Matrices:

For the following problems, solve the system of equations.

1. \(3y = 7x\)
   \(2x + 3y = 30\)

   Substituting equation 1 into equation 2,
   \[2x + \frac{7x}{3y} = 30\]

   Combining terms, we can solve for \(x\),
   \[9x = 30\]
   \[x^* = \frac{30}{9} = 3.33\]

   Lastly, rearranging the first equation, we can obtain \(y\),
   \[y^* = \frac{7}{3}x^* = 7.77\]

2. \(q_1 = 90 - \frac{q_2}{2}\)
   \(q_2 = 90 - \frac{q_1}{2}\)

   Substituting equation 1 into equation 2,
   \[q_2 = 90 - \frac{1}{2}(90 - \frac{q_2}{2})\]
   \[q_2 = 90 - 45 + \frac{q_2}{4}\]

   Combining terms, we can solve for \(q_2\),
   \[\frac{3q_2}{4} = 45\]
   \[q_2^* = 60\]

   Lastly, we plug in the value for \(q_2^*\) into the first equation to obtain \(q_1^*\)
   \[q_1^* = 90 - \frac{q_2^*}{2} = 60\]

3. \(4q_1 + q_2 + q_3 = 60\)
   \(q_1 + 4q_2 + q_3 = 60\)
   \(q_1 + q_2 + 4q_3 = 60\)

   Converting this into a matrix,
Performing a series of operations to achieve reduced row echelon form, First, we divide the first row by 4 to set the first element of the trace equal to 1,

\[
\begin{bmatrix}
4 & 1 & 1 & 60 \\
1 & 4 & 1 & 60 \\
1 & 1 & 4 & 60
\end{bmatrix}
\]

Next, we subtract row 1 from rows 2 and 3 to obtain zeroes in the remainder of the first column,

\[
\begin{bmatrix}
1 & 0.25 & 0.25 & 15 \\
0 & 3.75 & 0.75 & 45 \\
0 & 0.75 & 3.75 & 45
\end{bmatrix}
\]

Now we divide row 2 by 3.75 to obtain a 1 in the second element of the trace,

\[
\begin{bmatrix}
1 & 0.25 & 0.25 & 15 \\
0 & 1 & 0.2 & 12 \\
0 & 0.75 & 3.75 & 45
\end{bmatrix}
\]

Subtracting a proportion of row 2 from rows 1 and 3 to obtain zeroes in the rest of the second column,

\[
\begin{bmatrix}
1 & 0 & 0.2 & 12 \\
0 & 1 & 0.2 & 12 \\
0 & 0 & 3.6 & 36
\end{bmatrix}
\]

Finally, we divide the third row by 3 to obtain a 1 in the final element of the trace,

\[
\begin{bmatrix}
1 & 0 & 0.2 & 12 \\
0 & 1 & 0.2 & 12 \\
0 & 0 & 1 & 10
\end{bmatrix}
\]

Subtracting a proportion of row 3 from rows 1 and 2 gives us zeroes in the remainder of the third column,

\[
\begin{bmatrix}
1 & 0 & 0 & 10 \\
0 & 1 & 0 & 10 \\
0 & 0 & 1 & 10
\end{bmatrix}
\]

And our reduced row echelon form is complete. Reading from this matrix,

\[q_1^* = q_2^* = q_3^* = 10\]
For the following matrices, calculate the determinant.

4. \[
\begin{bmatrix}
1 & 2 \\
3 & 4
\end{bmatrix}
\]

Using the determinant formula,

\[Det = 1(4) - 2(3) = -2\]

5. \[
\begin{bmatrix}
1 & 4 \\
2 & 8
\end{bmatrix}
\]

Using the determinant formula,

\[Det = 1(8) - 4(2) = 0\]

6. \[
\begin{bmatrix}
1 & 2 & 3 \\
8 & 9 & 4 \\
7 & 6 & 5
\end{bmatrix}
\]

Using the determinant formula,

\[Det = 1(9)(5) + 2(4)(7) + 3(8)(6) - 1(4)(6) - 2(8)(5) - 3(9)(7) = -48\]

Invert the following matrices,

7. \[
\begin{bmatrix}
1 & 2 \\
3 & 4
\end{bmatrix}
\]

\[Inv = \begin{bmatrix}
-2 & 1 \\
1.5 & -0.5
\end{bmatrix}\]

8. \[
\begin{bmatrix}
1 & 2 & 3 \\
8 & 9 & 4 \\
7 & 6 & 5
\end{bmatrix}
\]

\[Inv = \begin{bmatrix}
-0.4375 & -0.1667 & 0.3958 \\
0.25 & 0.3333 & -0.4167 \\
0.3125 & -0.1667 & 0.1458
\end{bmatrix}\]