EconS 305 - Intermediate Microeconomics without Calculus
Assignment 4 Homework Solutions

Assignment 4-1

1. Consider the following production function

$$q = K^{0.7}L^{0.3}$$

   (a) What is the marginal rate of technical substitution?

   To get the marginal rate of technical substitution, we need to apply the power rule to the production function for both capital and labor,

$$\begin{aligned} MP_K &= 0.7K^{-0.3}L^{0.3} \\ MP_L &= 0.3K^{0.7}L^{-0.7} \end{aligned}$$

   and then we can apply our formula to obtain the marginal rate of technical substitution,

$$MRTS = -\frac{MP_L}{MP_K} = -\frac{0.3K^{0.7}L^{-0.7}}{0.7K^{-0.3}L^{0.3}} = -\frac{3K}{7L}$$

   (b) I mentioned in the slides that this function has constant returns to scale. Use the technique I presented on slide 34 to prove this. Here are a few math tricks with exponents and parenthesis that might help:

$$\begin{aligned} (ab)^c &= a^c b^c \\ a^c b^c a^d e^d &= a^{c+d} b^c e^d \end{aligned}$$

   To prove constant returns to scale, we need to multiply both K and L by some common multiple, which we will call t

$$(tK)^{0.7}(tL)^{0.3}$$

   using the first math trick, we can expand the exponents

$$t^{0.7}K^{0.7}t^{0.3}L^{0.3}$$

   and using the second math trick, we can combine some terms

$$t^{0.7+0.3}K^{0.7}L^{0.3}$$

   from here, we can substitute our production function, $q = K^{0.7}L^{0.3}$, which ends our proof

$$tq$$

   since it implies that multiplying all the inputs by a common multiple gives us an output multiplied by the same multiple.

Assignment 4-2

1. A golf ball manufacturer faces a fixed cost of $F = 20$ for operating and a variable cost of $VC = 2q^2$ where q is the quantity of golf balls it produces

   (a) Derive the total cost function

   We just add the fixed cost to the variable cost to obtain the total cost function,

   $$TC = F + VC = 20 + 2q^2$$

   (b) Derive the marginal and average cost functions.

   For the marginal cost function, we apply the power rule to the total cost with respect to q,

   $$MC = 4q$$

   and for the average cost function, we divide the total cost function by q,

   $$AC = \frac{20}{2} + 2q$$

   (c) Are these short run or long run cost functions?

   Since there are fixed costs, these are short run cost functions.

Assignment 4-3

1. Consider a firm with the following production function

   $$q = K^{\frac{2}{3}} L^{\frac{1}{3}}$$

   (a) Let $w = 1$, $r = 4$ and $q = 10$. What are the optimal level of capital and labor?

   First, we want to find our marginal rate of technical substitution,

   $$MRTS = -\frac{MP_L}{MP_K} = -\frac{\frac{1}{3}K^{\frac{2}{3}}L^{-\frac{2}{3}}}{\frac{2}{3}K^{-\frac{1}{3}}L^{\frac{1}{3}}} = -\frac{K}{2L}$$

   and we can set this equal to the negative ratio of prices

   $$-\frac{K}{2L} = -\frac{w}{r} = -\frac{1}{4}$$

rearranging,

$$2K = L$$

Next, we substitute this back into our production function,

$$\underbrace{10}_{q} = K^{\frac{2}{3}} \left(\underbrace{2K}_{L} \right)^{\frac{1}{3}} = 1.26K$$

and solving for K,

$$K^* = \frac{10}{1.26} = 7.94$$

We can plug this back in to our above equation to obtain K

$$L^* = 2K^* = 15.88$$

(b) What is the minimum total cost to achieve the production level of $q = 10$?

To get our total cost, we just multiply the output levels by their prices and add them together

$$\bar{C} = wL + rK = 1(15.87) + 4(7.94) = 47.62$$

Assignment 4-4

1. Consider a firm with total cost function

$$TC = 50 + 10q + 2q^2$$

(a) Derive the average, average variable, and marginal cost functions

To get the average cost function, we divide the total cost function by q,

$$AC = \frac{50}{q} + 10 + 2q$$

To get the average variable cost function, we divide only the variable costs (Terms next to a q) by q,

$$AVC = 10 + 2q$$

and to ge tthe marginal cost function, we apply the power rule to the total cost function with respect to q,

$$MC = 10 + 4q$$

(b) As a function of price, what is the optimal output level, q?

In a perfectly competitive market, $p = MC$. Setting them equal to each other

$$p = 10 + 4q$$

and solving for q yields our solution

$$q = \frac{1}{4}p - 2.5$$

(c) Let $p = 50$. What is the equilibrium output level, q? How much does the firm make in profits (or loss)?

Our equilibrium output level is

$$q^* = \frac{1}{4}(5) - 2.5 = 10$$

which gives total revenue of

$$TR = p^* q^* = 50(10) = 500$$

and total costs of

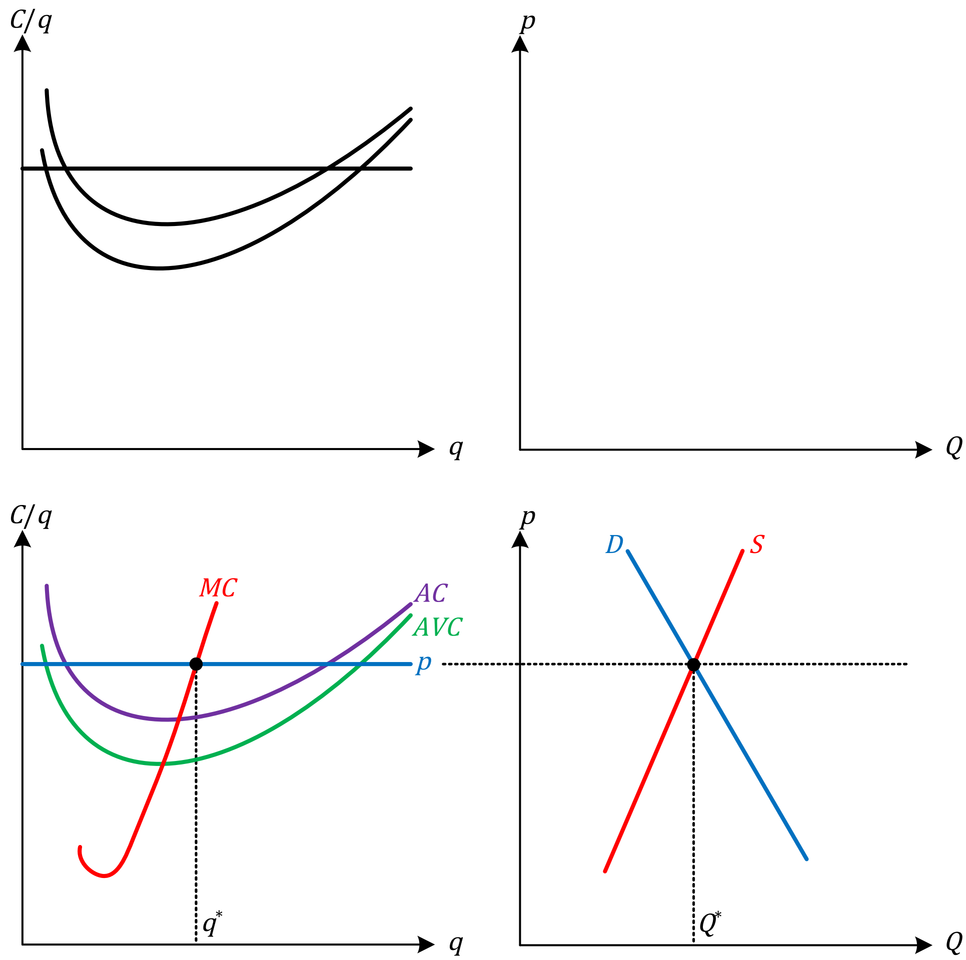
$$TC = 50 + 10(10) + 2(10)^2 = 350$$

yielding profits of

$$\pi = TR - TC = 500 - 350 = 150$$

Assignment 4-5

1. Consider the figure below. I have drawn an average cost and average variable cost curve for you, as well as a price level. But I seem to have forgotten my labels.



a) Label the average cost curve, average variable cost curve, and price level.

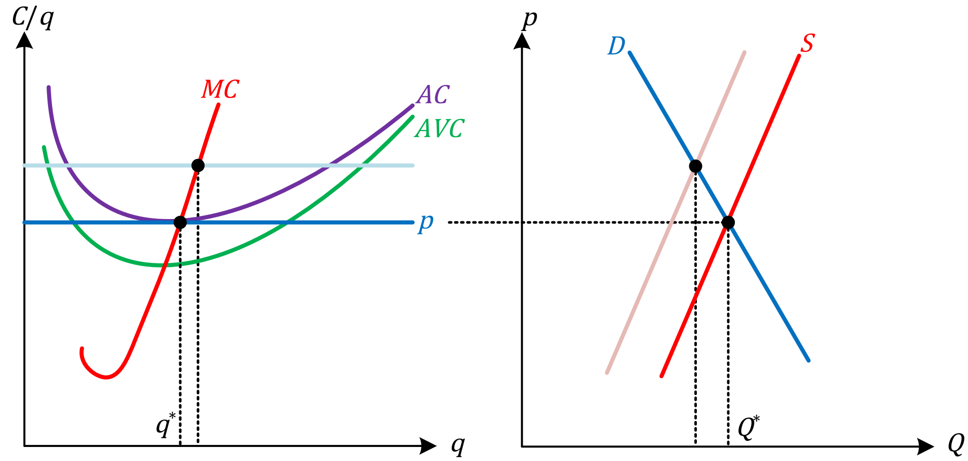
b) Draw the marginal cost curve on top of the figure and label it.

c) Draw the corresponding market supply and market demand curves and label them. (*Hint*: the steepness of the curves does not matter, only the intersection point).

d) Draw the short term profit level of the firm and label it.

e) Depict the short run equilibrium on the figure. Label both the equilibrium firm output level and the market quantity.

f) Depict what will happen in the market in the long run. Draw any extra appropriate curves and label them. Explain.



Parts (a)-(e) are depicted at top, with part (f) depicted in this section. In the long run, since the price is higher than the average cost, economic profits will cause firms to enter the market. This will shift the supply curve to the right and lower the price level until it is equal to the minimum of the average cost function.