Constraints

Basic Math for Economics – Refresher

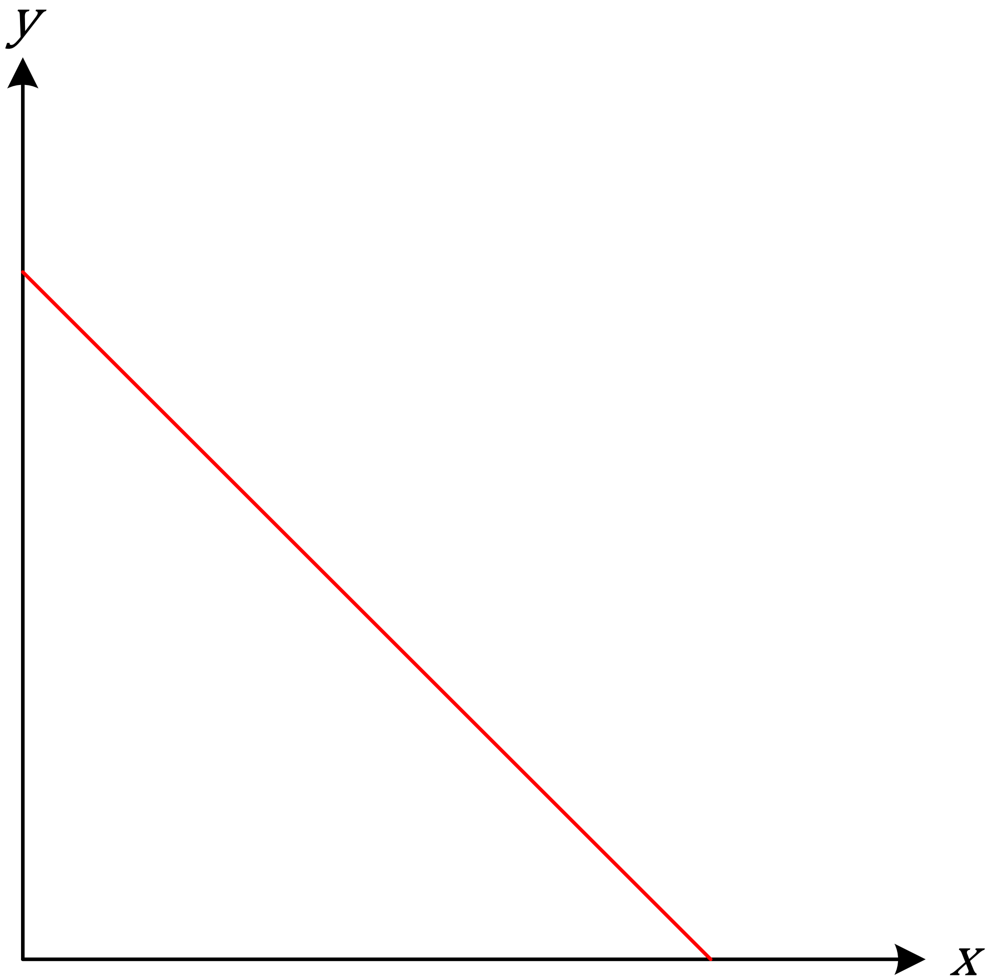
Introduction

- We all like nice things, but often we can not have everything that we want.
  - For example, if I gave someone a general utility function and asked them to maximize their utility with no restrictions, they would simply pick an infinite amount of both goods.
    - In the real world (and in almost every utility maximization problem), we deal with constraints, or limitations on what we can choose.
      - We need to incorporate these constraints into our optimization problems.

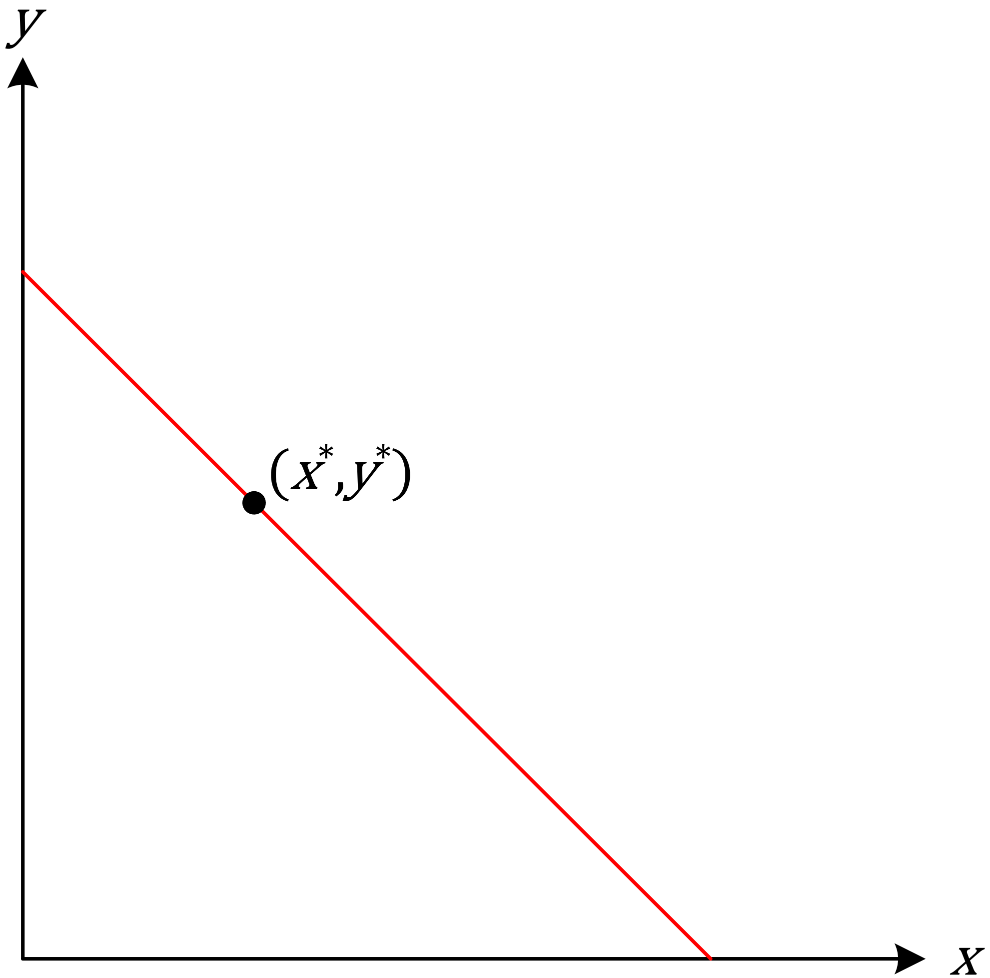
Constraints

- The problem with constraints is that they may not actually bind.
  - For example, suppose we have a budget constraint.
    - If the constraint binds, it means that we spend exactly as much money as we are allotted between the two goods.
    - On the other hand, there could exist a "bliss point" where a consumption level below the budget line maximizes that consumer's utility.
    - Using mathematical techniques, we can determine whether a constraint actually binds.

Constraints



Constraints



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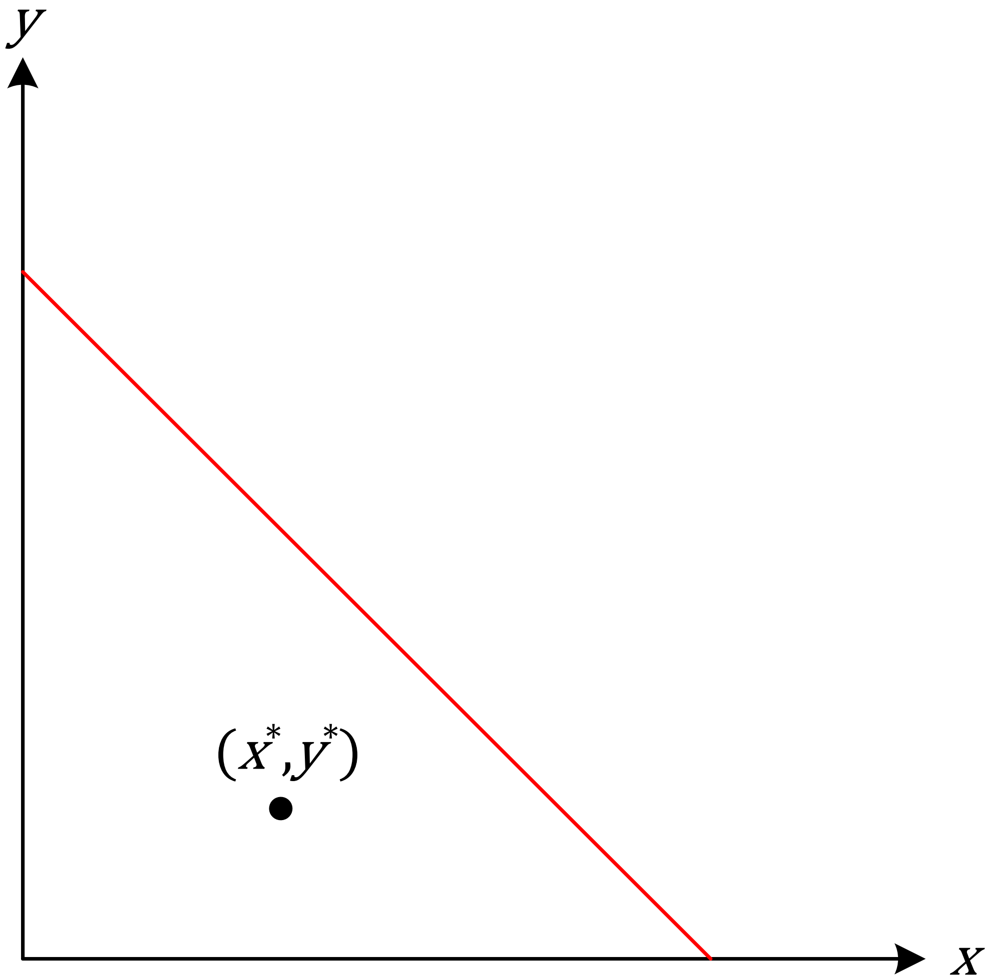

Constraints

- Suppose we were optimizing our consumption of goods x and y and dealing with the budget constraint

$$p_x x + p_y y \leq I$$

where p_x denotes the price of good x and p_y denotes the price of good y.

- To use this constraint properly in an optimization problem, we want to rearrange it such that it is greater than or equal to zero.
    - Rearranging, we have

$$I - p_x x - p_y y \geq 0$$

Constraints

$$I - p_x x - p_y y \geq 0$$

- Now, we incorporate this constraint into our problem through the use of a Lagrangian.
  - A Lagrangian simply takes our objective function, and adds in each constraint multiplied by their own Lagrange multiplier.
  - For a utility maximization problem with goods x and y, it would look like the following:

$$\max_{x,y,\lambda} u(x,y) + \lambda\left(I - p_x x - p_y y\right)$$

where λ denotes the Lagrange multiplier for our budget constraint.

Constraints

$$\max_{x,y,\lambda} u(x,y) + \lambda\left(I - p_x x - p_y y\right)$$

- The equilibrium value of λ is critical in explaining the role of the constraint in our optimization problem.
  - Essentially, the higher the equilibrium value of λ, the more the objective function is being restrained by the constraint.
    - Thus, if $\lambda^* > 0$, the constraint is binding, and we must constrain our optimization.
    - On the other hand, if $\lambda^* \leq 0$, the constraint does not bind and we can ignore it.

Constraints

- Otherwise, we calculate the equilibrium values from our optimization problem in the exact same way as before.
  - We just have an extra first-order condition to consider now.
    - We'll analyze this more thoroughly in the next lesson.
- This is a simplified explanation of what is known as Kuhn-Tucker conditions.
  - They are very powerful tools to explained constrained optimization in an economic context.
    - Naturally, some details have been left out for simplicity, but warrant further study.