Matrix Conformability

Basic Math for Economics – Refresher

Introduction

- For larger systems of equations, it's much more efficient to analyze them using a matrix.
  - To help with these situations, let's review some matrix operations.
- The first thing to recall about matrix algebra is that matrices must be conformable.
  - Recall that conformability involves matrices having the appropriate number of rows and columns depending on the desired operation.
    - The conformability requirements differ based on what we do to the matrices.

Conformability

- Our first conformability rule to consider is when we add two matrices together.
  - In this case, both matrices must have the same number of rows and columns.
    - Recall that rows go from left to right, whereas columns go from top to bottom.
    - For example, the follow matrix has 2 rows and 3 columns.

$$\begin{bmatrix} 1 & 6 & 2 \\ 4 & 3 & 5 \end{bmatrix}$$

Conformability

- If the matrices are additively conformable, we simply add each element of the matrices together to obtain our new matrix.
- Consider matrices A and B below,

$$A = \begin{bmatrix} 1 & 6 & 2 \\ 4 & 3 & 5 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 1 & 4 \\ 5 & 6 & 3 \end{bmatrix}$$

- Adding them together,

$$A + B = \begin{bmatrix} 1 & 6 & 2 \\ 4 & 3 & 5 \end{bmatrix} + \begin{bmatrix} 2 & 1 & 4 \\ 5 & 6 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 7 & 6 \\ 9 & 9 & 8 \end{bmatrix}$$

Conformability

- Suppose instead we had matrices C and D below,

$$C = \begin{bmatrix} 1 & 6 & 2 \\ 4 & 3 & 5 \end{bmatrix} \qquad D = \begin{bmatrix} 2 & 6 \\ 1 & 4 \\ 5 & 3 \end{bmatrix}$$

- In this case, we cannot add the matrices together because they are not conformable.
  - Matrix C has 2 rows and 3 columns, while matrix D has 3 rows and 2 columns.
    - Thus, matrices C and D are not additively conformable.

Conformability

- Now let's talk about multiplying matrices.
  - First of all, if we multiply a matrix by a scalar (a single number), we simply multiply each element of the matrix by the value of the scalar.
    - (recall that a scalar is just a number outside of a vector – most of the numbers we use are scalars.)
    - After all, you're just adding the matrix to itself a number of times equal to the scalar.
  - For example, suppose we multiplied matrix A by 3.

$$3A = 3 * \begin{bmatrix} 1 & 6 & 2 \\ 4 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 18 & 6 \\ 12 & 9 & 15 \end{bmatrix}$$

Conformability

- When a matrix is multiplied by another matrix, it's important to remember that **the order matters**.
  - This is in contrast to when we multiply with scalars.
  - For instance, multiplying matrix C by matrix D is not the same thing as multiplying matrix D by matrix C.
- In addition, the matrices must be multiplicatively conformable.
    - The number of columns in the first matrix must equal the number of rows in the second matrix.

Conformability

$$A = \begin{bmatrix} 1 & 6 & 2 \\ 4 & 3 & 5 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 1 & 4 \\ 5 & 6 & 3 \end{bmatrix}$$

- Going back to matrices A and B, recall that they were additively conformable.
  - They are not, however, multiplicatively conformable.
    - If we calculate AB (matrix A times matrix B), matrix A has 3 columns while matrix B has two rows.
      - Thus, they cannot be multiplied.
    - The same result holds if we calculate BA. Matrix B has 3 columns while matrix A has two rows.

Conformability

$$C = \begin{bmatrix} 1 & 6 & 2 \\ 4 & 3 & 5 \end{bmatrix} \qquad D = \begin{bmatrix} 2 & 6 \\ 1 & 4 \\ 5 & 3 \end{bmatrix}$$

- While matrices C and D were not additively conformable, they are multiplicatively conformable.
  - In fact, they are multiplicatively conformable in both directions.
    - If we calculate CD, matrix C has 3 columns while matrix D has 3 rows.
    - Likewise, if we calculate DC, matrix D has 2 columns while matrix C has 2 rows.

Conformability

$$C = \begin{bmatrix} 1 & 6 & 2 \\ 4 & 3 & 5 \end{bmatrix} \qquad D = \begin{bmatrix} 2 & 6 \\ 1 & 4 \\ 5 & 3 \end{bmatrix}$$

- Let's calculate CD.
  - We must calculate each element of the matrix individually.
    - To find the first row and first column of matrix CD, we multiply the elements of the first row of matrix C by the elements of the first column of matrix D, then add them together.
      - That's a big step. Let's walk through that one.

Conformability

$$C = \begin{bmatrix} 1 & 6 & 2 \\ 4 & 3 & 5 \end{bmatrix} \qquad D = \begin{bmatrix} 2 & 6 \\ 1 & 4 \\ 5 & 3 \end{bmatrix}$$

- Looking at the first row of matrix C and the first column of matrix D, we can multiply the elements together, then add them up, yielding,

$$1 * 2 + 6 * 1 + 2 * 5 = 18$$

- Thus, the element in the first row and first column of matrix CD is 18.
  - We can repeat this process to find all of the elements of matrix CD.

Conformability

$$C = \begin{bmatrix} 1 & 6 & 2 \\ 4 & 3 & 5 \end{bmatrix} \qquad D = \begin{bmatrix} 2 & 6 \\ 1 & 4 \\ 5 & 3 \end{bmatrix}$$

- To find the element in the second row and first column of matrix CD, we use the second row in matrix C and the first column in matrix D.
  - And so on. Repeat for each possible combination of rows and columns for the two matrices.
  - This leads us to matrix CD,

$$CD = \begin{bmatrix} 18 & 36 \\ 36 & 51 \end{bmatrix}$$

    - Note that the resulting matrix will contain the same number of rows as the first matrix and the same number of columns as the second.

Conformability

$$C = \begin{bmatrix} 1 & 6 & 2 \\ 4 & 3 & 5 \end{bmatrix} \qquad D = \begin{bmatrix} 2 & 6 \\ 1 & 4 \\ 5 & 3 \end{bmatrix}$$

- What if instead we calculated DC?
  - Now we use the rows from matrix D and the columns from matrix C. This yields,

$$DC = \begin{bmatrix} 26 & 30 & 34 \\ 17 & 18 & 22 \\ 17 & 39 & 25 \end{bmatrix}$$

    - Notice that not only did we obtain a different result from matrix CD, the matrices do not even have the same dimensions!

Conformability

- Lastly, we have inversion conformability.
  - For a matrix to be inversion conformable, it must be square.
    - A square matrix is simply a matrix with the same number of rows and columns.
  - We'll go over matrix inversion in a later lesson, however.
    - We need to talk about a few more things before we get there!