Derivatives and Integrals:

For each utility function below, calculate the marginal rate of substitution.

1. $u(x,y) = x^{0.7}y^{0.3}$

First, we calculate marginal utilities by differentiating the utility function with respect to x and y,

$$MU_x = \frac{\partial u(x,y)}{\partial x} = 0.7x^{-0.3}y^{0.3}$$
$$MU_y = \frac{\partial u(x,y)}{\partial y} = 0.3x^{0.7}y^{-0.7}$$

Combining these into the marginal rate of substitution by using our formula,

$$MRS = -\frac{MU_x}{MU_y} = -\frac{0.7x^{-0.3}y^{0.3}}{0.3x^{0.7}y^{-0.7}}$$

and simplifying by combining exponents,

$$MRS = -\frac{3y}{7x}$$

2. $u(x,y) = 2x + 3y$

Once again, we calculate marginal utilities by differentiating the utility function with respect to x and y,

$$MU_x = \frac{\partial u(x,y)}{\partial x} = 2$$
$$MU_y = \frac{\partial u(x,y)}{\partial y} = 3$$

Combining these into the marginal rate of substitution,

$$MRS = -\frac{MU_x}{MU_y} = -\frac{2}{3}$$

Interestingly, this marginal utility does not depend on either x or y. This is due to our utility function expression a perfect substitutes relationship between our goods.

3. $u(x,y) = (x-2)^3(y-1)$

Calculating marginal utilities,

$$MU_x = \frac{\partial u(x,y)}{\partial x} = 3(x-2)(y-1)$$

$$MU_y = \frac{\partial u(x,y)}{\partial y} = (x-2)^2$$

Combining these into the marginal rate of substitution,

$$MRS = -\frac{MU_x}{MU_y} = -\frac{3(x-2)(y-1)}{(x-2)^2} = -\frac{3(y-1)}{(x-2)}$$

Differentiate the following functions.

4. $u(c) = \frac{1}{1-\theta}\left(c^{1-\theta} - 1\right)$

   This is simply an application of the power rule, just a little more complicated.

$$u'(c) = c^{-\theta}$$

5. $\pi(q) = (250 - q)q - 50q$

   This can be differentiated either using the power or product rules.

$$\pi'(q) = 200 - 2q$$

6. $TC(q) = 10 + 40q - 16q^2 + 20q^3$

   This is a straightforward application of the power rule.

$$TC'(q) = 40 - 32q + 60q^2$$

For the following problem, calculate the Consumer Surplus for the given inverse supply and demand functions:

7. Demand: $p = \frac{25}{q+1}$
   Supply: $p = q + 1$

   Since these are inverse supply and demand functions, we can calculate the area underneath the curve by integrating the inverse demand function minus the equilibrium price from $q = 0$ to $q = q^*$ (the equilibrium quantity). First, let's calculate the equilibrium price and quantity from the supply and demand curves,

$$\frac{25}{q+1} = q + 1$$
$$(q+1)^2 = 25$$
$$q + 1 = 5$$

$$q^* = 4$$

From the inverse supply curve, $p^* = q^* + 1 = 5$

Setting up our integral,

$$CS = \int_0^4 \left(\frac{25}{q+1} - 5\right) dq$$

Integrating,

$$CS = [25\log(q+1) - 5q]|_0^4$$

Plugging in our endpoints,

$$CS = 25\log(4+1) - 5(4) - [25\log(0+1) - 5(0)] = 20.24$$