What is a Derivative?

Basic Math for Economics – Refresher

Introduction

- One of the most common quantitative tools used in economics is the derivative.
  - It allows us to solve the vast majority of our economic models.
  - We use them everywhere; from utility maximization, to cost minimization, or anywhere the word marginal appears.
    - For those that haven't used these skills for a while, let's review how derivatives work and how they are applied in an economic context.

Derivatives

- A derivative measures the rate of change.
  - Specifically, it gives us the rate of change of a function for any given ***point*** in that function's domain.
    - (Recall that if we have y as a function of x, $y = f(x)$, the domain is all of the values that x can take, while the range is all of the values that y can take.)
  - This is important in economics as we find that our values change depending on where we start our measurements.
    - For example, without derivatives, we'll obtain different values for the price-elasticity of demand depending on which way the price changes.
      - Derivatives fix this problem.

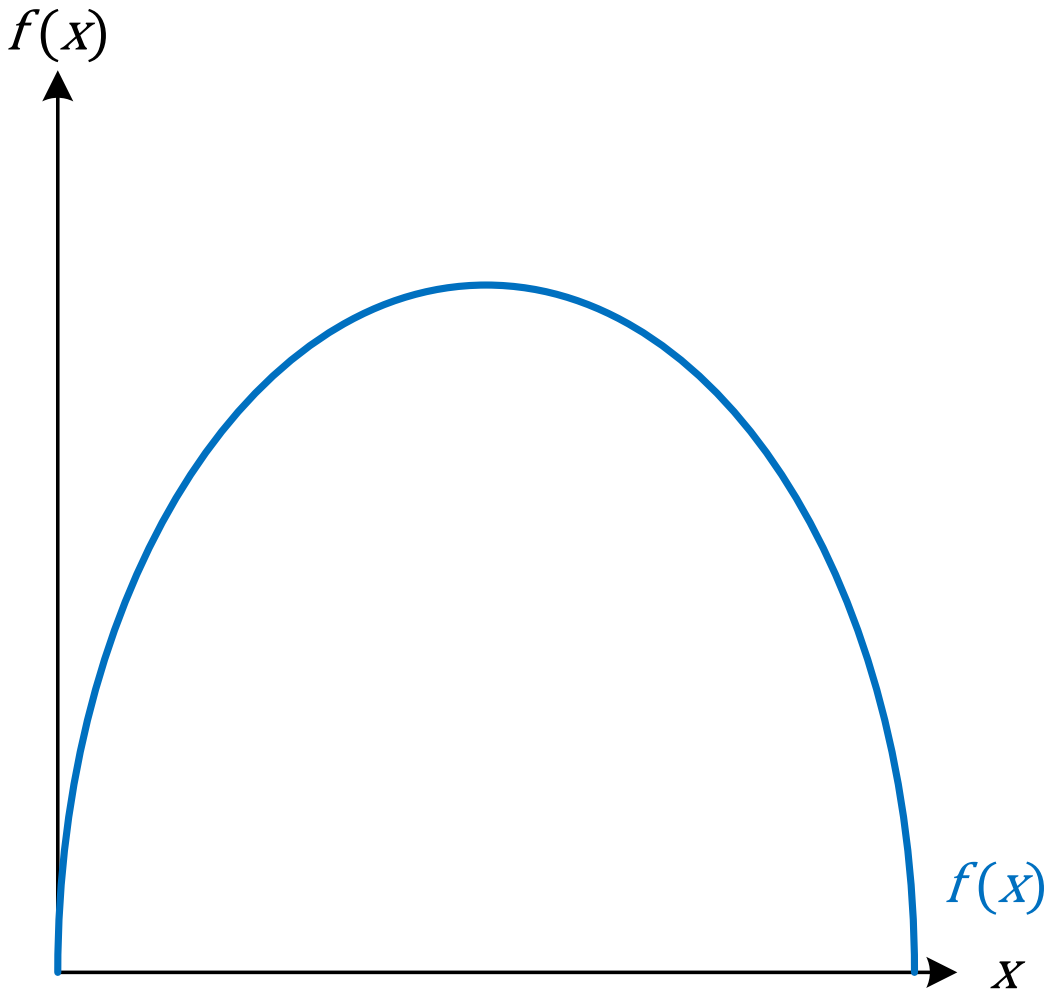
Derivatives

- Visually, we can see the rate of change (derivative) of a function by looking at its slope,

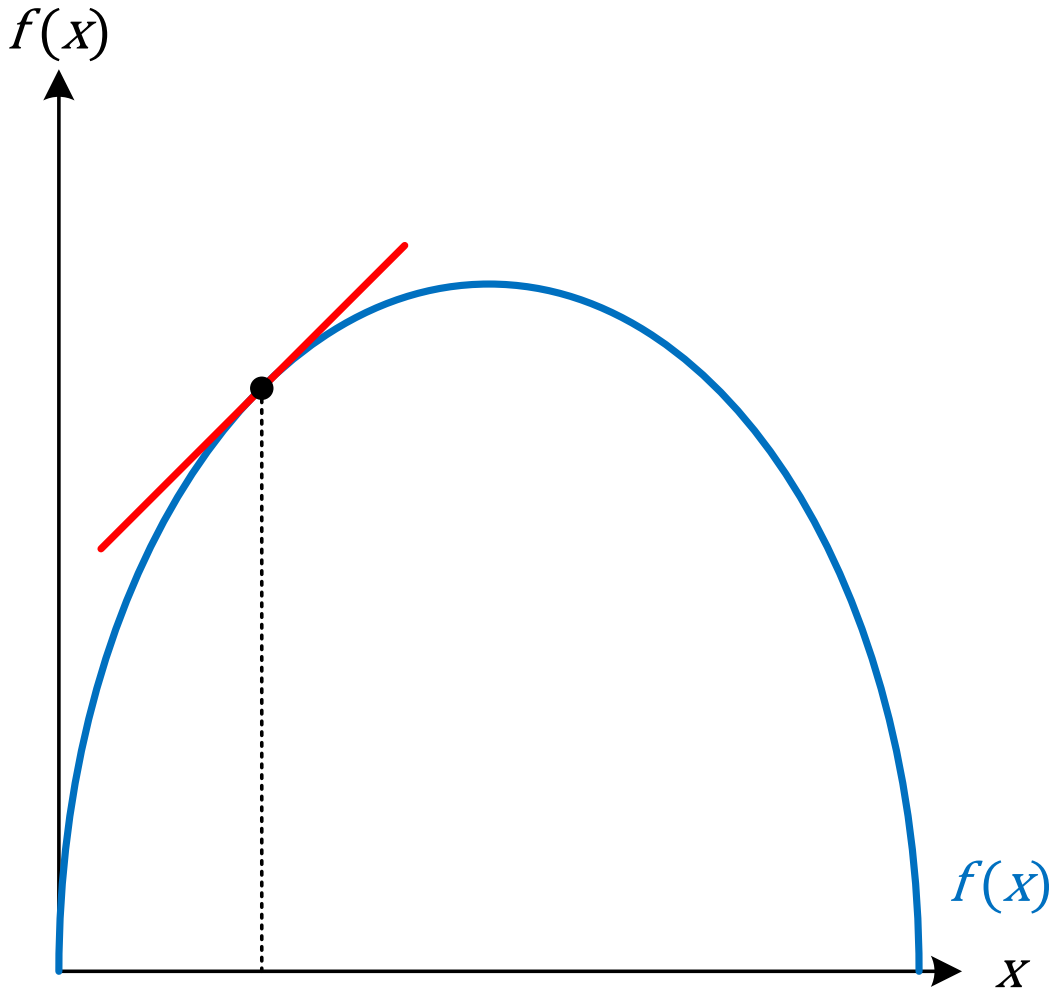
$$\text{Slope} = \frac{\Delta f(x)}{\Delta x}$$

- In this regard, the derivative of a function provides the slope of the line that is tangent to the function at point x.
  - (Recall that a tangency point is a point where two lines touch exactly once.)
    - We can connect the values of these slopes into a new function, $f'(x)$, which represents the derivative of x.

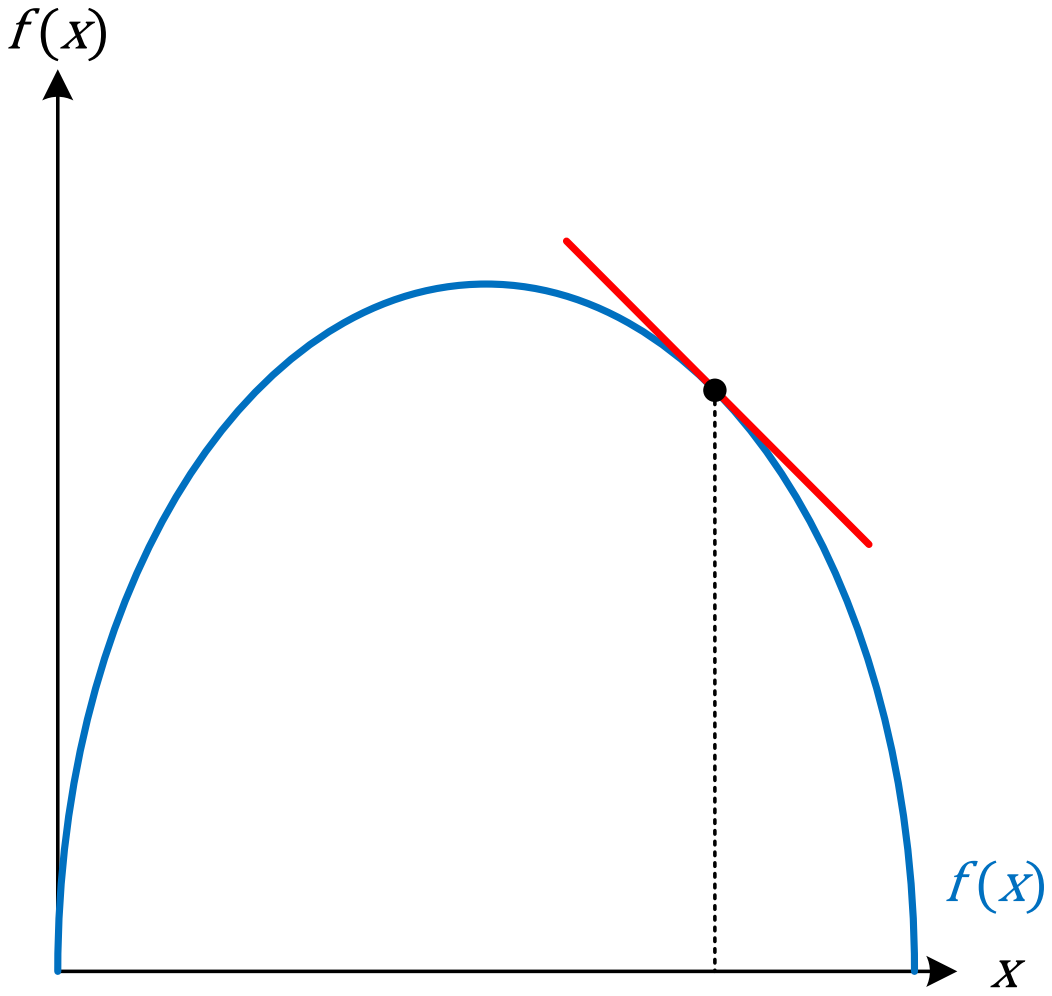
Derivatives



Derivatives



Derivatives



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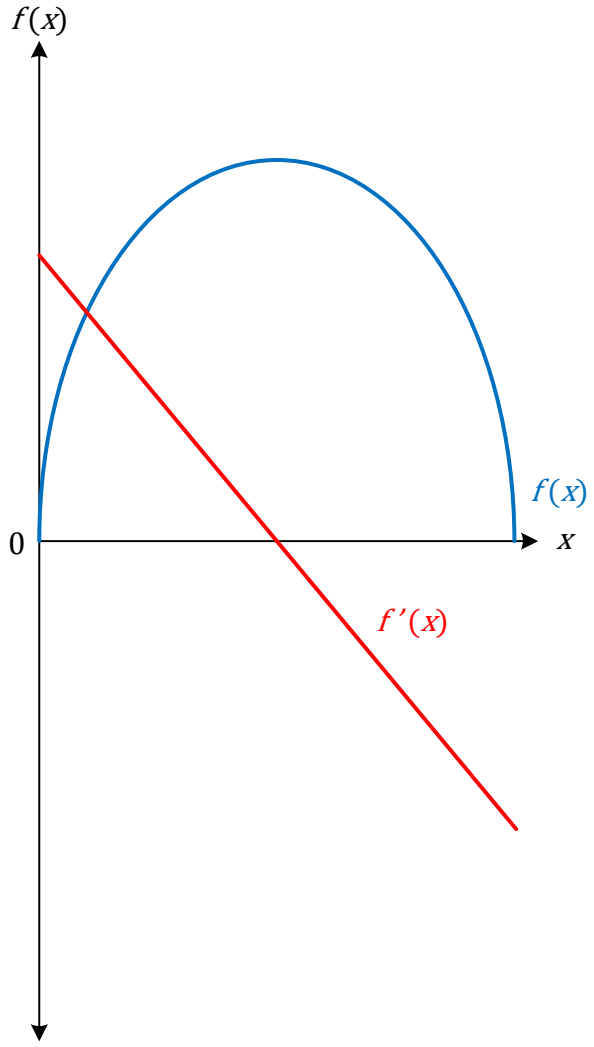


Derivatives

- Mathematically, the formula for a derivative is

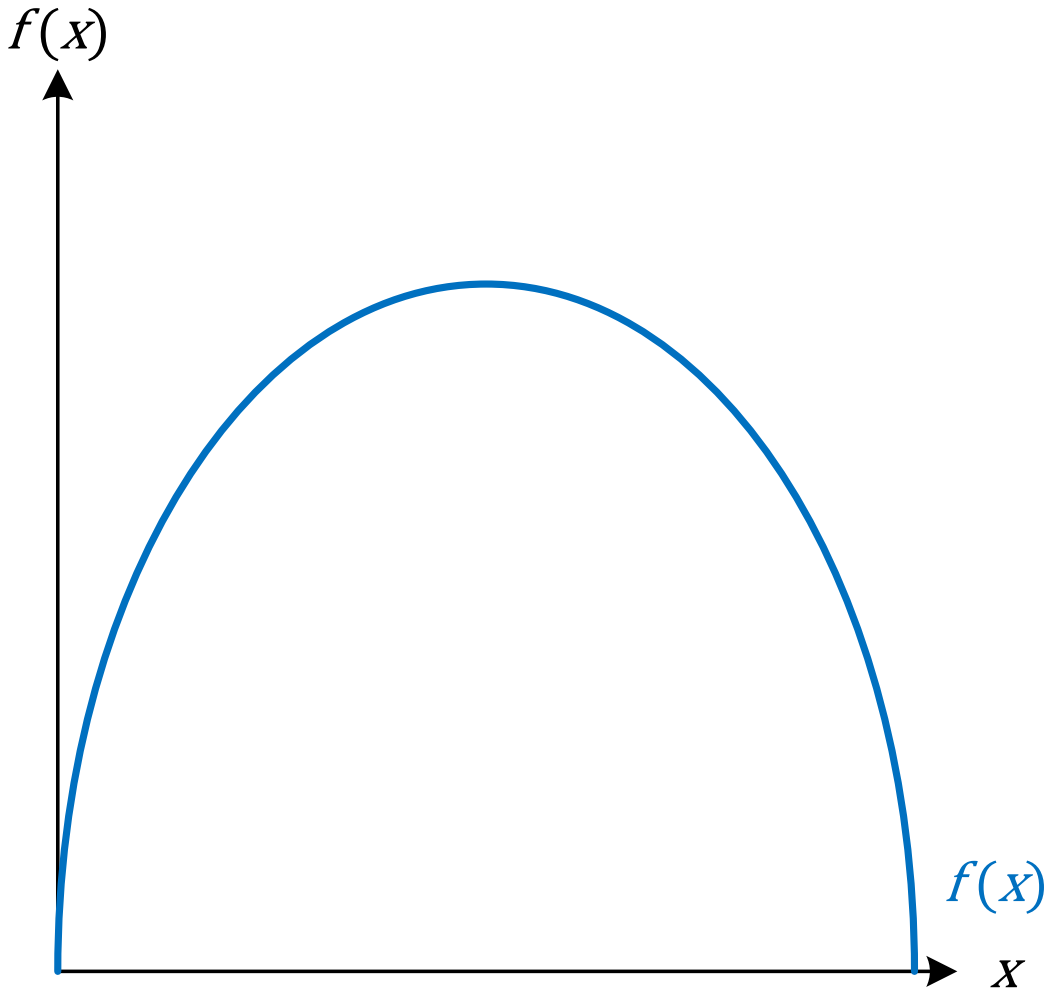
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{(x+h) - x} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

- Notice that this formula is essentially just the slope formula.
  - The numerator represents $\Delta f(x)$ and the denominator represents Δx when x is increased by h.
  - All we are doing to calculate the derivative is making the distance between x and h arbitrarily small.
    - This removes those precision errors with several calculated variables that we saw before.

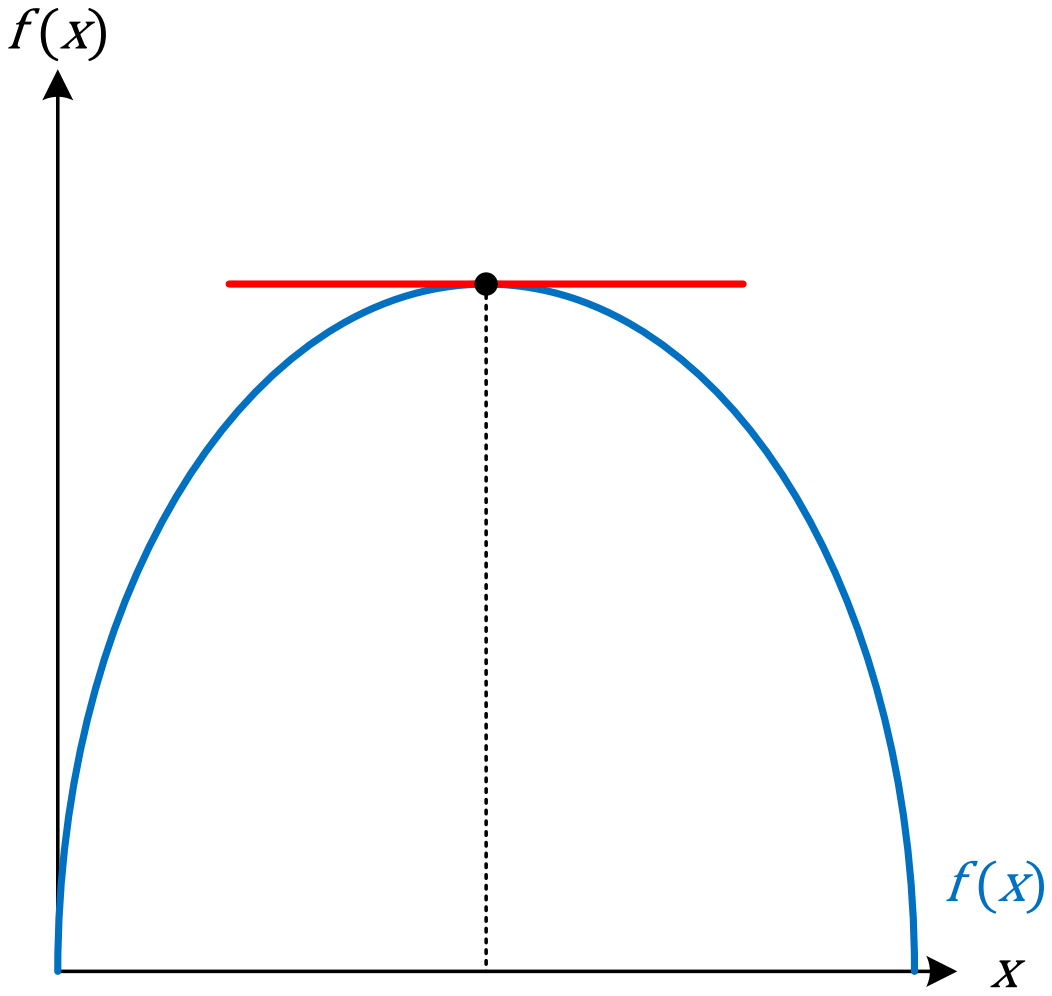
Using Derivatives

- We use derivatives primarily in two ways in economics.
  - **Conversion to marginals**: We can convert any variable into its marginal by simply taking the derivative of its base function.
    - Examples: Marginal revenue, marginal cost, marginal utility, etc.
  - **Maximization or minimization**: By setting a derivative equal to zero, we can determine a maximum or minimum of a function.
    - This is due to the fact that the rate of change of a function is zero at a maximum or minimum.

Using Derivatives



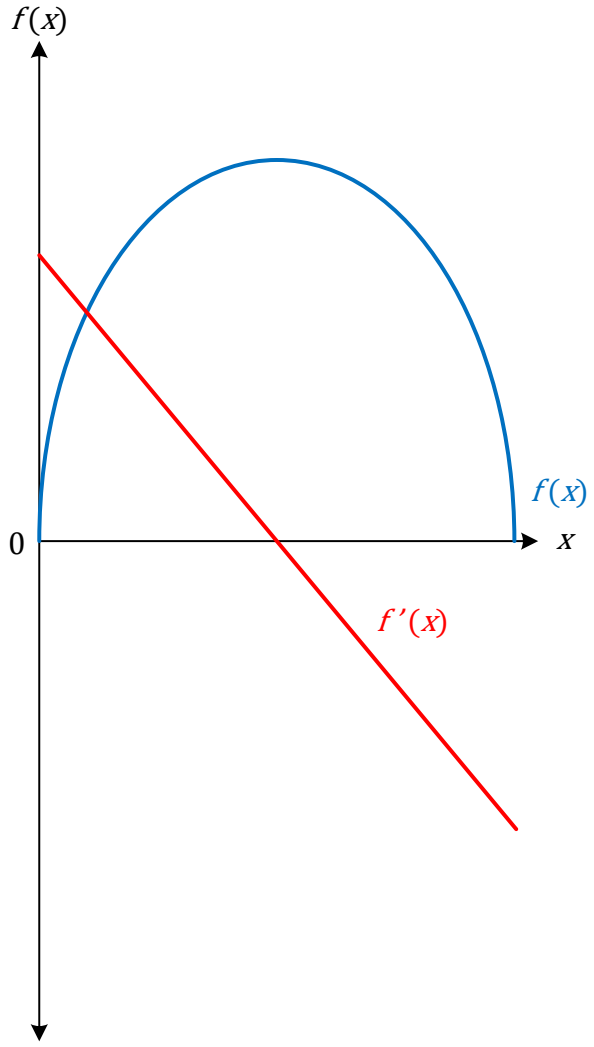
Using Derivatives



Using Derivatives

- An important thing to remember about setting a single-variable derivative equal to zero is that it does not specify whether it is a maximum or a minimum; just that it is one of the two.
  - In order to determine whether we have a maximum or a minimum, we need to take a look at the second derivative of the function, $f''(x)$.
    - This is just the derivative of the derivative; we use the exact same steps as before.
    - The sign of the second derivative tells us whether we have a maximum or minimum.
      - A positive second derivative corresponds with a minimum, while a negative second derivate corresponds with a maximum.

Using Derivatives



Using Derivatives