Derivatives and Integrals:
For each utility function below, calculate the marginal rate of substitution.

1. \( u(x, y) = x^{0.7} y^{0.3} \)

First, we calculate marginal utilities by differentiating the utility function with respect to \( x \) and \( y \),

\[
MU_x = \frac{\partial u(x, y)}{\partial x} = 0.7x^{-0.3}y^{0.3} \\
MU_y = \frac{\partial u(x, y)}{\partial y} = 0.3x^{0.7}y^{-0.7}
\]

Combining these into the marginal rate of substitution by using our formula,

\[
MRS = -\frac{MU_x}{MU_y} = -\frac{0.7x^{-0.3}y^{0.3}}{0.3x^{0.7}y^{-0.7}}
\]

and simplifying by combining exponents,

\[
MRS = -\frac{3y}{7x}
\]

2. \( u(x, y) = 2x + 3y \)

Once again, we calculate marginal utilities by differentiating the utility function with respect to \( x \) and \( y \),

\[
MU_x = \frac{\partial u(x, y)}{\partial x} = 2 \\
MU_y = \frac{\partial u(x, y)}{\partial y} = 3
\]

Combining these into the marginal rate of substitution,

\[
MRS = -\frac{MU_x}{MU_y} = -\frac{2}{3}
\]

Interestingly, this marginal utility does not depend on either \( x \) or \( y \). This is due to our utility function expression a perfect substitutes relationship between our goods.

3. \( u(x, y) = (x - 2)^3(y - 1) \)

Calculating marginal utilities,

\[
MU_x = \frac{\partial u(x, y)}{\partial x} = 3(x - 2)(y - 1)
\]
\[ MU_y = \frac{\partial u(x,y)}{\partial y} = (x-2)^2 \]

Combining these into the marginal rate of substitution,

\[ MRS = -\frac{MU_x}{MU_y} = -\frac{3(x-2)(y-1)}{(x-2)^2} = -\frac{3(y-1)}{(x-2)} \]

Differentiate the following functions.

4. \[ u(c) = \frac{1}{1-\theta} \left(c^{1-\theta} - 1 \right) \]

This is simply an application of the power rule, just a little more complicated.

\[ u'(c) = c^{-\theta} \]

5. \[ \pi(q) = (250 - q)q - 50q \]

This can be differentiated either using the power or product rules.

\[ \pi'(q) = 200 - 2q \]

6. \[ TC(q) = 10 + 40q - 16q^2 + 20q^3 \]

This is a straightforward application of the power rule.

\[ TC'(q) = 40 - 32q + 60q^2 \]

For the following problem, calculate the Consumer Surplus for the given inverse supply and demand functions:

7. Demand: \[ p = \frac{25}{q+1} \]
   Supply: \[ p = q + 1 \]

Since these are inverse supply and demand functions, we can calculate the area underneath the curve by integrating the inverse demand function minus the equilibrium price from \( q = 0 \) to \( q = q^* \) (the equilibrium quantity). First, let’s calculate the equilibrium price and quantity from the supply and demand curves,

\[ \frac{25}{q+1} = q + 1 \]
\[ (q + 1)^2 = 25 \]
\[ q + 1 = 5 \]
\[ q^* = 4 \]

From the inverse supply curve, \( p^* = q^* + 1 = 5 \)

Setting up our integral,

\[ CS = \int_{0}^{4} \left( \frac{25}{q+1} - 5 \right) dq \]

Integrating,

\[ CS = \left[ 25 \log(q+1) - 5q \right]_0^4 \]

Plugging in our endpoints,

\[ CS = 25 \log(4+1) - 5(4) - [25 \log(0+1) - 5(0)] = 20.24 \]