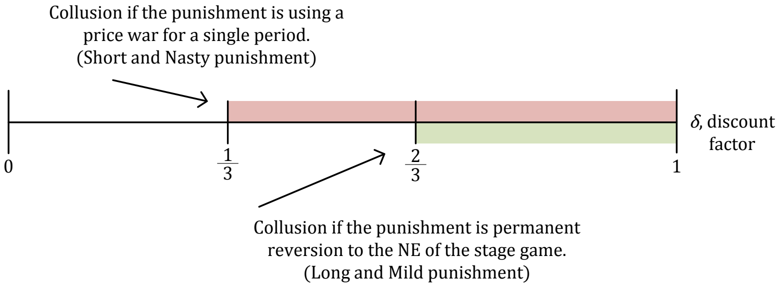
Both conditions hold if $\delta \geq \frac{2}{5}$.

Two important points:

1. During a price war, a house can raise its current profit from zero to 1 by setting a rate of 6% rather than 4%.
   - It is however induced to go along with the "stick" of the price war by the lure of the "carrot" of a high commission rate of 8% tomorrow.
2. Collusion is easier to sustain with the threat of a one-period price war than with the threat of reverting to regular competition *forever*.
   - Indeed, under price-war collusion can be supported if $\delta \geq \frac{2}{5}$, while...
   - under permanent reversion to the NE of the stage game (charging 6%), collusion is sustained if $\delta \geq \frac{2}{3} \left(> \frac{2}{5}\right)$.
3. **Intuitively:** a "short and nasty" punishment can then be more effective in order to sustain cooperation than a "long and mild" punishment.

Two important points:

- Discount factors supporting cooperation:



More equitable punishments:

1. The cheated party is victimized by the cheater.
   1. Indeed, the cheated party obtains a payoff of $1 during the cheating period...
   2. While the cheater obtains $7.
2. Let's search for a more equitable punishment, whereby the cheater must allow to be undercut by the other player for one period if he ever deviates.
3. *Example*:

1st) I cheat: choosing 6% rather than 8%
2nd) I allow to be undercut for one period.
3rd) We return to cooperation, 8%.$\longrightarrow$

More equitable punishments:

		Sotheby's 4%	Sotheby's 6%	Sotheby's 8%	
Christie's	4%	0, 0	2, $\underline{1}$	4, -1	
Christie's	6%	$\underline{1}$, 2	$\underline{4}$, $\underline{4}$	$\underline{7}$, 1	1st
Christie's	8%	-1, 4	1, $\underline{7}$	5, 5	
			2nd	3rd	

1. Then, cooperation can be sustained if

$$5+\delta 5+\cancel{\delta^2 5+\ldots}\geq \underbrace{7}_{\text{I cheat}}+\underbrace{\delta 1}_{\text{I am undercut}}+\underbrace{\cancel{\delta^2 5+\ldots}}_{\text{Back to coop.}}$$

2. Rearranging,

$$5+\delta 5\geq 7+\delta \implies \delta\geq\frac{1}{2}$$

More equitable punishments:

- In addition to the above condition, we must show that, after deviating, I will allow to be undercut before we go back to cooperation.
- That is,

$$\underbrace{1}_{\text{I am undercut}} + \underbrace{\delta 5 + \delta^2 5 + \ldots}_{\text{Back to coop.}} \geq \underbrace{4}_{\text{I stay at 6\%}} + \underbrace{\delta 1}_{\text{I am undercut}} + \underbrace{\delta^2 5 + \ldots}_{\text{Back to coop.}}$$

- **Legend**:
  1. When I allow myself to be undercut, I charge 8% while the "victimized" party charges 6% now.
     1. After being undercut, the GTS prescribes we go back to coop.
  2. Rather than allowing myself to be undercut, I could stay at 6%, obtaining $4.
     1. In that case, I am undercut the following period, and we go back to coop afterwards.

More equitable punishments:

- Simplifying the above expression:

$$1 + \delta 5 + \cancel{\delta^2 5 + \delta^3 5 + \ldots} \geq 4 + \delta 1 + \cancel{\delta^2 5 + \delta^3 5 + \ldots}$$

  Rearranging,

$$1 + \delta 5 \geq 4 + \delta 1$$

  which is more restrictive than our previous condition $(\delta \geq \frac{1}{2})$.
- Therefore, cooperation can be sustained under this punishment scheme if $\delta \geq \frac{3}{4}$.

More equitable punishments:

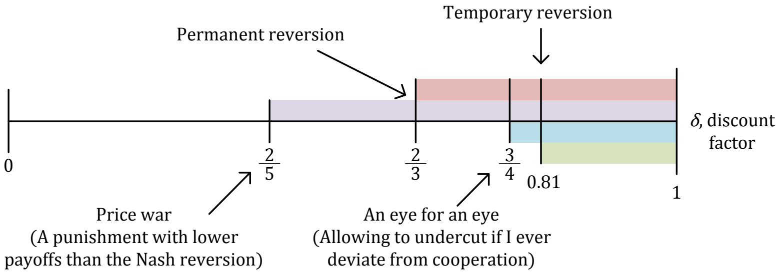
1. This type of punishment boils down to a simple (and old!) adage:

   "an eye for an eye"

2. Interesting:
   1. The punishment is more equitable, since the cheated party is compensated in the following period, and then cooperation is re-established.
   2. But it can only be supported if players assign relatively high values to future payoffs, i.e., $\delta \geq \frac{3}{4} \longleftarrow$ Because I must allow myself to be undercut after deviating.
3. Figure (next slide) $\longrightarrow$

More equitable punishments:

- Discount factors supporting cooperation:



Cooperation in distant periods

- So far all the applications we considered had players cooperating in the same period of time.
- However, in some instances, people take costly actions in order to benefit another individual (i.e., "kind" actions) even if the other individual does not reciprocate such "kind" action in the same period...
  - but at some unspecified point in the future.
- We refer to these cases in which *"I scratch your back and then you will scratch mine"* as "Quid pro Quo."

Cooperation in distant periods

- Two examples where Quid pro Quo emerges as part of the SPNE in the infinitely repeated game:
    - Pork-barrel spending in the Congress.
    - Vampire bats helping each other in a time of need by regurgitating blood, messy!

Application: Pork-barrel spending - "Quid Pro Quo" in Congress.

1. Three senators, A, B, C, proposing bills in the Congress during alternating periods.
   1. *Example*: Senator Stevens and the “bridge to nowhere” (Expected cost: $398 million, Population: 50).
   2. You can find more information here: http://en.wikipedia.org/wiki/Gravina_Island_Bridge.

Application: Pork-barrel spending - "Quid Pro Quo" in Congress.

2. **Payoffs:**
   1. if the bill you propose passes: \$100 (benefits for your district)
   2. if the bill that another senator proposes passes: -\$25 (taxes for your district, but no benefits)
   3. if no bill passes: \$0

Application: Pork-barrel spending

Time period:	$t=1$	2	3	4	5	6	...
Senator Proposing:	A	B	C	A	B	C	...

Proposing cycle

1. **GTS Strategy with temporary punishments**
    1. vote each other's projects (cooperation) as long as everybody votes for each others' projects...
    2. if someone deviates: vote "No" for his project, and then start cooperating again
    3. (This is a usual temporary reversion, but with punishments that last for only one period)

Application: Pork-barrel spending

- Let's analyze Senator B's incentives to stick to this cooperative strategy.
- We will need to separately examine his incentives to cooperate
- in periods 1, 4, 7, . . . (where A proposes)
- in periods 2, 5, 8, . . . (where B proposes)
- in periods 3, 6, 9, . . . (where C proposes)

Application: Pork-barrel spending

- Senator B, at period 1 (senator A is proposing):

$$
\begin{aligned}
&\overbrace{-25}^{A} + \overbrace{\delta 100}^{B} + \overbrace{\cancel{\delta^2(-25)}}^{C} + \overbrace{\cancel{\delta^3(-25)}}^{A} + \overbrace{\cancel{\delta^4 100}}^{B} + \overbrace{\cancel{\delta^5(-25)}}^{C} + \cancel{\ldots} \\
\geq\ &\underbrace{0}_{A} + \underbrace{\delta 0}_{B} + \underbrace{\cancel{\delta^2(-25)}}_{C} + \underbrace{\cancel{\delta^3(-25)}}_{A} + \underbrace{\cancel{\delta^4 100}}_{B} + \underbrace{\cancel{\delta^5(-25)}}_{C} + \cancel{\ldots}
\end{aligned}
$$

  where the first term on the right side of the inequality represents Senator B voting down Senator A's proposal, then the second term represents Senator B's proposal being voted down by Senators A and C in retaliation.

- Rearranging,

$$-25 + \delta 100 \geq 0, \text{ which implies } \delta \geq \frac{1}{4}$$

Application: Pork-barrel spending

- Senator B, at period 2 (he is proposing):

$$\overbrace{100}^{B}+\overbrace{\delta(-25)}^{C}+\overbrace{\cancel{\delta^2(-25)}}^{A}+\overbrace{\cancel{\delta^3 100}}^{B}+\cancel{\ldots}$$
$$\geq \underbrace{0}_{B}+\underbrace{\delta 0}_{\substack{C\\ \text{(Retaliation)}}}+\underbrace{\cancel{\delta^2(-25)}}_{A}+\underbrace{\cancel{\delta^3 100}}_{B}+\cancel{\ldots}$$

Rearranging,

$$100+\delta(-25)\geq 0, \text{ which implies } \delta < \frac{100}{25}=4$$

which holds by definition, since $\delta \in (0,1)$.

Application: Pork-barrel spending

- Senator B, at period 3 (when senator C proposes):

$$\underbrace{-25}_{C} + \underbrace{\delta(-25)}_{A} + \underbrace{\delta^2 100}_{B} + \underbrace{\cancel{\delta^2(-25)}}_{C} + \underbrace{\cancel{\delta^2(-25)}}_{A} + \cancel{\ldots}$$

$$\geq \underbrace{\underbrace{0}_{C} + \underbrace{\delta 0}_{A}}_{\text{(Optimal deviation)}} + \underbrace{\underbrace{\delta^2 0}_{B}}_{\text{(Retaliation)}} + \underbrace{\cancel{\delta^2(-25)}}_{C} + \underbrace{\cancel{\delta^2(-25)}}_{A} + \cancel{\ldots}$$

1. The optimal deviation is to vote "no" for two periods, when C and A propose. (Otherwise, senator B would be obtaining -25 rather than 0) Rearranging,

$$-25 - 25\delta + 100\delta \geq 0$$

solving for δ we obtain $\delta \geq 0.64$.

Application: Pork-barrel spending

1. We therefore found two conditions for cooperation: $\delta \geq 0.64$ and $\delta \geq \frac{1}{4}$.
2. Since $\delta \geq 0.64 > \frac{1}{4} = 0.25$, then $\delta \geq 0.64$ is the condition we need to support cooperation
    1. (More formally, it is a sufficient condition for cooperation).
3. Note that the condition on δ when you make proposals tomorrow ($\delta \geq \frac{1}{4}$), is less demanding (less stringent, i.e., lower values of δ) than
    1. when you will make proposals two periods from today ($\delta \geq 0.64$).
4. **Intuition**: Rewards are further away, As a consequence, I will only maintain our cooperation if I strongly care about future payoffs (higher δ).

Reputation

- Reputation is a valuable asset:
  - A good feedback score on eBay can increase your sales.
  - A reputation of hard work can increase the chances of getting a job.
  - A reputation for paying your debts on time may secure you a loan.
- But reputation is fragile...
  - A single day leaving work early can label you as a slacker...
  - A single unpaid debt can ruin your credit score.
- However, the fragility of reputation can provide the right incentives for people to behave properly.

Application: Lending to kings

1. In premodern Europe, taxation was not widespread and kings needed to borrow from private lenders significant amounts of money.
2. In some cases, a king reneged on his loans.
   1. He is the king, and can do whatever he wants!
   2. Why would he repay? He might need another loan in the future.
3. This situation is not just an example:
   1. Countries constantly default on their debt: Argentina, Russia, etc.
   2. Even some economists were suggesting it for the US now...
4. What's the cost of doing that? Ruining your reputation. Let's see how.

Application: Lending to kings

1. *King/Country*: needs to get a loan of \$100, with interest rate of 10%, repaying \$110.
    - Repay, or not repay?
    - He might need another loan in the future, with probability *b*.
2. *Lender:* payoff of \$110 if the king pays him back, 0 otherwise.
    1. "if this king ever fails from paying back his debt, I will never given him a loan again!"

Application: Lending to kings

Value (utility) from the loan to the king: = \$125

- \$110

\$15 Net Benefit

1. Payoff from *repaying* the loan:

$$-110 + \delta b15 + \delta^2 b15 + \ldots = -110 + \delta \frac{b15}{1-\delta}$$

Since the king has already benefited from the value of the load (\$125) and today: he is paying it back (-\$110).

Application: Lending to kings

2. Payoff from *not paying* the loan:

$$\overbrace{0}^{\text{No cost}} + \overbrace{\delta 0b + \delta^2 0b}^{\text{No loans ever again}} + ... = 0$$

3. Hence, the king should repay the loan if:

$$-110 + \delta \frac{b15}{1-\delta} \geq 0$$

Application: Lending to kings

1. Rearranging,

$$\begin{aligned} -110 + \delta\frac{b15}{1-\delta} &\geq 0 \implies -110(1-\delta) + \delta b15 \geq 0 \\ &\iff -110 + 110\delta + \delta b15 \geq 0 \end{aligned}$$

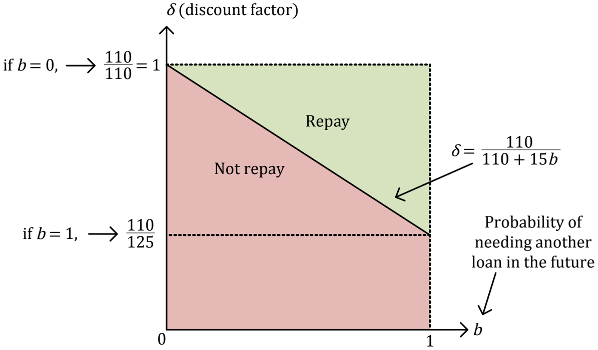
2. And solving for δ,

$$\delta(110 + 15b) \geq 110$$

$$\delta \geq \frac{110}{110 + 15b}$$

(Figure)$\longrightarrow$

Application: Lending to kings



Application: Lending to kings

1. **Intuition:**
   1. As the probability of needing another loan in the future, b, increases, the repayment of the current loan (cooperation) can be supported for a larger set of discount factors.
      - Graphically, the height above the line describing cutoff discount factor $\delta = \frac{110}{110+15b}$ increases.
   2. That is, reputation might be valuable in the future, and for this reason the king repays his loan today.

Imperfect Monitoring

- So far, every player could perfectly observe other players' actions.
  - e.g., I could observe if you stick or deviate from the agreement.
- But in several cases such monitoring is imperfect.
  - *Example*: Production levels in a cartel.
  - *Example*: Antiballistic Missile treaty between the US and USSR in 1972 (ABM treaty).
  - Every country can imperfectly observe each other's compliance (despite spies, satellites, etc.)

Imperfect Monitoring

- Nixon and Brezhnev signing the Antiballistic Missile treaty treaty in 1972

Imperfect Monitoring

- Before examining cooperation (No ABMs) in the infinitely repeated game...
  - we must specify how we introduce the fact that the ABM treaty cannot be monitored perfectly.

Imperfect Monitoring

- We introduce imperfect monitoring with the following probabilities

Number of ABMs	Probability of Detecting ABMs
None	0
Low	.10
High	.50

- How to read this table? As conditional probabilities:
  - If a country has no ABMs, then the prob. that my satellite detects ABMs is zero.
  - If a country has a low level of ABMs, then the prob. that my satellite detects ABMs is 10%.
  - If a country has a high level of ABMs, then the prob. that my satellite detects ABMs is 50%.

Imperfect Monitoring

- Let us first analyze the unrepeated game:

		USSR		
		No ABMs	*Low ABMs*	*High ABMs*
USA	*No ABMs*	10, 10	6, 12	0, $\underline{18}$
	Low ABMs	12, 6	8, 8	2, $\underline{14}$
	High ABMs	$\underline{18}$, 0	$\underline{14}$, 2	$\underline{3}$, $\underline{3}$

- Hence the unique psNE is (High,High).
  - However, (Low,Low) is more efficient, and (No,No) is the most efficient!
  - Can we cooperate playing (No,No) in the SPNE of the infinitely repeated game?

Imperfect Monitoring

1. **Antiballistic Missiles:** GTS
2. In period $t = 1$, choose No ABMs (i.e., cooperate).
3. In subsequent periods $t > 1$, choose:
    1. No ABMs if neither country has observed ABMs in other countries during previous periods, or
    2. High ABMs if either country has observed ABMs in other countries during previous periods.
        1. Reversion to the psNE of the unrepeated game thereafter.

Imperfect Monitoring

1. At any given period t, if no country has detected ABMs, the payoff from sticking to the agreement is:

$$10 + \delta 10 + \delta^2 10 + ... = \frac{10}{1-\delta}$$

Imperfect Monitoring

2. In contract, the payoff from deviating to **Low** ABMs during one period,

$$12 + \delta \left[\underbrace{0.1 \frac{3}{1-\delta}}_{\text{Detected}} + \underbrace{0.9 \frac{10}{1-\delta}}_{\text{Undetected}} \right]$$

3. And the payoff from deviating to **High** ABMs during one period,

$$18 + \delta \left[\underbrace{0.5 \frac{3}{1-\delta}}_{\text{Detected}} + \underbrace{0.5 \frac{10}{1-\delta}}_{\text{Undetected}} \right]$$

Imperfect Monitoring

1. Hence, we need Coop $\geq$ Low

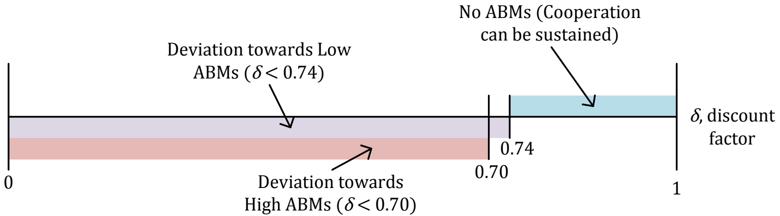
$$\frac{10}{1-\delta} \geq 12 + \delta\left[0.1\frac{3}{1-\delta} + 0.9\frac{10}{1-\delta}\right] \implies \delta \geq 0.74$$

and Coop $\geq$ High

$$\frac{10}{1-\delta} \geq 18 + \delta\left[0.5\frac{3}{1-\delta} + 0.5\frac{10}{1-\delta}\right] \implies \delta \geq 0.70$$

Figure $\longrightarrow$

Imperfect Monitoring



- We hence need $\delta \geq 0.74$ as a sufficient condition to support cooperation.

Imperfect Monitoring

1. *Practice exercise:*
   - Consider a technological improvement that increases the probability of detecting another country's ABMs. (Better monitoring)
   - This expands the set of discount factors for which cooperation can be sustained.

Number of ABMs	*Probability of Detecting ABMs*
None	0
Low	.30
High	.75

   - You can see the answer to this exercise Review session #7-8 in the EconS 424 website.

Cooperation in a public project

- Consider two individuals forming a partnership.
- Each player i exerts an amount of effort x_i, which benefits the partnership.
- Player i's utility function is

$$u_i(x_i, x_j) = x_j^2 + x_j - x_i x_j$$

and if $x_i = x_j = 0$, utility levels are $u_i = 0$ and $u_j = 0$.

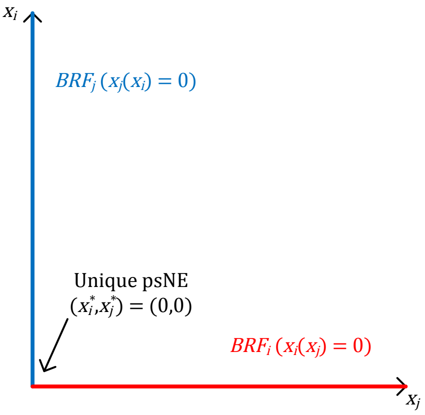
Cooperation in a public project

- Let's start analyzing the unrepeated game.
  - Using $u_i(x_i, x_j) = x_j^2 + x_j - x_i x_j$ and taking FOCs with respect to x_i, we obtain

    $$-x_j \leq 0$$

    which implies that BRF_i is $x_i(x_j) = 0$.
  - Similarly for individual j, $x_j(x_i) = 0$.
- How to depict these BRF? Figure (next slide).$\longrightarrow$

Cooperation in a public project



Cooperation in a public project

- **But, is this psNE efficient? No!**
- Players could select a symmetric strategy profile $x_i = x_j = k$ (both exert k units of effort), yielding

$$u_i(k, k) = k^2 + k - k * k = k$$

which exceeds the utility from playing the psNE (where utility is zero).

Cooperation in a public project

- How can we support this cooperative outcome $x_i = x_j = k$ in the infinitely-repeated game?
  - Using the following **GTS**:
  - In period $t = 1$, choose $x_i = k$ (i.e., cooperate).
  - In period $t > 1$, choose $x_i = k$ as long as both players selected $x_i = x_j = k$ in the past.
    - Otherwise, revert to the psNE $x_i = x_j = 0$ forever thereafter.

Cooperation in a public project

- **By cooperating...**

$$k + \delta k + \delta^2 k + \ldots = \frac{1}{1-\delta} k$$

- **By deviating...**
  - Wait! What's my most profitable deviation if the other player still selects $x_j = k$?
  - My utility level for any value of x_i, is $u_i(x_i, k) = k^2 + k - x_i k$.
  - Taking FOCs with respect to x_i, we obtain $-k < 0$, indicating that we are in a corner solution, i.e., $x_i(k) = 0$ (zero effort).
  - Therefore, my instantaneous gain from deviating to my best deviation (a zero effort) is

$$k^2 + k - 0k = k^2 + k$$

Cooperation in a public project

- **By deviating...**

$$\underbrace{k^2 + k}_{\text{Current Gain}} + \underbrace{\delta 0 + \delta^2 0 + \dots}_{\text{Future Punishment}} = k^2 + k$$

- Hence, I prefer to cooperate if

$$\frac{1}{1-\delta} k \geq k^2 + k$$

Cooperation in a public project

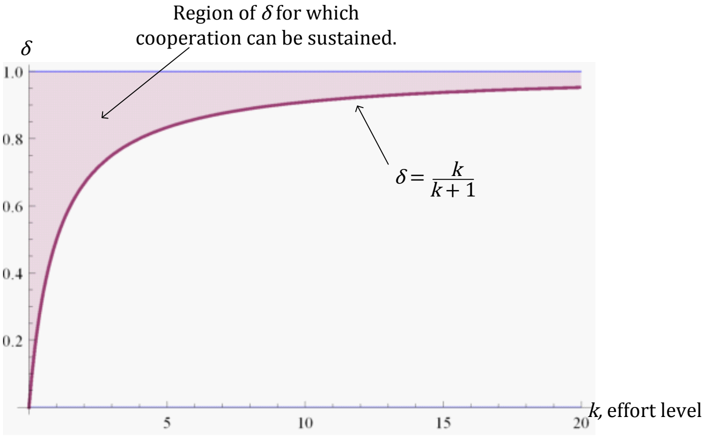
- Solving for δ,

$$\begin{aligned} k &\geq (1-\delta)k^2 + (1-\delta)k \\ \cancel{1} &\geq (1-\delta)k + \cancel{1} - \delta \\ \delta(k+1) &\geq k \\ \delta &\geq \frac{k}{k+1} \end{aligned}$$

- See figure (next slide).

Cooperation in a public project

- Minimal discount factor supporting cooperation in the infinitely repeated version of the public project game:



Cooperation in a public project

- Hence, the minimal cutoff δ is **increasing** in the effort level:
    - Intuitively, as individuals seek to coordinate on a larger effort level (higher k), their individual incentives to free-ride increase.
    - As a consequence, cooperation becomes more difficult to sustain.