Matrices:

For the following problems, solve the system of equations.

1. $3y = 7x$
   $2x + 3y = 30$

   Substituting equation 1 into equation 2,

$$2x + \underbrace{7x}_{3y} = 30$$

   Combining terms, we can solve for x,

$$9x = 30$$
$$x^* = \frac{30}{9} = 3.33$$

   Lastly, rearranging the first equation, we can obtain y,

$$y^* = \frac{7}{3}x^* = 7.77$$

2. $q_1 = 90 - \frac{q_2}{2}$
   $q_2 = 90 - \frac{q_1}{2}$

   Substituting equation 1 into equation 2,

$$q_2 = 90 - \frac{1}{2}\underbrace{\left(90 - \frac{q_2}{2}\right)}_{q_1}$$
$$q_2 = 90 - 45 + \frac{q_2}{4}$$

   Combining terms, we can solve for q_2,

$$\frac{3q_2}{4} = 45$$
$$q_2^* = 60$$

   Lastly, we plug in the value for q_2^* into the first equation to obtain q_1^*

$$q_1^* = 90 - \frac{q_2^*}{2} = 60$$

3. $4q_1 + q_2 + q_3 = 60$
   $q_1 + 4q_2 + q_3 = 60$
   $q_1 + q_2 + 4q_3 = 60$

   Converting this into a matrix,

$$\begin{bmatrix} 4 & 1 & 1 & 60 \\ 1 & 4 & 1 & 60 \\ 1 & 1 & 4 & 60 \end{bmatrix}$$

Performing a series of operations to achieve reduced row echelon form, First, we divide the first row by 4 to set the first element of the trace equal to 1,

$$\begin{bmatrix} 1 & 0.25 & 0.25 & 15 \\ 1 & 4 & 1 & 60 \\ 1 & 1 & 4 & 60 \end{bmatrix}$$

Next, we subtract row 1 from rows 2 and 3 to obtain zeroes in the remainder of the first column,

$$\begin{bmatrix} 1 & 0.25 & 0.25 & 15 \\ 0 & 3.75 & 0.75 & 45 \\ 0 & 0.75 & 3.75 & 45 \end{bmatrix}$$

Now we divide row 2 by 3.75 to obtain a 1 in the second element of the trace,

$$\begin{bmatrix} 1 & 0.25 & 0.25 & 15 \\ 0 & 1 & 0.2 & 12 \\ 0 & 0.75 & 3.75 & 45 \end{bmatrix}$$

Subtracting a proportion of row 2 from rows 1 and 3 to obtain zeroes in the rest of the second column,

$$\begin{bmatrix} 1 & 0 & 0.2 & 12 \\ 0 & 1 & 0.2 & 12 \\ 0 & 0 & 3.6 & 36 \end{bmatrix}$$

Finally, we divide the third row by 3 to obtain a 1 in the final element of the trace,

$$\begin{bmatrix} 1 & 0 & 0.2 & 12 \\ 0 & 1 & 0.2 & 12 \\ 0 & 0 & 1 & 10 \end{bmatrix}$$

Subtracting a proportion of row 3 from rows 1 and 2 gives us zeroes in the remainder of the third column,

$$\begin{bmatrix} 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 1 & 10 \end{bmatrix}$$

And our reduced row echelon form is complete. Reading from this matrix,

$$q_1^* = q_2^* = q_3^* = 10$$

For the following matrices, calculate the determinant.

4. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Using the determinant formula,

$$Det = 1(4) - 2(3) = -2$$

5. $\begin{bmatrix} 1 & 4 \\ 2 & 8 \end{bmatrix}$

Using the determinant formula,

$$Det = 1(8) - 4(2) = 0$$

6. $\begin{bmatrix} 1 & 2 & 3 \\ 8 & 9 & 4 \\ 7 & 6 & 5 \end{bmatrix}$

Using the determinant formula,

$$Det = 1(9)(5) + 2(4)(7) + 3(8)(6) - 1(4)(6) - 2(8)(5) - 3(9)(7) = -48$$

Invert the following matrices,

7. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$$Inv = \begin{bmatrix} -2 & 1 \\ 1.5 & -0.5 \end{bmatrix}$$

8. $\begin{bmatrix} 1 & 2 & 3 \\ 8 & 9 & 4 \\ 7 & 6 & 5 \end{bmatrix}$

$$Inv = \begin{bmatrix} -0.4375 & -0.1667 & 0.3958 \\ 0.25 & 0.3333 & -0.4167 \\ 0.3125 & -0.1667 & 0.1458 \end{bmatrix}$$