Invoking Symmetry

Basic Math for Economics – Refresher

Introduction

- Another commonly used trick when analyzing economic situations is invoking symmetry.
  - Symmetry is when we face agents who have symmetric objective functions.
    - Essentially, if all the people making choices are the same, we can assume that their choices will also be the same.
  - This greatly eases our analysis.
    - That being said, symmetry must be invoked correctly.

Invoking Symmetry

- Suppose we had three firms that competed in a Cournot context and faced the following inverse demand function,

$$p = 100 - q_1 - q_2 - q_3$$

where q_1, q_2, and q_3 are firm 1, firm 2, and firm 3's, quantities, respectively.

- In addition, firms face a constant marginal cost of $MC = 20$.
  - Let's calculate the equilibrium output level for each firm.

Invoking Symmetry

- Setting up firm 1's profit maximization problem, we have,

$$\max_{q_1} \ (100 - q_1 - q_2 - q_3)q_1 - 20q_1$$

- Note that the profit maximization problems for firms 2 and 3 would look very similar (they're symmetric). I just switch q_1 and their respective quantity.

$$\max_{q_2} \ (100 - q_1 - q_2 - q_3)q_2 - 20q_2$$

$$\max_{q_3} \ (100 - q_1 - q_2 - q_3)q_3 - 20q_3$$

  - This is a requirement for invoking symmetry (the agents must, in fact, be symmetric).

Invoking Symmetry

- From here, I can calculate a first-order condition for each problem to obtain,

$$\frac{\partial \pi_1}{\partial q_1} = 100 - 2q_1 - q_2 - q_3 - 20 = 0$$
$$\frac{\partial \pi_2}{\partial q_2} = 100 - q_1 - 2q_2 - q_3 - 20 = 0$$
$$\frac{\partial \pi_3}{\partial q_3} = 100 - q_1 - q_2 - 2q_3 - 20 = 0$$

- Once again, if I switched all the subscripts around, these three equations could all rearrange into one another.
  - This is, however, a system of three equations and three unknowns. It would take a while to solve.

Invoking Symmetry

- Since the firms are identical, however, we know that their solutions should also be identical.
  - Intuitively, they are all going to produce the same quantity, so why not assume it in the first place?
- Suppose we let $q_1 = q_2 = q_3 = q$ *in equilibrium*. If I substitute this into any of the three first-order conditions we have, they collapse into a single equation.

$$100 - 2q - q - q - 20 = 0$$

$$80 - 4q = 0$$

  - This is just one equation and one unknown, and is much easier to solve.

Invoking Symmetry

$$80 - 4q = 0$$

- Solving this expression for q, we obtain $q^* = q_1^* = q_2^* = q_3^* = 20$, and we're all done!
- This is a very powerful technique that can save you a lot of time when analyzing this type of problem, but it has two **significant** requirements.
  - First, the agents (firms in this case) must have symmetric objective functions.
  - Second, don't invoke symmetry until **after** you calculate first-order conditions.
    - Doing it beforehand will give you a different solution!