Systems of Equations

Basic Math for Economics – Refresher

Introduction

- Algebra is one of the critical skills for success in economics.
- We often work with systems of several equations and several unknowns.
  - We have a few different ways of organizing these types of problem.
    - We can use standard algebra to solve simple situations.
    - When situations are less simple, we must rely on matrix techniques.

Systems of Equations

- Let's examine some of the simple situations.
- Suppose we had a situation where you had calculated two best response functions for a Cournot duopoly of

$$q_1 = 90 - \frac{1}{2}q_2$$
$$q_2 = 90 - \frac{1}{2}q_1$$

where q_1 and q_2 represent the quantities for firms 1 and 2, respectively.

  - Solve this system of equations for q_1 and q_2.

Systems of Equations

$$q_1 = 90 - \frac{1}{2} q_2$$

$$q_2 = 90 - \frac{1}{2} q_1$$

- There are multiple ways to do this (Do whichever you are most comfortable with).
  - I am going to use the substitution technique.
- First, I substitute the first equation into the second for q_1,

$$q_2 = 90 - \frac{1}{2} \underbrace{\left(90 - \frac{1}{2} q_2 \right)}_{q_1}$$

Systems of Equations

$$q_2 = 90 - \frac{1}{2}\left(90 - \frac{1}{2}q_2\right)$$

- Next, we distribute through the parenthesis,

$$q_2 = 90 - 45 + \frac{1}{4}q_2$$

- Combining terms,

$$\frac{3}{4}q_2 = 45$$

- And multiplying both sides of this equation by $\frac{3}{4}$ leads to our solution,

$$q_2^* = 60$$

Systems of Equations

$$q_1 = 90 - \frac{1}{2}q_2$$

- Lastly, we plug in our result for q_2 into the equation for q_1 to obtain our other firm's quantity,

$$q_1^* = 90 - \frac{1}{2}(60) = 90 - 30 = 60$$

- We can extend this to any number of equations and unknowns.
  - As long as we have as many equations as we do unknowns (and the equations are appropriately identified), we can solve any system.
    - It's usually easier to use matrices beyond two equations and two unknowns, however.

Systems of Equations

- When we do not have the same number of equations and unknowns, certain problems can arise.
  - If there are more unknowns than equations, we have what is called an underidentification problem.
    - In these situations, there are usually an infinite number of solutions.
  - If there are more equations than unknowns, we have what is called an overidentification problem.
    - In these situations, there could be no solution.